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2022 Edition

GRE
Study Guide

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GRE® Study Guide

LESSONS, STRATEGIES, AND DIAGNOSTIC TESTS

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STUDY GUIDE

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How to Use This Book

Welcome to the 2022 edition of Varsity Tutors' *GRE Study Guide: Lessons, Strategies, and Diagnostic Tests*. We hope that you'll find the lessons contained herein useful for improving your scores on the Revised General GRE, which can help you gain admission to the postgraduate program of your choice.

While you will find some advice for test-taking in this book, its primary focus is skills-based. In other words, if you can learn how to do the math or how to read passages effectively, just to name two specific examples, you'll do better on the test. Being able to apply your skills in a timely fashion is the main consideration in approaching the GRE, but the skills you learn and use are broadly applicable, and will help you succeed even after you're done taking your exam. What that means for you is that we're not trying to sell you on a proprietary "method" or some made-up approach. This book is designed to teach you the skills you don't know and remind you of the skills you have, but may have forgotten.

This is an e-book, and the version of this book you are reading is a Kindle textbook, which is a feature-rich format. This book's content may be available in other digital formats, like as a PDF or iBook, so be sure to check for availability on your preferred device.

In this book, you will find things that are familiar to you from other books, both of the physical and the electronic variety, but you may come across some features you haven't seen before. We're striving to push the electronic format in ways that are useful to you, our reader, while maintaining a recognizable and clean layout.

The first thing you might notice is that this book is heavily linked to itself. If you browse the Table of Contents, you'll see that its entries serve as active links that can get you to other parts of this book quickly and accurately. These links are not always perfectly obvious, so you should always try hovering your mouse over anything that might interest you, in order to see if you can click on it.

We have formatted in-paragraph links as text highlighted in blue. Try clicking the following link:

This is an example of a link. This link takes you back to the Table of Contents.

It's a big book, and there's a lot to cover: explore and have fun. We certainly had fun writing this for you!

- The Learning Tools Team

Introduction

When preparing to take a test as extensive as the GRE, it's crucial that you know what you'll be up against. Mastering the content that appears on the exam isn't the entire battle: you also need to familiarize yourself with the test's idiosyncrasies, as these can have significant effects on the your approach. In "About the GRE," we briefly consider the major changes that were made to the GRE in August 2011 to help you orient yourself to the most up-to-date view of the test and plan how to make the most out of each of the specific tools at your disposal, such as the onscreen calculator and the ability to go back to previous questions in a section. We take an in-depth look at the Verbal Reasoning and Quantitative Reasoning sections in lessons focused on each. In "GRE Scoring," we go over the nuts and bolts of how your exam is graded. Familiarity with such details can prepare you to make informed decisions on test day about whether it's a good idea to guess on a question you're not sure about. We conclude with "GRE Testing Tips," in which we offer advice about how to manage your stress and anxiety leading up to and on test day. The information contained in the next few sections can easily seem extraneous compared to the content coverage that we later delve into; however, conscientiously reviewing the test's structure and rules can help you establish strategies early on, and the earlier you understand the specifics of the GRE, the easier it is to apply them to the rest of your review.

Section Outline

About the GRE

Verbal Reasoning

Quantitative Reasoning

GRE Scoring

GRE Testing Tips

About the GRE

   When you set out traveling with a destination in mind, it's important to have a map. The same can be said of the GRE, so in the next few lessons, we take an in-depth look at the terrain of each of the test's three sections, considering which types of problems you'll face when on exam day. While jumping directly into reviewing content might seem appealing, slowing your approach and taking time to understand the structure of each section can inform the way in which you spend your time studying. Identifying which particular types with which you struggle and those with which you do not can help you allot your study time to the areas of the test where review can potentially have the greatest payoff for your particular knowledge and skill set.

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Section Outline

Verbal Reasoning

Quantitative Reasoning

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Verbal Reasoning

The Verbal Reasoning score of your GRE is derived from your performance in two thirty-minute sections containing twenty questions each. Your particular exam may include three Verbal sections; if it does, one of those sections is unscored and used for research purposes so that ETS can measure the difficulty of new questions.

The twenty questions in each of the two Verbal Reasoning sections are composed of questions of three different types: Text Completion, Sentence Equivalence, and Reading Comprehension. About ten questions on each section are Reading Comprehension and the rest are Text Completion and Sentence Equivalence questions in about equal proportion to one another.

None of the questions on the GRE's Verbal Reasoning section test grammatical correctness, error identification, or the effects of proposed changes to a passage. While you may have seen such questions on other standardized tests' language-related sections (e.g. the ACT English section and the SAT Writing section), the GRE does not test any of these editorial skills. Its questions can instead be divided into those that test your vocabulary and ability to mine for context clues (Text Completion and Sentence Equivalence questions), and those that test your ability to comprehend short but complex reading passages (Reading Comprehension questions).

Note also that analogies are not present on the GRE Verbal section. While analogies (e.g. "blue is to color as popcorn is to _____" (food)) used to appear on the previous version of the GRE, the exam was revised in August 2011 and this question type was omitted from the current version. Make sure that you don't accidentally study and expect this question type if you're using any test prep materials published before 2011 that don't refer to the Revised version of the exam.

Text Completion questions test your ability to use the context of a selection to reverse-engineer the word that makes the most sense in a blank. These questions provide you with a sentence or short passage, up to five sentences long, that contains between one and three blanks. Each blank is associated with a list of five answer choices, each a word or short phrase. Your job is to select the word or words that best complete(s) the blank(s), namely those that create a comprehensive text that is logical and unified in terms of content, tone, and style. It's worth noting that each blank in a multiple-blank Text Completion question amounts to a distinct decision you have to make: no matter what you choose for the first blank, it won't affect the second. Additionally, you need to select the correct response for each blank to earn credit for a given question; partial credit is not given, even if you get one of two or two of three blanks correct. You can expect to encounter one-, two-, and three-blank Sentence Completion, though it's likely that you'll see more one-blank Sentence Completions than three-blank Sentence Completions as the difficulty of such questions increases the more blanks that are involved. We discuss strategies applicable to this question type in our Single-Blank Sentence Completion lesson and Multiple-Blank Sentence Completion lesson.

Sample Text Completion Question

Even though the plot was heralded by many critics as _____, it was actually _____ to last summer's box office hit.

Blank (i)
singular
exemplary
plebian

Blank (ii)
unwitting
analagous
extraneous

Correct Answers: "singular," "analagous"

Sentence Equivalence questions also test your ability to pick up on context clues and determine the best word for a blank in a sentence, but they also test your ability to compare the meanings of various terms to select synonymous or near-synonymous words. When you encounter a sentence equivalence question, you're given one prompt sentence or short passage that contains a single blank, and a list of not five, but six, words. From these six terms, you need to select the two that, when substituted into the blank in the sentence, create sentences that are similar in meaning. So, the aim here is not just to pick out words that create logical and coherent sentences—all of the answer choices might accomplish that. Your goal is more specific: identifying the words that not only make logical sentences, but also make logical sentences that match in meaning. This second level of consideration can make these questions seem counterintuitive and difficult, so, if you're apprehensive about these questions, we recommend checking out our two lessons on strategies specific to *Sentence Equivalence* questions: *Deriving Meaning from Context* and *Equivalent Vocabulary*.

Sample Sentence Equivalence Question

He started the job enthusiastically, but his _____ waned the longer he worked there.

- ☐ concentration
- ☐ zeal
- ☐ skill
- ☐ ambivalence
- ☐ fervor
- ☐ abilities

Correct Answers: "zeal," "fervor"

Reading Comprehension questions form the rest of the GRE's Verbal sections. They make up a significant portion of its questions—about half. On your entire exam, you can expect to encounter between fifteen and a dozen short passages drawn from a variety of nonfictional texts. Some of the passages you encounter may be notably academic in tone and subject material, whereas others might not. The vast majority of GRE passages span a single paragraph; between both Verbal Reasoning sections, a maximum of two of them will span several paragraphs. The test requires you to answer between one and six questions about each of these passages. These questions test a panoply of concepts, including the following:

- Main Idea, Meaning, and Purpose

- Strength of Support and Evidentiary Choices

- Word Usage in Context

- Summary and Paraphrase

- Text Structure

- Tone and Style

Reading Comprehension questions come in three formats: Single-Answer, Multiple-Answer, and Select-in-Passage. Single-Answer questions ask you to pick out one correct answer from four choices. Multiple-Answer questions present three choices and require you to pick out any that are correct. It's possible that all three might be correct, two might be correct and one might be incorrect, or that only one is correct. You need to pick out the exact combination of correct answers to receive credit for the question. As with multiple-blank Text Completion questions, the test doesn't give any partial credit for mostly-correct answer combinations. Select-in-Passage questions don't list answer choices with the question; instead, they give you a question stem describing a specific sentence's function in the passage and ask you to identify it by clicking on it, which highlights it as your answer choice.

With its wide variety of different question types and formats, the Verbal Reasoning section can be intimidating. By breaking it down into manageable pieces and reviewing each of its component parts one at a time, you can carefully compile a specific skill set specific to its particular challenges and be confident in your knowledge when you sit for your exam.

Quantitative Reasoning

The GRE tests your mathematical acumen in two Quantitative Reasoning sections. Each of these sections contains twenty questions, and you're given thirty-five minutes in each section to answer them. On test day, you may take a third Quantitative Reasoning section, but if you do, one of those is an unscored section for research purposes for ETS.

We can analyze the questions on the Quantitative Reasoning section in two ways: their format and their content. No matter what content they test or what form they take, some of the GRE's Quantitative Reasoning questions are presented as story problems, while others are presented as abstract algebraic equations. In addition, you have access to an on-screen calculator for each question on the exam, and you will be provided with scratch paper on which to diagram problems and perform calculations by hand. (We go over the details of the on-screen calculator in our Calculator Use lesson.) Some Quantitative Reasoning questions contain visual graphics such as graphs, charts, tables, or geometric diagrams, and several questions may relate to a single question stem. The screen will be divided in half for such problems, with the common question stem showed at the top and each problem shown below it.

You'll encounter three distinct formats of question on Quantitative Reasoning sections: Single-Answer multiple-choice questions, Multiple-Answer multiple-choice questions, and Numeric Entry questions. Single-Answer multiple-choice questions form the majority of the exam and present you with a question stem and four answer choices, asking you to select the correct one. Multiple-Answer multiple-choice questions present five answer choices and require you to select the correct ones, of which there may be multiple. No partial credit is given if you select some, but not all, of the correct answers, or if you select an incorrect answer choice along with the correct ones. Numeric Entry questions don't present you with any answer options: you're given a blank text box in which to enter your response to the question stem.

Each of these question formats can be used to test any of the areas of mathematical knowledge covered on the Quantitative Reasoning section: arithmetic, algebra, geometry, and data analysis. Arithmetic questions focus on fundamental mathematical operations, definitions of different types of numbers, and elementary numeric representations such as fractions, decimals, and percents. Algebra questions test your understanding of linear and quadratic algebraic equations and inequalities we well as graphical representations of them. Geometry questions ask you to calculate various measurements of common shapes, and Data Analysis questions test your skills in calculating probability and making sense of graphs, tables, and charts.

Considering the array of mathematical concepts tested on the GRE and the different formats in which the exam presents them, it can be easy to feel overwhelmed by the prospect of studying each detail, especially if you're not confident in your mathematics abilities. Taking your review one step at a time and not attempting to focus on form and content simultaneously can help you partition your attention efficiently and make the most of your studying time.

GRE Scoring

Understanding exactly how the GRE is scored can not only prepare you for the exam by affecting your test-taking strategies, but it can also save a great deal of stress and hassle after you exit the testing center. This lesson will offer a brief but detailed consideration of how the various parts of the Revised GRE are scored.

If you take one thing away from this lesson, it is this:

> There is no guessing penalty on the GRE, so you should submit an answer for each question even if you're not 100% sure it's the correct one.

Revised Score Scales

Each GRE score report consists of three scores: a Verbal Reasoning score based on your performance on the two Verbal sections, a Quantitative Reasoning score based on your performance on the two Quantitative sections, and an Analytical Writing score that is a composite of the scores you receive for your Analyze an Issue and Analyze an Argument tasks.

You may have heard people discuss GRE scores in terms of scores out of 1600. Before the test was revised in August 2011, test-takers received Verbal and Quantitative scores between 200 and 800, measured in ten-point units. The updated version of the exam uses a new scale: test-takers' scores on the Verbal and Quantitative sections are reported as numeric values between 130 and 170, measured in one-point units. The scale of the Analytical Writing score did not change when the test was revised; Analytical Writing scores are between 0 and 6 and are reported using half-point units.

Adaptive Testing and Scoring Processes

In order to understand how the test-makers arrive at your scores, we need to provide some context about how the test is given. Before its significant August 2011 revision, the test was adaptive at the question level, but this is no longer the case. The Verbal Reasoning and Quantitative Reasoning sections of the GRE are now adaptive at the section-level. This means that how well you do on your first section of either type determines how relatively difficult the second section of that type will be. Thus, if you do excellently on your first Verbal Reasoning section, you'll be presented with a second Verbal Reasoning section that will be more relatively difficult than the one presented to someone who did poorly.

That's not fair, you might be thinking: someone might do better than someone else just because they received easier questions on the second portion of a section! This is not the case. Neither section

is weighted more than the other, and each one produces a raw score: the number of questions you answered correctly. (If you don't provide any response for a section, you'll receive an "NS" ("No Score") score on it.) No partial credit is given for mostly correct responses on questions that involve multiple parts. Your raw score is then converted to a scaled score in order to eliminate variation in test edition and test-section difficulty. Thus, it doesn't end up mattering to your score whether you take a test with relatively easy or relatively hard questions—the GRE is presented this way so that in two sections it can narrow-in on your score as accurately as possible. If you're answering all of the easy questions correctly, there's little sense in asking you a bunch more easy questions, but asking you more difficult ones can help differentiate the top scores from the merely good ones.

Since the Analytical Writing section consists of two written responses instead of answer choices, necessarily it is graded differently from the Verbal Reasoning and Quantitative Reasoning sections. After your written responses pass the anti-plagiarism measures ETS has in place to ensure essays are your original compositions, they're each scored by a professional reader and by a computer program designed by ETS; if the scores are close, they're averaged. If they're not close, a second reader score is given and the two reader scores are averaged. Keep in mind that your Analytical Writing section tasks are scored based on how well you perform the specific task at hand. There are two distinct tasks, so make sure to review them each and understand specifically what is being asked of you on test day. Writing an essay with good flow and proper grammar isn't enough: you need to complete the specific assignment you're given.

Score Report Strategies On and After Test Day

The scores you receive after taking the GRE are valid for five years; however, it's worth noting that ETS defines a year as a "testing year" that spans from July 1 to June 30th. You get to send four free score reports to schools on the day of your exam. If you choose to do this, pick out the schools in advance so you don't get flustered before you even start the exam by having to choose schools without prior consideration. You have two options at this point: you can send only the scores associated with the test you will take that day (the "Most Recent" option), or you can send all of the GRE scores you've received in the past five years from ETS (the "All" option). Of course, you don't have to agree to send the scores of an exam you have yet to take to the schools of your choice; you can decline this free offer. Depending on your confidence levels, you may want to do so, if only to omit another potential source of test stress.

After you take your exam and view your scores, you can send score reports to schools for $27 per school. Previously, when you sent your GRE scores to a school, you had to send all of them. If you took the GRE, didn't do as well as you wanted to, retested, and got a better score, schools would still be able to see your initial scores that didn't measure up to your standards. ETS has since initiated a policy that lets you have more control over which scores you send to schools, for a small fee. Three options are available: you can pick and choose the scores to send to a school, send just the scores you earned the last time you took the GRE, or send all of your GRE scores earned in the last five years.

The caveat to be aware of in working with these options is that it functions at the level of tests: you cannot pick and choose section scores. For example, if you took the GRE once and got a great Verbal Reasoning score but thought you could do better in Quantitative Reasoning, you might retest. If you got a better Quantitative Reasoning score on your second try but your Verbal Reasoning score dropped, you wouldn't be allowed to send the Verbal Reasoning score from the first exam and the Quantitative Reasoning score from the second. You'd need to decide whether to send the first exam's scores, the second's, or both.

The way in which the GRE is scored can be complex, but keep in mind that it is done this way in order to be as accurate as possible in providing a snapshot of your language and math skills. Familiarizing yourself with the scoring structure can help you decide on reasonable goals for your testing session and to better understand how the scores you receive rank you against other test-takers.

GRE Testing Tips

While Studying

No Noise: This point probably seems commonsensical, but it is important to mention. Do not have any noise in the background when you are working. In almost all cases, we process information best when we do not multitask or experience multiple sources of stimulation. Your attention should be focused only on GRE preparation work and nothing else. You only have a limited time to work. Make the most of every minute!

Start Early: Do not wait until a month before the exam to begin your study program. The best preparation will take you between three and four months, depending on how much time you can devote per week. It is possible to prepare in less time, especially if you have an extensive vocabulary and have used mathematics regularly in your undergraduate major; however, even in the best circumstances, you need to familiarize yourself with the test's idiosyncrasies, so do not underestimate the amount of preparation time needed!

Schedule Well: Hopefully you will only need to take the GRE once; however, bad test days happen, and you should be ready for that possibility. Make sure that you schedule your time so that you can retake the exam. You can retake the exam five times per twelve months, though you need to wait at least twenty-one days between reexaminations. Reduce the pressure you put on yourself by assuring that you can retake the exam in time for your application.

Make a Weekly Schedule: Ideally, you should do some work every day—at least review vocabulary! In any case, you need to decide which days of the week work for your preparation time. Stick to your schedule. It is very easy to convince yourself that you can take a day off here or there. It is best that you avoid this. The more disciplined that you are with your study time, the more likely you will put in all the time that you need—and the less likely it is that you will need to retake the exam!

Order Your Studies: Be intelligent about how you study the material—especially the mathematics. Start with basic concepts and build up to more complex ones. Use online resources and get a tutor if necessary.

Study Vocabulary Every Day: Vocabulary is critical for success on the Verbal section. Unless you are an experienced linguist and literary critic, it is unlikely that your vocabulary will be adequate for the exam. Prepare with phone apps, flash cards, and online exercises. Do this daily! Vocabulary takes time to "sink in."

Don't Forget About Verbal Practice: It is very easy for students to focus on math but forget to work on Verbal practice problems. Do not succumb to this temptation. Be sure to schedule Verbal practice problems alongside your Quantitative ones.

Don't Forget About Writing Practice: Likewise, it is very easy to forget about the Analytical Writing sections. Don't! You need to hone your ability to respond quickly and thoughtfully to a prompt. Factor this practice into your schedule early enough so that you have time to augment your skills at argumentation and logical self-expression.

Use a Comprehensive Study Guide: Especially in preparation for the Quantitative section, make sure that you work through questions of every type. Do not limit yourself to what is comfortable. Instead, make sure you can answer a variety of math questions, applying your skills readily to new kinds of questions.

Get General Evaluations: When preparing for the GRE, it is tempting to stay with one comfortable question type. When working on probability questions, you can feel like a master! There will, however, be many other types of questions on exam day. You need to make sure you can switch your mind from one question type to another; therefore, every so often, you should work on a mixed set of questions just to guarantee that you have a mastery of all the topics on which you have worked.

Work on Timing: In addition to content mastery, learn how to quicken your pace. You can do this through smart elimination of wrong answers. Also, learn tricks for math shortcuts. Use online resources as well as a tutor for these tools.

Before the Exam (Morning of Exam)

Know Your Center's Location: Don't wait until test day to figure out where you need to go. Have your route (or routes) planned well in advance. You don't need the stress of being lost before getting to the exam!

Sleep Well: Make sure you are well-rested. We are often more sleep-deprived than we think, so plan to rest well for several days in advance.

Stay Hydrated: Stay hydrated for several days before the exam. This will help your mental acuity (and your health in general)!

Eat A Balanced Breakfast, But Not Too Much: Don't just have a bunch of sugar for breakfast. Have a mixture of long-lasting carbohydrates and proteins to help you make it through the long exam. Of course, don't eat so much that you feel ill!

Leave Early: Don't risk the stress of running late. Leave adequate time before the exam. You never know what traffic will be like. Don't let something so small ruin your test day!

Do Not Worry: Don't add stress and difficulty by worrying about the examination. This will only distract you and set you up to make unwanted mistakes! You have been prepping for this. Be confident!

During the Exam

Return to a Question Later When Necessary: Do not be afraid to mark a question for your return at the end of the exam section. Always guess—just in case you cannot get back to the question. Be sure to make it obvious on your work paper exactly where this question is. You will want to return to your notes when you come back to the question at the end of the section. Of course, do not skip too many questions. You will likely only have several minutes at the end of the section.

On Writing—Just Choose: On the issue essay, choose the position that feels most comfortable. You really can take either side of the question. You are more likely to come up with examples based on what you know well. Of course, do brainstorm—just in case you are fooling yourself!

Guess Intelligently: Use elimination to your advantage. It is much better to guess for the answer out of three possible choices than out of five.

Don't Guess About Difficulty: Do not guess how you are doing while you are taking the test. This will only add stress and likely make you do irrational things, like skipping questions that you could actually answer.

After the Exam

Reward Yourself: No matter how you do, reward yourself. Plan to be able to take off the rest of the test day if at all possible. If you cannot do this, take time in the next couple of days. You have worked hard and should allow yourself to decompress. Even if you do not do as well as you hoped you would, this reward time will help to keep your spirits high. You will do great the next time!

Don't Worry About Retesting Immediately: Even if you need to retake the exam, do not worry about that immediately. Focus on the positives—you took the exam; you know where you stand; you are better prepared now than you were when you started this whole process! Keep your spirits high. This will help you to prepare for the next go at the exam.

Assess What Went Well and What Needs Improvement: Do not focus too much on the negatives right after the exam; however, you do need to figure out what you did well and what you need to improve. You will best know and recall this immediately after the exam. Take notes as soon as you can so that you can focus any future preparation work on trouble areas. Still, try to stay positive; (Note that this is an important and repeated point!).

Verbal Reasoning

Even if you've breezed through the English, Reading, and Writing sections of other standardized tests before, the GRE Verbal section tests different skills in different ways. Many students who have met with success before on other similarly-themed test sections find that the GRE's Verbal Reasoning section takes a great deal of work to prepare for. For many, it is the test's most difficult section, regardless of their performance on previous examinations.

In this section, we'll work through each topic and question format that can appear on the Verbal Reasoning section. We'll begin with a review of two broad skills that you'll need throughout your entire review process: close reading and vocabulary-memorization tactics. After practicing these crucial skills, we'll consider Reading Comprehension content, first through a content-focused perspective detailing topics about which questions might ask, then from a format-focused view considering strategies you can use when answering Single-Answer and Multiple-Answer Multiple Choice questions, as well as Text Selection problems.

Then, we turn our attention to the test's two vocabulary-heavy question formats: Text Completion and Sentence Equivalence. Again, we split our review into content- and format-focused considerations. In "Context Clues and Agreement," we examine different common patterns of context clues. We devote a "Question Strategies" section to each question type so that you can practice tactics specific to each format's demands.

The GRE Verbal Reasoning section may be tough, but part of its difficulty stems from the wide range of ways in which problem content can be combined with different challenging question formats. We've broken this section into detailed subsections and lessons in order to make it as easy as possible for you to study in detail while focusing on whichever aspects of the section you find most difficult. After reviewing all of the lessons and practice content in this section, the Verbal Reasoning section shouldn't seem so intimidating!

Section Outline

GRE Verbal Review

Reading Comprehension

Question Strategies: Reading Comprehension

Text Completion and Sentence Equivalence

Question Strategies: Text Completion

Question Strategies: Sentence Equivalence

GRE Verbal Review

The questions found on the Verbal Reasoning section can be broken into two general categories: Text Completion and Sentence Equivalence questions test the breadth of your vocabulary, whereas Reading Comprehension questions test your ability to understand prose passages. While the vocabulary-focused question types function at the sentence level and the Reading Comprehension questions focus on longer passages, both question formats require you to be able to read material closely for context clues. Before we focus on the specific concepts that these questions might test and how they might test them, we begin the section with a review of the general but core skills that these questions test. Our "Close Reading" lesson functions as a primer on how to read a sentence or a passage for all of the information it contains; such a skill is invaluable given how focused the test is on your ability to spot a term's subtle meanings which are often influenced or determined by surrounding material—its context. Our "GRE Vocabulary" lesson offers suggestions on how to tackle the gargantuan task of learning hundreds of complex new terms in a relatively short period of time. In it, we direct you to our GRE Word List, a resource available at the back of this book. In it, we've compiled many of the difficult terms with which you'll want to make sure you're familiar when you sit for your exam. It may seem like a waste of time to review these general skills before diving into the specifics covered in the rest of this section, but we can assure you it is not: these two lessons help ground your review in the skills that you'll need in many if not all of the lessons that follow, and help you consciously consider how you'll design your study schedule to meet the demands of the exam. These two lessons form the crucial first steps to mastering GRE Verbal content, so don't skip them!

Section Outline

GRE Vocabulary

Close Reading

GRE Vocabulary

Facing down hundreds of GRE terms without a concrete plan is a recipe for disaster. In this lesson, we've collected a number of strategies that you can use when creating your own personalized plan for learning GRE Vocabulary. Need some words to start learning? Check out our GRE Word List found at the end of this lesson.

Group Words Together Categorically . . .

When trying to memorize a large number of words, group terms together that relate to one another in some general way, e.g. "words about communication" or "words that describe appearance." You can make these categories as specific or as a general as is helpful to you, but merely categorizing words like this can help you begin to sketch valuable interconnections that will be extremely helpful when you go about memorizing each one. After all, if you can remember that two words are very close in meaning, you effectively have only one definition to remember, not two! For words that are a bit more distant, if you can consider the group into which you've assigned a term, it can be helpful for narrowing your focus when selecting terms for blanks based on the context of a sentence. For example, if you know you need a word about communication but you recognize one of the answer choices as being about appearance, it doesn't matter if you remember more about that terms' definition: you know it's not the correct word for the context.

. . . and Identify Synonyms

As you group words into broad categories, note any that have definitions close enough that the terms qualify as synonyms, even distant ones. Sentence Equivalence questions require you to select two terms that when used to complete a sentence produce sentences with similar meanings. In short, you'll be asked to identify synonyms. Knowing this ahead of time can help you structure your review.

Learn to Differentiate Between Words That Sound Similar

The GRE isn't above trickery such as asking you to choose a term between terms that are nearly homonyms. For example, "tepid," "torpid," "turgid," "turbid," and "turpid" can form a very confusing continuum if you're not familiar with their specific meanings. Prepare for these sorts of questions so that if you do encounter one, it doesn't disrupt your composure. You can turn groups of close-sounding terms to your advantage by memorizing them as a list, thus using their close pronunciations as a memorization tool.

Pay Attention to Small Distinctions in Tone, Meaning, and Connotation

Pay good attention to small distinctions in connotation and meaning when a bunch of words mean essentially the same thing. Text Completion questions may present answer choices that differ only at the level of subtle connotations in order to test the depth of your knowledge. A surface-level understanding of what each term means won't be sufficient to get these questions right. Analyze words that qualify as synonyms with microscopic consideration of their meanings. Considering the etymologies (historical development of forms) of such terms can help bring some differentiating aspects to words that at first glance seem to state the exact same thing. If you're asked to differentiate between close terms, you'll be prepared to do so.

Learn Prefixes, Roots, and Suffixes, But Be Aware of False Cognates

For some reviewers, prefixes, roots, and suffixes provide the structure they need to make sense of the onslaught of terms they need to retain. Learn the meanings of common morphemes so that you better understand the words you do know and are more prepared to tackle any unfamiliar terms that use familiar parts. Be careful, though: certain words can act as false cognates. While they may appear to use a familiar root, their etymology actually does not involve it and their meaning has nothing to do with it. Check the definitions of words that have significant roots carefully so that you don't end up confusing yourself.

Use Mnemonics to Help with Mental Recall

Mnemonic devices are anagrams, acronyms, or other easy-to-remember mental reminders that can serve as scaffolding for other, more complex knowledge. Did you ever use the sentence "My very eager mother just served us nine pizzas" to remember the order of the planets (when Pluto was still included), or "PEMDAS" or "SOHCAHTOA" to recall order of operations or trigonometric functions, respectively? These are very common mnemonic devices, and while there aren't any pre-made ones specific to GRE vocabulary, you can make up your own to remember words that just won't stick. Remember: they don't have to be realistic or even make sense, as long as you remember them! Let's say we wanted to remember the difference between "tepid" and "torpid." Well, "tepid" starts with "te," like "tea," and you wouldn't want tepid (lukewarm) tea—you'd probably want hot tea. "Torpid," on the other hand, starts with "tor," like "torpedo." You wouldn't want to use a torpid (slow, lethargic) torpedo either! Single-sentence mnemonic devices like this can be very useful when memorizing GRE vocab, but be careful not to rely on them for more meaning than simple recall unless you designed them that way. For instance, part of the meaning of "torpid" is lethargic, which doesn't make sense when used to describe a torpedo, a non-living object. Just don't rely on mnemonics for more information than they contain, and they can be very useful tools!

Use New Words in Conversation and Writing to Cement Them

It's difficult to retain any information you've memorized but aren't actively using, so as you add new terms to your vocabulary, go out of your way to use them in conversation and in writing where appropriate. Such use can help you get the most out of the work you've done to memorize the words in the first place by helping them stick in your memory. This way, you avoid having to learn them multiple times to no avail. While using pre-made vocabulary flashcards can be helpful, you may find that writing the words out is even more useful at improving your retention. If you do this, put each word in a sentence to give it context.

Drill Vocabulary Cards: Repetition is Key

No matter whether you write them yourself or use some that someone else has put together, drilling vocabulary cards can help you incorporate the repetition necessary to remember hundreds of new words. The benefits of vocab cards are maximized when you drill them often, so consider carrying a 3x5 ring of cards with you as you go about your daily activities. Whenever you have downtime—on lunch breaks, on buses, or while waiting in lines—you can quickly study by learning new words, reviewing old ones, and giving extra consideration to those you find most difficult.

GRE Word List

Abeyance
Abrogate
Abstemious
Adumbrate
Ait
Alacrity
Alloy
Ambivalent
Ambrosial
Anathema
Apocryphal
Apogee
Apostate
Apotheosis
Apposite
Apprise
Approbation
Apropos
Arcane
Arch
Arrant
Artful
Artless
Ascetic
Asperity
Aspersion
Assiduous
Assignation
Astringent
Attenuate
Augury
August
Auspicious
Autochthonous
Autodidact
Aver
Axiom
Axiomatic

Baleful
Banausic
Beatific
Beatify
Beatitude
Bedizen
Belie
Bellicose
Benighted
Bilious
Bilk
Blandishment
Bombast
Boor
Brook
Bucolic
Bulwark
Burgeon
Byzantine
Cadge
Cajole
Calumny
Caparison
Capricious
Castigate
Catharsis
Celerity
Censorious
Chary
Chauvinist
Chicanery
Chimera
Choleric
Churlish
Cogent
Complacent
Complaisant
Complicit

Consequential
Consonant
Contemn
Contrite
Contumacious
Corrigible
Countenance
Cozen
Craven
Cynicism
Cynosure
Dais
Daunt
Debacle
Declamatory
Defenestrate
Deliquesce
Demur
Denigrate
Depredate
Depredations
Deracinate
Desuetude
Desultory
Detraction
Detumescence
Diaphanous
Diaspora
Diatribe
Diffident
Dilatory
Dilettante
Disabuse
Discomfit
Disingenuous
Disinterested
Dissolution
Distrait

Divagate
Doctrinaire
Ebullience
Éclat
Effrontery
Effulgence
Embargo
Emollient
Empyrean
Encomium
Enervate
Enormity
Entelechy
Enumerate
Equanimous
Equivocal
Errant
Esoteric
Essay
Etymology
Exacerbate
Excoriate
Exculpate
Exemplum
Exhort
Exigent
Exonerate
Expatiate
Expurgate
Extemporaneous
Extirpate
Facetious
Fallow
Fatuous
Fauna
Feckless
Felicitous
Fell

Festoon	Homily	Isthmus	Nonplussed
Firmament	Hubris	Itinerate	Nostrum
Flag	Iconoclasm	Jaundiced	Numinous
Flip	Idyll	Jejune	Obdurate
Flora	Ignominous	Jibe	Obloquy
Flout	Imbroglio	Jocose	Obsequious
Foment	Immutable	Jocund	Obstreperous
Forbearance	Impassive	Ken	Obtain
Forestall	Impecunious	Kinetic	Obtest
Forswear	Imperious	Labile	Obtuse
Forward	Implacable	Laconic	Obviate
Fracas	Import	Limn	Occlude
Fractious	Importunate	Limpid	Occult
Frission	Importune	List	Officious
Froward	Impugn	Magnanimity	Oneiric
Fulminate	Impunity	Malevolent	Onerous
Gainsay	Impute	Malinger	Opprobium
Gambol	Inchoate	Manifold	Ossified
Garner	Incorrigible	March	Ostensible
Garrulous	Inculcate	Maunder	Ostentatious
Gauche	Indolent	Maverick	Otiose
Germane	Ineluctable	Melifluous	Ouevre
Glib	Inerrant	Mendacity	Overweening
Gloam	Ingenue	Mendicant	Overwrought
Gossamer	Ingenuous	Meretricious	Paean
Grandiloquence	Inimical	Mettlesome	Pale
Gregarious	Inimitable	Militate	Palliate
Grouse	Iniquitous	Milk	Palpable
Guile	Innervate	Minatory	Panegyric
Guileless	Insipid	Mince	Panoply
Guy	Insouciant	Mitigate	Paramour
Hackneyed	Insular	Mollify	Pariah
Halcyon	Interdict	Mordant	Parsimonious
Hale	Interdiction	Moribund	Partisan
Harangue	Interstice	Mulct	Peccadillo
Harrow	Intractable	Multifarious	Peckish
Hegemony	Intransigent	Nacreous	Pecunious
Hermeneutic	Inundate	Nadir	Pedantic
Hermetic	Inure	Natty	Pedestrian
Heterodox	Inveigh	Neologism	Pellucid
Hirsute	Inveigle	Nexus	Penurious
Homiletics	Inveterate	Nice	Peregrinate

Peremptory	Proscribe	Riparian	Steep
Perfidious	Providential	Roil	Stentorian
Perfunctory	Prurient	Rubric	Stint
Peripatetic	Puerile	Salacious	Stoic
Pernicious	Pugilism	Salient	Stolid
Persiflage	Pugnacious	Salubrious	Striated
Perspicacious	Puissance	Salutary	Strut
Pertinacious	Pulchritude	Sanctimonious	Stupefy
Petrous	Punctilious	Sanction	Stygian
Petulant	Pundit	Sanguine	Stymie
Philistine	Pungent	Sate	Subfusc
Phlegmatic	Pusillanimous	Saturnine	Sublimate
Pied	Quaff	Scintilla	Subpoena
Pillory	Quail	Scintillating	Succor
Pique	Qualify	Scrivener	Supercilious
Placebo	Querulous	Scurvy	Supine
Plaintive	Quintessence	Sedulous	Supperate
Plangent	Quixotic	Seine	Supplant
Plastic	Quotidian	Seminal	Suppliant
Platitude	Raffish	Sententious	Supplicant
Pleonastic	Rail	Sere	Surfeit
Plumb	Ramifications	Shibboleth	Svengali
Polemical	Ramify	Simper	Sybarite
Portentous	Rarefy	Sinecure	Sycophant
Prate	Rash	Sinuous	Sycophantic
Prattle	Reactionary	Slake	Table
Precept	Recalcitrant	Sobriquet	Taciturn
Precipitate	Recant	Sodden	Tamp
Preempt	Recondite	Solder	Tandem
Prescience	Reconnoiter	Solicitous	Tautology
Prevaricate	Recreant	Somnolent	Tawdry
Probity	Recumbent	Sophistry	Tendentious
Proclivity	Redolent	Sophomoric	Tender
Procrustean	Redoubtable	Sordid	Timorous
Prodigious	Refractory	Specious	Tirade
Profligate	Refulgent	Spendthrift	Toady
Profuse	Regale	Splenetic	Torpid
Proleptic	Remonstrate	Spurious	Torpor
Prolix	Renege	Squalid	Torrid
Prone	Repine	Stanch	Tortuous
Propinquity	Repudiate	Static	Torturous
Propitiate	Rescind	Staunch	Tout

Tractable
Travesty
Trenchant
Truculent
Tryst
Tumescence
Tumid
Turbid
Turgid
Turpid
Tyro
Unfeigned
Unmitigated
Untenable
Untoward
Urbane
Vacuous
Vagaries
Vaunt
Venal
Verisimilar
Verisimilitude
Veritable
Villify
Virulent
Vitiate
Vituperate
Volatile
Voluble
Waft
Welter
Wend
Whimsical
Whinge
Winnow
Wry
Xenophobic
Zealous

Close Reading

Everyone taking the GRE has certainly been reading for many years. This familiarity with the art of understanding a passage is both an asset and a danger to the aspiring graduate student. You must be aware of the need to give a close reading of the passages presented in the GRE Verbal section. It is tempting to review the passage in a cursory manner, getting the "general details" and little else. Indeed, given your long familiarity with reading and deciphering textual selections, such a quick content review will be implicitly tempting—but do not immediately give in to the temptation to just skim over details! As you review, it is critically important that you hone the specific ability to read passages on unfamiliar content in a manner that is both rapid *and* close.

The reading passages in the GRE Verbal section require the most time of any question type in the section. Unlike the Sentence Equivalence and Sentence Completion questions, the reading passages will require you to process a good deal of text before even being able to attempt to answer the questions. When reading the passages, you will need to get the general idea of the passage, noting the overall paragraph structure, the order of the arguments, and the specific details of the most important points in the passage. This last point is most important. As you read over the passage for the first time, you cannot get bogged down trying to understand completely everything that is said. Given that many of these passages will be from fields in which you do not have expertise, such exhaustive understanding will take too long; however, you do need to understand the critical junctures in the passage's argument, as these will be central when the time comes for answering questions. Thus, be sure to note occasions of examples, evidence, and explicit argumentation. Your first read should not be ponderously slow, but it must be incisive enough to note these details.

When you are answering your questions for the passage, you will need to close read carefully. You must return to the passage for each and every question. This point cannot be emphasized enough: **If you cannot point directly to the place in the passage that answers the question, you are not ready to answer the question.** Your initial reading should enable you to return to the passage in this thoughtful manner. When you then return to the passage, you will need to ask two questions:

1. Is there anywhere else in the passage that helps me to answer this question?
2. Based on the information gathered, do I need to infer something in order to answer the question? If so, is it valid for me to make that inference?

The second question is very important. Answering it will help you to avoid pitfalls. Many questions try to make you infer more from the passage than is actually possible. A close reading of the relevant passage sections will help you see just how the writers may be trying to get you to infer more than is justified. Be aware of this kind of trickery. It will help you to eliminate numerous wrong answers.

Close reading is crucial when reading Reading Comprehension passages, but it's also a skill useful for answering other types of Verbal questions. When you can expertly look for clues in a passage, you may find it easier to find relevant hints in Sentence Completion and Sentence Equivalence question stems.

How to Close Read a Passage: A Play-by-Play

Here, we present an example of how to gather close details for each paragraph in a selection. Every reading selection is unique, as are the questions that will be asked for every passage. You should, however, be able to undertake this kind of two-pronged close reading of every passage with which you are presented. The following passage is adapted from *Beyond Good and Evil* (1886) by Friedrich Nietzsche (trans. Zimmern 1906).

A species originates, and a type becomes established and strong in the long struggle with essentially constant unfavorable conditions.[1] On the other hand, it is known by the experience of breeders that species which receive superabundant nourishment, and in general a surplus of protection and care, immediately tend in the most marked way to develop variations.[2]

Now look at an aristocratic commonwealth, say an ancient Greek polis, or Venice, as a voluntary or involuntary contrivance for the purpose of rearing human beings;[3] there are there men beside one another thrown upon their own resources, who want to make their species prevail, chiefly because they must prevail, or else run the terrible danger of being exterminated.[3] The favor, the superabundance, the protection are there lacking under which variations are fostered; the species needs itself as species,

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Paragraph 1

1. In the first sentence, the author is focused on the fact of how biological species originate. Struggle is said to help strengthen and establish a given line of species.

2. The second sentence is about how favorable conditions and a supportive environment help encourage variation. The important thing is to note that the focus is different—origination and variation. Though minute, this difference is key. The author can combine these two ideas, as they are not contradictory, though they may appear to be so at a cursory reading. It is possible that there can be very favorable conditions early in the lives of various organisms, while also

allowing for the fact that true testing (and long-term differentiation) is only possible when struggle has occurred. Thus, on this close reading, you see that the author is not providing mutually exclusive alternatives so much as he is providing two particular insights that require discussion.

Paragraph 2

3. Notice that the aristocratic commonwealth is viewed in terms of rearing humans and survival. The author is not really worried about the freedom of association in political life (or any other matter akin to that).

as something which, precisely by virtue of its hardness, its uniformity, and simplicity of structure, can in general prevail and make itself permanent in constant struggle with its neighbors, or with rebellious or rebellion-threatening vassals.[4] The qualities to which it principally owes the fact that it still exists, in spite of all Gods and men, and has hitherto been victorious, it calls virtues, and these virtues alone it develops to maturity.[5] It does so with severity, indeed it desires severity; every aristocratic morality is intolerant in the education of youth, in the control of women, in the marriage customs, in the relations of old and young, in the penal laws (which have an eye only for the degenerating): it counts intolerance itself among the virtues, under the name of "justice."[6] A type with few, but very marked features, a species of severe, warlike, wisely silent, reserved and reticent men… is thus established, unaffected by the vicissitudes of generations; the constant struggle with uniform unfavorable conditions is, as already remarked, the cause of a type becoming stable and hard.[7]

Paragraph 2 (Cont.)

4. Notice that from the first sentence, Nietzsche makes a parallel with the case of species in their struggle for prevailing and living. Then, in the second sentence, he makes clear that hardness, uniformity, and simplicity help to ensure the endurance of the species. Notice that he mentions two kinds of struggle—of one commonwealth with its neighbors and of the commonwealth with vassals who are unruly.

5. Notice then that he discusses qualities that arise in these circumstances. In the first paragraph, you were likely expecting a discussion of biology. Here, however, the qualities will be slightly different than physical traits. These qualities ensure survival—in spite of every adverse condition (whether from the gods or of human origin).

6. Then, he notes the role of severity and one important example (according to him) of virtue in aristocratic morality (i.e. the qualities helping aristocratic commonwealths survive): Intolerance. He follows this with examples listed. He is saying that in every single kind of relationship—whether toward children, in the interactions we have with the elderly, or even in marriage—severity and intolerance are key for aristocratic commonwealths. Indeed, this character quality they call a virtue, namely justice. Thus, he is trying to argue that justice is a name given to something that is really a survival quality for the aristocratic commonwealth—i.e. severity and intolerance.

7. Then, when he speaks of a "type with few, but very marked features," he is showing that there is a parallel between the character of the member of such a commonwealth and a species that is a particular type of creature, having very certain features. He then describes this character and states that it is established as a kind of unchanging type—much like a biological species.

Finally, however[8], a happy state[9] of things results, the enormous tension is relaxed; there are perhaps no more enemies among the neighboring peoples, and the means of life, even of the enjoyment of life, are preset in superabundance. With one stroke[10] the bond and constraint of the old discipline severs: it is no longer regarded as necessary, as a condition of existence. Variations, whether they be deviations[11] (into the higher, finer, and rarer), or deterioration[11] and monstrosities[11], appear suddenly on the scene in the greatest exuberance and splendor; the individual dares to be individual and detach himself.[12] At this turning-point of history there manifest themselves, side by side, and often mixed and entangled together, a magnificent, manifold, virgin-forest-like up-growth and up-striving, a kind of tropical tempo[13] in the rivalry of growth, and an extraordinary decay and self-destruction, owing to the savagery opposing and seemingly exploding egoisms, which strive with one another "for sun and light,"[13] and can no longer assign any limit, restraint, or forbearance[14] for themselves by means of the hitherto existing morality.

Paragraph 3

8. The paragraph opens with a transition, indicated by the word "however."

9. The new state of affairs is referred to as being a "happy state." There is an implied tone of the loosened conditions being a positive thing

10. This happens "with one stroke." Notice the immediacy involved in the transition from scarcity to abundance.

11. Notice that the variations are called "deviations" and "deterioration / monstrosities." "Deviation" is thus not necessarily something negative. Implicitly, it is something positive contrasted to deterioration. To "deviate" from the old norm is thus presented as the possible path to new, positive traits.

12. Notice that he wants to stress that in these cases the individual can exist *qua individual*. The exuberance and development thus occurs as a removal of the stifling effects of corporate existence in the commonwealth.

13. Notice that he clearly pushes the analogy with biology by the expression, "A magnificent, manifold, virgin-forest-like up-growth and up-striving, a kind of tropical tempo." Also, note his comparison to individual humans as struggling "for sun and light," as though they were plants in the forest.

14. Finally, notice that the former morality is presented as a kind of limitation on the individual, striving now to become something new. The overall tone is thus that the "happy state" of superabundance allows for absolutely unlimited aspirations for individual genius.

Reading Comprehension

When reviewing for the GRE Verbal section, it's easy to get "vocabulary tunnel vision," focusing on studying for the section's characteristic vocab-heavy question formats while ignoring Reading Comprehension questions. After all, many students assume that they know what reading comprehension entails, having encountered similarly named test sections on previous standardized exams. Oftentimes, however, test-takers are surprised to find that the GRE Verbal's most difficult questions are those that test reading comprehension skills. The difficulty of the reading material on the GRE Verbal section and the rigor with which the test examines reading comprehension skills are unmatched by tests like the ACT and the SAT. Don't make the mistake of assuming that you're prepared for the GRE's reading questions just because you did well on a previous exam. Some efficient but careful review—and a bit of time spent refreshing rusty skills—can spell the difference between an excellent Verbal score and a passable one.

This section will guide you through the catalog of different concepts likely to be in play in GRE Reading Comprehension questions. Because successfully fielding these questions relies on a combination of reading ability and test-taking skills, each lesson examines a facet of a reading passage and looks at how to deduce efficiently what that literary element is accomplishing in the text and how to apply your deductions to a variety of question types.

In its own way, and to the chagrin of many while testing, the GRE's Reading Comprehension questions are just as notorious as those focused on vocabulary. By taking the time to carefully prepare yourself to answer GRE-level reading questions on test day, you can face your exam knowing that your review was well-rounded and feel prepared to succeed on the entire section, not just parts of it.

Section Outline

Establishing Main Idea, Meaning, and Purpose (Part I)

Establishing Main Idea, Meaning, and Purpose (Part II)

Strength of Support and Evidentiary Choices

Word Usage in Context

Summary and Paraphrase

Inferences, Conclusions, and Applications

Authorial Choices: Text Structure

Authorial Choices: Tone and Style

Authorial Choices: Perspective and Bias

Establishing Main Idea, Meaning, and Purpose (Part I)

Anything that is written well is written as a unified whole. The principle of unity for a passage is governed by its main idea or general thesis. This concept provides all of the reasons for the passage's content and structure—at least so long as it is written well! The excerpts for the GRE are presented as little unified wholes in this manner, so it is important that you look for the main idea of each passage you encounter. Do not go looking for an explicit statement like, "Therefore, I will argue . . ." or "My thesis is . . ." The passages will not be this open about their main ideas; you will need to do some interpretive work to accurately identify the main idea of any complex passage. Once you uncover and express the main idea or thesis of a passage, however, you will have a clearer vision of its various sub-elements.

Consider two examples. For instance, in a selection about the importance of learning logic, the author may state that he or she is *not* speaking of advanced mathematical logic. Why would the author include this remark? Well, if the overall thesis is about the importance of logic, the qualification *is not* a repudiation of the thesis. Instead, it is an effort to help qualify the claims (and to define the sense of "logic" being discussed more sharply). If you do not pay attention to the main idea, you may well misread this qualification as a statement about the unimportance of logic—quite the opposite of the author's intent! Another example benefit of "thesis awareness" is that you are able to focus on the important aspects of details. Thus, if the author lists various logicians, you will filter out the unnecessary details as being secondary. Though these details are not worthless, your primary concern should be to answer the question, "What in this fact is directly related to the thesis about the importance of learning logic?"

As you read a selection, you should quickly summarize the passage in your mind as you progress through it. More than likely, the main idea will be formed and expressed in the first paragraph. However, you need to pass through the whole selection to be sure of this. Read the following selection on your own, adapted from "The Influence of the Conception of Evolution on Modern Philosophy" by H. Höffding (1909) in *Evolution in Modern Thought* (1917 ed.). Afterward, we'll give an example walkthrough of how to read the passage to identify its main idea. Ignore the underlining for now; we will return to discuss that sentence later.

When *The Origin of Species* appeared fifty years ago, Romantic speculation, Schelling's and Hegel's philosophy, still reigned on the continent, while in England, Positivism, the philosophy of Comte and Stuart Mill, represented the most important trend of thought. German speculation had much to say on evolution; it even pretended to be a philosophy of evolution. But then the word 'evolution' was to be taken in an ideal, not in a real, sense. To speculative thought, the forms and types of nature formed a system of ideas, within which any form could lead us by continuous

transitions to any other. It was a classificatory system which was regarded as a divine world of thought or images, within which metamorphoses could go on—a condition comparable with that in the mind of the poet when one image follows another with imperceptible changes.

Goëthe's ideas of evolution, as expressed in his *Metamorphosen der Pflanzen und der Thiere*, belong to this category; it is, therefore, incorrect to call him a forerunner of Darwin. Schelling and Hegel held the same idea; Hegel expressly rejected the conception of a real evolution in time as coarse and materialistic. "Nature," he says, "is to be considered as a system of stages, the one necessarily arising from the other, and being the nearest truth of that from which it proceeds; but not in such a way that the one is naturally generated by the other; on the contrary [their connection lies] in the inner idea which is the ground of nature. The metamorphosis can be ascribed only to the notion as such, because it alone is evolution.... It has been a clumsy idea in the older as well as in the newer philosophy of nature, to regard the transformation and the transition from one natural form and sphere to a higher as an outward and actual production."

In the first paragraph, begin by noting that the context is the publication ("origin") of *Origin of Species*. Two schools of philosophy are presented—romantic / continental and English / positivist. Notice that the second sentence is about German speculation on evolution. Scanning ahead, you should note that the English positivism is not really discussed. Instead, German speculation is presented as a general philosophy of evolution. The remainder of this paragraph explains the general sense of this assertion. What we know thus far is that the passage seems to be about German philosophy and its character as a kind of philosophy of evolution; however, notice that the remaining portion of the paragraph is a kind of long explanation that the word "evolution" is not being used here in a kind of "real" sense. These philosophers are more interested in the evolution of ideas than in biological evolution.

The second paragraph provides two examples of such German philosophies. The author opens with Goëthe, whose philosophy is classed in this general evolutionary trend. Note, however, the important remark: "It is, therefore, incorrect to call him a forerunner of Darwin." So, as we will see in the next section about deriving meaning in context, the general idea is that German philosophy is evolutionary in nature during this period, but it is not like Darwin's biological / materialist evolution. Hegel is cited as another example of this same point. Notice the emphasis here—it is not a matter of real evolution in the material world.

With all of this in view, you have the general idea of the passage, which you can summarize as, "When Darwin's *The Origin of Species* appeared, German philosophy was marked by its own awareness of evolution as a general aspect of any theory of reality; however, it was not concerned with explicating material evolution as such." Notice how this helps you to see the structure of the entire passage too. It opens with the general idea, gives a qualification, and then explains that qualification a bit. Then, two examples are provided to help you understand (at least to a degree) just in what way we should call German philosophy of this period a "philosophy of evolution."

Sentence Meaning and Purpose in Context

Some questions on the GRE Verbal section might direct you to a particular phrase or sentence in the passage and ask you to describe its importance in the context of the main idea of the whole presented text. The most important part of addressing such a question is reading the sentence in the context of the passage, not as a stand-alone statement. The question may be trying to test whether you can take note of how the sentence in question functions as one cog in the rhetorical machinery of the author's argument, and is likely to draw your attention to sentences that, when read independently, don't carry the same import or meaning that they do as part of the passage. Let's take a look at one such sentence, the underlined one in the passage as presented earlier.

"But then the word 'evolution' was to be taken in an ideal, not in a real, sense."

As soon as you see concessive words like "but," "however," or "although," you should be on the lookout for some kind of qualification. In the selection above, the author states in the first sentence that German philosophy at the time of the publication of *The Origin of Species* was a kind of philosophy of evolution. You may likely be rushing to the conclusion that it was a kind of forerunner to Darwin's thought. The indicated sentence, however, states that the German sense of "evolution" was *ideal*, not *real*. What does this mean?

Read outside of the passage's context, it might be feasible to conclude that "ideal" is being contrasted against "real" in the sense that "ideal" things are idealistic fantasies nonexistent in reality. This is not at all what the passage intends to mean by this sentence! Let's consider how this statement interacts with the rest of the author's argument and note how we can capture a more accurate snapshot of its meaning.

After the statement is given in the passage, you get some good qualifying remarks and examples that help you see the general point more clearly. Various German thinkers held that nature was really just a system of ideas or thoughts that were interconnected. These thinkers were not interested in a kind of material or biological evolution. Instead, they held a much more abstruse position regarding evolution—namely, that the truly evolving reality is a system of ideas that constitute the fundamental ground of nature itself. The selection from Hegel, though difficult, is a great example of this assertion. He held that Nature was really just a whole that *itself* evolved. It is not a matter of stages as much as

it is a kind of inner development of the one *idea* of nature itself. This is a wild-sounding philosophy; however, the point is clear: these German thinkers weren't really interested in the natural / biological evolution of the kind expressed in Darwin.

Thus, we can see what is meant by the sentence, "But then the word 'evolution' was to be taken in an ideal, not a real, sense." This means, "Such German thinkers used the word 'evolution' to refer to the development of ideas (which they also believed to be the ultimate reality of nature) and not to the evolution of physical entities."

This interpretive work also allows you to grasp the purpose of the sentence, another topic about which the Verbal section can easily quiz you. The indicated sentence helps to make precise the claims about German philosophy being a "philosophy of evolution." It explains the sense of "evolution" so as to prevent the reader from misinterpreting the claim, thinking that the German philosophers in question were interested in the same thing as Darwin. This definitely helps you to interpret the rest of the selection and understand why the authors freight the second paragraph with the weighty prose of Hegel. This text describes this "ideal" kind of focus on the abstract concept of evolution, not its biological realities.

"Goffman's *Asylums*" by Joseph Ritchie (2015)

Sociologists such as Goffman revolutionized the field by introducing ethnographic methodologies that sought to understand social phenomena through direct observations and interactions. Prior to this, sociologists conducted what has been satirically referred to as "armchair sociology." Pioneers of the field engaged with society in a philosophical manner that left them disengaged from their targets of investigation. Sociologists of the ethnographic school, however, actively engage in activities of participant observation with the subcultures under investigation. Instead of theorizing from a distance, Goffman carried out his fieldwork for *Asylums* within an actual asylum. Immersion into the institution and social embeddedness with its actors enabled Goffman to accurately document and theorize the characteristics of and methods of socialization used by total institutions.

Sample Question #1

Which of the following best describes the passage's main idea?

A. Goffman's *Asylums* was a landmark work in the study of culturally ostracized individuals.

B. Sociology based on sociological observations is far superior to "armchair sociology."

C. Goffman's *Asylums* represents the culmination of "armchair sociology."

D. Goffman was an innovative sociologist who helped usher in a major change in the way the field conducts research.

To determine the main idea of the passage, it helps to take a conceptual step back and consider what general topics the passage is discussing. The first sentence introduces us to the main topic—sociology—by talking about a revolution in the field that certain figures like one Goffman helped bring about. The passage then talks about the previously dominant methodology, the distant approach of "armchair sociology," before contrasting it against the direct observation of the "ethnographic school." The passage concludes by discussing Goffman's research as an example of direct observation fieldwork.

Based on this quick skim for major topics, we have identified some that are core to the passage's discussion of sociology: the contrast between "armchair sociology" and the ethnographic school, and Goffman, who is categorized as part of the latter. Since the passage spends time talking both about Goffman's individual work and about the abstract sea change in sociological methodology, we need to pick out an answer choice that presents both topics as part of the passage's main idea. Answer choice A only talks about Goffman individually, whereas B only talks about the two sociological methodologies. Both thus end up missing a major part of the passage. Answer choice C looks good in contrast: it mentions both Goffman's *Asylums* as well as "armchair sociology." But wait! It relates the two by claiming that *Asylums* was "the culmination of" "armchair sociology." That's not correct—the passage constantly alligns Goffman with the ethnographic school of sociology, not "armchair sociology." D is thus the best answer: Goffman was an innovative sociologist who helped usher in a major change in the way the field conducts research. This answer encapsulates the passage well because it conveys the relationship between the passage's two main topics: Goffman and the contrast between "armchair sociology" and the ethnographic school.

Sample Question #2 directs us to the sentence, "Prior to this, sociologists conducted what has been satirically referred to as 'armchair sociology.'" It immediately follows the sentence in which Goffman is introduced as part of the ethnographic school. While the chronological focus of the phrase "Prior to this" may make answer choice A look potentially correct, this answer choice derails when it claims that ethnographic methodologies and "armchair sociology" are "so similar." They're presented as practically opposites. This leads us to the correct answer, B: the underlined sentence "introduces a contrasting sociological methodology." Goffman's

Sample Question #2

What purpose does the underlined excerpt serve in the passage?

    A. It describes the chronology of the field in sociology so that the reader can understand why ethnographic methodologies and "armchair sociology" were so similar

    B. It introduces a contrasting sociological methodology

    C. It provides information about what prompted Goffman to study asylums

    D. It conveys how new methodologies resulted in sociology temporarily losing its cultural status as a widely respected, legitimate field of study

work in studying asylums isn't mentioned in the passage before the indicated sentence, and while D hooks onto the satirical name "armchair sociology," it includes a logical misstep: "new methodologies" would not be associated with "armchair sociology" in the chronology we're given; ethnographic methodologies would be, and there's no indication that the field's cultural status suffered as a result of this shift.

Sample Question #3 asks us to identify the purpose of the passage. Again, let's return to the general topics it addresses: Goffman and his work, and the transition from the methodologies of "armchair sociology" to ethnographic methodologies. D doesn't deal with these topics at all. While the passage does discuss the history of sociology, it's not meant as a general introduction for the reader to this topic; instead, it discusses one significant change in methodologies. It doesn't aim to tell us anything about what preceded "armchair sociology," so D isn't the best answer. Does the passage point out flaws in ethnographic methodologies? No. It casts ethnographic methodologies in a very positive light when contrasting them against "armchair sociology." Similarly, the text does not critique Goffman's *Asylums*; the last sentence discusses how Goffman's ethnographic approach allowed

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Sample Question #3

Which of the following best states the purpose of this passage?

A. To point out the flaws inherent in ethnographic methodologies

B. To critique the overly narrow focus of Goffman's *Asylums*

C. To discuss a sociologist's work in the context of the field during his era

D. To serve as a general introduction for the reader to the history of sociology

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him to "accurately document" various apsects of asylums, which has a positive connotation. Thus, the best answer is C: the purpose of the passage is to discuss a sociologist's work (Goffman's) in the context of the field during his era. The latter half of this answer choice applies to the contrasting methodologies that the passage presents.

Establishing Main Idea, Meaning, and Purpose (Part II)

Meaning and Purpose in Context

Although each GRE passage is a selection, it is also a united whole, selected to make a general point. We cannot understand a part of a given whole unless we actually understand the whole itself. Imagine a hand separated from a living person's body. We really do not know all of the various uses of a hand unless we experience it as part of a whole person. It is only a "piano playing hand" when it is part of the body of a living, acting pianist. The same is true of the parts of a written passage. They only make sense when you see the overall form of the whole.

The particular details of the argument will build around the author's central thesis. As you read along, you need not be confused about why a particular remark has been made—at least if you know and understand the main idea of the passage. You always should be able to ask, "How does this shed light on the overall project? What is its role in the overall scheme of things?" Thus, if an author makes a concession to an opposing argument, you will see it for exactly what it is—a concession. If you didn't grasp the overall argument, you might well think that the author is actually stating his or her opinion in such a case; however, with the view of the whole, you understand well the role of the part!

Also, a grasp of the overall theme will help you to retain the structural elements in memory. If you know what is being argued, you will be more likely to recall the basic outline of the passage. This will be very helpful when you have to return to the passage for the questions that you are asked. For example, you might notice that an author wishes to argue that America shouldn't have dropped atomic bombs at the end of World War II. In the course of the selection, you notice that she states her argument, explains briefly why she believes it was not a good moral choice, provides two parallel examples to shed light on her position, and finally ends with a concession that she is not condemning the people who made the decision. By knowing the main idea and thesis of the passage, you will quickly see everything in a quick outline: position statement, explanation, parallel examples, softening of condemning language. A grasp of the main idea will help you see the role of each and every part in the passage. Thus, you will quickly return to the passage when you are asked specific questions—all enabled by your grasp of the overall form of the passage.

Identifying Argument/Thesis

Let's practice identifying the general argument being made in the following passage, adapted from *Walden* by Henry David Thoreau (1854). Pay no particular attention to the underlined sentence, as we will address it later.

> Our life is like a German Confederacy, made up of petty states, with its boundary forever fluctuating, so that even a German cannot tell you how it is bounded at

any moment. The nation itself, with all its so-called internal improvements, which, by the way are all external and superficial, is just such an unwieldy and overgrown establishment, cluttered with furniture and tripped up by its own traps, ruined by luxury and heedless expense, by want of calculation and a worthy aim, as the million households in the land; and the only cure for it, as for them, is in a rigid economy, a stern and more than Spartan simplicity of life and elevation of purpose. It lives too fast. Men think that it is essential that the Nation have commerce, and export ice, and talk through a telegraph, and ride thirty miles an hour, without a doubt, whether they do or not, but whether we should live like baboons or like men is a little uncertain. If we do not get out sleepers, and forge rails, and devote days and nights to the work, but go to tinkering upon our lives to improve them, who will build railroads? And if railroads are not built, how shall we get to heaven in season? But if we stay at home and mind our business, who will want railroads? We do not ride on the railroad; it rides upon us. Did you ever think what those sleepers are that underlie the railroad? Each one is a man, an Irishman, or a Yankee man. The rails are laid on them, and they are covered with sand, and the cars run smoothly over them. They are sound sleepers, I assure you.

For this passage, it is important that you pay attention to the opening sentence. Otherwise, you are very likely to miss the main idea of Thoreau's passage. Notice that he is making a parallel between our lives and the German Confederacy. He provides a description of the German Confederacy at his time of writing, as a changing nation, with boundaries that are never the same over the passage of time. Notice, however, that he continues to speak of the "nation" (or a nation)—but he is really speaking about our lives. (Perhaps, too, he is speaking of the particular lives of Americans in his day.) Pay heed to this as you go through the details that follow in the selection.

Thus, he describes the nation (i.e. our lives) as being cluttered. The "furniture" and luxury of life requires a "rigid economy" to support such luxuries. From what he says after this, it is clear that he thinks that the desires (for the various kinds of "furnishings" of life) require a life that is fast-paced and is focused on practical, economic gains; however, in the midst of all this, the people involved in this stern economy are not even sure whether or not they should live in a truly human manner (or merely as baboons).

The details can sound, of course, like he is making a critique of a particular nation; however, he clearly wants to present these images of the "modern" life of his day as examples of the ways in which humans become mired in their pursuits of wealth and various "creatures comforts." It is a critique of a very way of life. Indeed, he expresses the idea that we are slaves to this—as though the railroad itself rides upon human persons. He clearly has a different view of human life, namely one that is

simpler than all of these various examples that he provides (and implicitly critiques). If you were not aware of the main idea, however, this passage would be abuzz with a number of inexplicable examples and remarks that may well confuse you. You can, however, understand these remarks so long as you hold fast to the fact that Thoreau is providing an image for how human life and culture in his day is external and superficial—overgrown and shifting without end, just like the German Confederacy as regards its political life.

Deriving Purpose in Context

Consider the underlined sentence in the passage presented on the preceding page. This sentence can be very confusing if you do not have an idea concerning the overall point of the passage. Thoreau's style alternates back and forth quite a bit, with a kind of wild cadence; however, we can tame some of this prose if we hold fast to the main argument that Thoreau is making: human life in his day is generally lived in a superficial and external way. Always remember this main point, and you will understand this passage well.

Now, bearing in mind our main theme, let's actually break down each part of this sentence. This close reading will help us to get a grasp on what is going on:

"If we do not get out sleepers, and forge rails . . .": That is, if we do not push out into the workforce those who are merely "sleeping"—people who are outside of the system of commerce and not worried with "getting ahead" and being "productive."

". . . and devote days and nights to the work . . .": Notice how strongly Thoreau wants to emphasize how work becomes a day-and-night affair. It takes overall aspects of one's life, enslaving one to the speed and requirements of the market—all so that one can live life with "improvements" that are really mere "furniture" and "traps."

". . . but [instead] go to tinkering upon our lives to improve them . . .": Note that we have added "instead." This helps you see the structure of the sentence. It basically reads, "If we don't do A but instead do B . . ." What is the alternative that Thoreau presents? It is the promise of living our lives by improving ourselves—instead of just becoming cogs in the overall economy. Here, we see what people who are "serious" think of those who are focused on self-improvement: they merely tinker; however, for Thoreau, this is the main point of existence, though it seems that we all miss it as we bustle about. People should spend their lives in improving their individual selves in more profound ways than mere externals.

". . . who will build railroads?" This is a sarcastic ending to the sentence. In the sentence, Thoreau is speaking from the perspective of one who sneers at "idlers" who merely want to "tinker" on their lives. Likely, it will be very difficult to catch this sarcasm if you aren't always aware of his main idea in this passage; however, with this viewpoint in mind, you will finally see the overall point of the sentence.

Thus, we can see the purpose of this selection. Thoreau wants to put a kind of sarcastic phrase on the lips of those who would think that his vision of life is unimportant. Those who are caught up in the bustle of the economy (which he critiques) only can think about railroads (i.e. the life of economic advancement). If people worry about improving themselves, being mere "sleepers" who "tinker" on themselves, what good can they do for the railroads? (Indeed, as such, they seem to be tied up in something very unimportant—very detached from the "true affairs" of quick living and making technological progress.) Thus, by understanding his main idea, you see exactly what kind of remark he is making here—all without disrupting or ruining the point of the passage.

Let's now read through the following sample passage, adapted from *Extemporary Essays* by Maurice Hewlett (1922), and answer the few questions about main idea, thesis, and purpose that follow.

In days of more single purpose than these, young men and maidens, in the first flush of summer, set up a maypole on the green; but before they joined hands and danced round about it they had done honor to what it stood for by draping it with swags of flowers and green-stuff, hanging it with streamers of diverse colors, and sticking it with as many gilt hearts as there were hearts among them of votive inclination. So they transfigured the thing signified, and turned a shaven tree-trunk from a very crude emblem into a thing of happy fantasy. That will serve me for a figure of how the poet deals with his little idea, or great one; and in his more sober mood it is open to the essayist so to deal with his, supposing he have one. He must hang his pole, or concept, not with rhyme but with wise or witty talk. He must turn it about and about, not to set the ornaments jingling, or little bells ringing; rather that you may see its shapeliness enhanced, its proportions emphasized, and in all the shifting lights and shadows of its ornamentation discern it still for the notion that it is. That, at least, is my own notion of what the essayist should do, though I am aware that very distinguished practitioners have not agreed with me and do not agree at this hour. The modern essayist, for reasons which I shall try to expound, has been driven from the maypole to the column.

Certainly, the parent of the Essay draped no maypoles with speech. Montaigne was a sedentary philosopher, of the order of the post-prandials; a wine-and-walnuts man. One thing would open out into another, and one seem better than the other, at the time of hearing. To listen to him is a liberal education; yet you can hardly think of Montaigne footing it on the green. Bacon's line, again, was the aphoristic. He shreds

off his maypole rather than clothes it: but he has one set up. He can give his argument as witty a turn as the Frenchman when he pleases—"There is no man doth a wrong for the wrong's sake, but thereby to purchase himself profit, or pleasure, or honor, or the like. Therefore why should I be angry with a man for loving himself better than me?" That is the turn his thoughts take upon Revenge, and a fair sample of his way with an abstract idea—shredding off it all the time, getting down to the pith. But he can be very obscure: "A single life doth well with Churchmen; for charity will hardly water the ground where it must first fill a pool." That is proleptic reasoning. We are to caper about the pole before the ornaments are on.

Picking out the thesis of the preceding passage is not a walk in the park. The author uses a lot of complex language to discuss essay-writing, and he begins with and maintains an overarching metaphor about maypoles that can be elusive on the first read. Taking a step back and considering the passage's general movements, he starts off the first paragraph by talking about an old maypole celebration, and then compares the maypole with ornaments on it to a good essay before claiming that writers have been "driven

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Sample Question #1

Which of the following best conveys the passage's thesis?

   A. The styles of essayists like Montaigne and Bacon are each ones that overly ornate writers would do well to imitate.
   B. Cultural traditions like the maypole celebration have a lot in common with the writing process.
   C. The quality of essays has suffered as they have recently become stylistically austere, though exemplary essays in this style are praiseworthy.
   D. Whereas Montaigne's works may seem to have no main point, Bacon's essays are perhaps excessively to-the-point.

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from the maypole to the column." In the second paragraph, he talks about two essayists: Montaigne and Bacon. Neither of them is associated with excessively florid language. Taking another step back, how can we distill all this? What's the author arguing? B may seem appealing, but it's pretty general and doesn't have anything to do with the second paragraph's discussion. Let's see if any of the other answers are better. D can't be correct, because it's too focused on the second paragraph to the exclusion of the first one. Is the writer arguing that overly ornate writers should imitate Montaigne and Bacon? This gets tricky: the answer choice mentions the same things the passage concerns itself with, but the writer isn't actually making this particular argument. Instead, he seems to think that ornate language can be a positive thing; he associates it with the ornamentation on the maypole, which has positive connotations. Consider this excerpt from the first paragraph:

[The essayist] must hang his pole, or concept, not with rhyme but with wise or witty talk. He must turn it about and about, not to set the ornaments jingling, or little bells ringing; rather that you may see its shapeliness enhanced, its proportions emphasized, and in all the shifting lights and shadows of its ornamentation discern it still for the notion that it is. That, at least, is my own notion of what the essayist should do . . .

Nowhere in these statements does the author make the ornamentation or "witty talk" out to be a bad thing; on the contrary, in his opinion, it helps form part of a good essay. Montaigne and Bacon are mentioned in the second paragraph to demonstrate that such rhetoric isn't absolutely necessary to a good essay, but that doesn't mean he's arguing that people should eschew it.

The best answer is C, as it accurately reflects this point ("though exemplary essays in this [stylistically austere] style are praiseworthy"), but also states the main gist of the entire passage ("The quality of essays has suffered as they have recently become stylistically austere"). We find the author's thesis statement at the end of the first paragraph, which serves as an introduction: "The modern essayist, for reasons which I shall try to expound, has been driven from the maypole to the column." Answer choice C restates this while explicating the meaning of the author's metaphors.

Sample Question #2

In the context of the passage, which of the following is NOT an example of what the ornaments placed on the maypole signify in the passage?

    A.   A thesis statement
    B.   Supporting ideas
    C.   Descriptive language
    D.   Rhyming language

Let's now consider Sample Question #2. While reading the passage, you likely had an inkling that the complex metaphors would be the subject of a question or two. Here's another question that hinges on them, though it is quite a straightforward one that just asks us to pick out the meaning of one particular facet of the author's "maypole" metaphor. What are the ornaments on the maypole associated with? Let's reread the relevant section of the passage. The maypole metaphor is introduced in the first paragraph:

. . . they had done honor to what it stood for by draping it with swags of flowers and green-stuff, hanging it with streamers of diverse colors, and sticking it with as many gilt hearts as there were hearts among them of votive inclination. So they transfigured the thing signified, and turned a shaven tree-trunk from a very crude emblem into a thing of happy fantasy. That will serve me for a figure of how the poet deals with his little idea, or great one; and in his more sober mood it is open to the essayist so to deal with his, supposing he have one. He must hang his pole, or concept, not with rhyme but with wise or witty talk.

"Not with rhyme" sticks out because "rhyming language" is an answer choice. But is that "not" excluding this as part of the metaphor? No—it's just the author specifying that for poets, the linguistic "ornaments" on their work might be rhyme, whereas for essayists, the "ornaments" would be "wise or witty talk." That alligns well with "descriptive language" (C). So, two down and two to go: is it A or B, and how do we tell? Look at how the author says that the writer "must hang his pole, or concept . . ." So let's look at this another way: what's the maypole in the author's metaphor? It's the "concept," or in other words, the main idea—perhaps a thesis statement. A is the correct answer. "Supporting ideas" could fall under the author's label of "wise and witty talk" (the ornaments), but the "concept" (the maypole) is a different thing.

Sample Question #3

What is the meaning of the underlined sentence "He shreds off his maypole rather than clothes it: but he has one set up" in the context of the passage?

A. He uses straw arguments to tear down his own, but these are consciously set up so that his main point is left intact.

B. His work does not have an obvious argument, but if you consider how he deconstructs the text about which he's writing, one becomes apparent.

C. His drafts of the same text get progressively more concise, but he retains a particular metaphor in each one.

D. His writing is succinct and avoids rhetorical flourishes, but it still has a point.

Sample Question #3 involves more maypole metaphors. Let's situate this sentence in context before we attack it. Who's "he," the subject? Bacon, an essayist. The author mentions him after Montaigne as another example of a writer who didn't use a lot of flowery language but still wrote essays the writer thinks are good:

Bacon's line, again, was the aphoristic. He shreds off his maypole rather than clothes it: but he has one set up.

After a quotation to serve as an example, the author continues,

That is . . . a fair sample of his way with an abstract idea—shredding off it all the time, getting down to the pith.

Those sentences may be separated by a lengthy quotation in the passage, but did you catch their parallelism? The author says that Bacon "shreds off his maypole rather than clothes it," and after the quotation, he reprises the word "shred": "shredding off [an abstract idea] all the time, getting down to the pith." The "pith" of something is the crux or core of it, so Bacon is shredding at an idea and getting down to its crux. This "idea" is the "maypole" in our sentence. In the rest of the passage, remember how the author associates ornamentation on the maypole with flowery language? That's the opposite of what's going on here, apparently: instead of "clothing" his idea with flowery langauge, Bacon shaves away at it to get to its core. This makes sense, as the author says that Bacon is "aphoristic." An "aphorism" is a concise, witty phrase—the key word for us here is "concise."

So what's the best answer? There's no mention of straw arguments (A) or multiple drafts of work (C). B may seem like it could be correct beacuse it upholds the maypole-main idea metaphor, but it gets the quotation's logic wrong: in saying that Bacon "has [a maypole] set up," the author definitively states that Bacon's essays have main points. This leaves us with D, the correct answer: our sentence is best paraphrased as "Bacon's writing is succinct and avoids rhetorical flourishes, but still has a point."

Let's start answering Question #4 by looking at where in the passge the indicated sentence appears.

But [Bacon] can be very obscure: "A single life doth well with Churchmen; for charity will hardly water the ground where it must first fill a pool." That is proleptic reasoning. We are to caper about the pole before the ornaments are on.

The subject "we" refers to readers of Bacon's sentence. Even if you don't know what "proleptic" means, consider its connotation. Before the quotation, the author says that Bacon "can be very obscure." This follows an example of a pretty positive characteristic: that he can be "aphoristic," or *concise and witty*. The author seems to be suggesting that Bacon can take his concision a step too far so that it confuses readers. Remember that the author is using the maypole to represent the main idea or point of a text, and the ornaments to represent the "wise and witty talk" that essayists use to "decorate" their main idea. So, when the author says that "we are to caper about the pole before the ornaments are on," he's saying that we're supposed to appreciate Bacon's main idea before he's "decorated" it with language. This has nothing to do with critics and reputations of writers (C) or the context of the author's original statement (A). The answer choice about readers understanding the writer's metaphor (D) doesn't follow the logic of the statement as closely as does B: the writer expects the reader to allow a conclusion before it has been explained. (For those curious, this also fits with the meaning of "proleptic": "prolepsis" refers to a rhetorical move in which something is described in terms of how it will be after the action described has taken place. An example would be "She was found guilty as soon as she stepped in the courtroom.")

Sample Question #4

What is the meaning of the underlined sentence "We are to caper about the pole before the ornaments are on" in the context of the passage?

A. We are expected to appreciate the writer's statement out of its original context, and as a result, it makes little sense.

B. The writer expects the reader to allow his conclusion before it has been explained.

C. Critics are expected to praise the writer based on his reputation, before actually having read his work.

D. Readers can only understand the writer's metaphor after having been told to what it refers.

Strength of Support and Evidentiary Choices

Once you determine the main idea of your passage, it is important to note how the author supports this topic with various forms of evidence and reasoning. Strong evidence should always address the exact scope of the main idea. If the author is making a general claim, it is not enough for him or her to present anecdotal evidence to support his or her contention. Of course, an anecdote may introduce a more detailed set of evidence; however, a mere passing statistic or point cannot justify wide-ranging claims. For example, if the author argues that all Western philosophy has been biased against women but then only cites an example text from Aristotle, this would be a very weak set of data. Indeed, to make that large of a claim, it would be necessary to provide a very long discussion—or to limit the discussion merely to Aristotle (or even to merely one text of Aristotle's). Likewise, evidence should be rigorously selected. Thus, it might be argued, "The United States is falling behind in education. At every level of examination, students are failing in mathematics throughout the United States." This argument requires a long history of data and not just one data point (or set of contemporary data points). This argument also limits its example to one case—mathematics—while making a general claim about the state of education in the US.

Likewise, you need to watch the reasoning that is presented by the author. Sometimes, the author will argue by using an enthymeme—an argument that does not explicitly state a premise. For example, it might be written, "It is critical that people have jobs that pay well. Therefore, we need to spend more on education." This argument presupposes an unstated premise, namely that education is the best way to help people achieve high-paying jobs. While this may be true, the overall argument has its strength diminished by the author's failure to discuss this matter openly and in detail. Additionally, you should look for any logical fallacies in the passage's argument. There are many possible fallacies, so we will not list them all here. Nonetheless, you should always ask yourself, "Do those premises really require me to reach the conclusion (or sub-conclusion) that the author presents?"

Let's practice answering some sample questions about the claims and support found in the following paragraph, adapted from *The Extermination of the American Bison* by William T. Hornaday (1889).

From observations made upon buffaloes reared in captivity, I am firmly convinced that confinement and semi-domestication are destined to effect striking changes in the form of Bison americanus. While this is to be expected to a certain extent with most large species, the changes promise to be most conspicuous in the buffalo. Both the live buffaloes in the National Museum collection of living animals are developing the same shortness of body and lack of muscle, and when fully grown will but poorly resemble the splendid proportions of the wild specimens in the Museum mounted group, each

of which has been mounted from a most careful and elaborate series of post-mortem measurements. It may fairly be considered, however, that the specimens taken by the Smithsonian expedition were in every way more perfect representatives of the species than have been usually taken in times past, for the simple reason that on account of the muscle they had developed in the numerous chases they had survived, and the total absence of the fat which once formed such a prominent feature of the animal, they were of finer form, more active habit, and keener intelligence than buffaloes possessed when they were so numerous. Under such conditions it is only natural that animals of the highest class should be developed. On the other hand, captivity reverses all these conditions.

Luckily, this passage has a very clearly stated main idea in its first sentence. What remains, however, is the need to assess the strength and weakness of various claims made in support of this idea. It is easiest to do this by considering one sentence at a time:

Sentence 2 ("While this . . ."): This sentence is a kind of qualification, not really evidence. The author states that such confinement would cause similar changes in any other large species. The author is adding a bit to the claim in the passage. Unlike other such animals, we can expect that buffalo will show such changes in a fuller manner than in other species. While this point marks an important development in the expression of the overall main idea, it is not evidence.

Sentence 3 ("Both the live . . ."): This sentence cites a particular example based on the National Museum's animals. Now, notice that this is based upon two sample specimens—a very small set of examples for supporting his claim. Yes, these are not resembling the mounted (i.e. "dead and stuffed") animals. Certainly, this provides a bit of support for his claims, but it is a wild leap to go from two particular specimens living in a very limited form of captivity to making claims regarding captivity in general on an entire species! This passage's logic is looking quite fallible.

Sentences 4 and 5 ("It may fairly . . ."): This is supporting the claim that the wild specimens were of the highest caliber. Apparently, the Smithsonian expedition into the bison herds took place after a diminution of the total number of animals. Once upon a time, the bison were fatter; however, the author argues that the animals found as the herds diminished were in greater need of being healthy and intelligent, since a larger herd would have protected some of the weaker animals from being killed. The implication is that the mounted animals are of these healthier, leaner, more intelligent variety of bison. Thus, the author claims that in the wild, the animals were developing into their finest possible form.

Sentence 6 ("On the other hand . . ."): This final sentence implies that the captive animals are

understandably weaker than the wild ones found in captivity. According to the author, the weak will survive, thus changing the overall gene pool.

Now, this selection is not the strongest of all arguments, though it presents a partial argument for alterations to the form of the buffalo population. The point is not so much that the buffalo are altered by captivity as the fact that they do not have strong characteristics promoted by such captivity. In the wild, they must fend for themselves. Indeed, given the changes in buffalo populations, only the fittest are able to survive. The only support given is the anecdote based upon two lonely animals in the national collections. This is somewhat supported by the fact that the harsher conditions seemed to make the animals more robust, even when they were in the wild. Therefore, it is to be expected that the less harsh conditions of captivity will do the same to the animals.

The passage makes some striking claims without completely supporting those claims in great detail. As mentioned above, it bases itself on two animals without really discussing many details about the lives of those animals in captivity. A much larger study would help to flesh out these details. Furthermore, the study bases itself on the fact that wild animals are quite hearty especially in light of changes in buffalo population. If the wild population were, however, to increase in numbers, it is quite possible that the wild animals will develop just as do the captive ones. Furthermore, from a scientific perspective, the passage is lacking in that it is merely speaking of how the animals develop during their lifetime. Perhaps the limited gene pool of the specimens in captivity will retain the possibility of developments into hearty animals—if the conditions are right. Thus, the species really would not change in its root, genetic foundation. It would merely change in its expression, due to the conditions. Certainly, this would mark an external alteration, but it would not justify the sweeping claims made by the author. Finally, these sweeping claims are problematic in themselves. The author never really says anything that justifies the implied claim that buffalo are going to change even more than other animals that are in captivity in a similar manner.

Let's conclude this lesson by taking a look at another passage and answering a few sample questions about its claims. The following passage is adapted from "Humming-Birds: As Illustrating the Luxuriance of Tropical Nature" in *Tropical Nature, and Other Essays* by Alfred Russel Wallace (1878)

The food of hummingbirds has been a matter of much controversy. All the early writers down to Buffon believed that they lived solely on the nectar of flowers, but since that time, every close observer of their habits maintains that they feed largely, and in some cases wholly, on insects. Azara observed them on the La Plata in winter taking insects out of the webs of spiders at a time and place where there were no flowers. Bullock, in Mexico, declares that he saw them catch small butterflies, and that he found many kinds of insects in their stomachs. Waterton made a similar statement. Hundreds and perhaps thousands of specimens have since been dissected by collecting

naturalists, and in almost every instance their stomachs have been found full of insects, sometimes, but not generally, mixed with a proportion of honey. Many of them in fact may be seen catching gnats and other small insects just like fly-catchers, sitting on a dead twig over water, darting off for a time in the air, and then returning to the twig. Others come out just at dusk, and remain on the wing, now stationary, now darting about with the greatest rapidity, imitating in a limited space the evolutions of the goatsuckers, and evidently for the same end and purpose. Mr. Gosse also remarks, " All the hummingbirds have more or less the habit, when in flight, of pausing in the air and throwing the body and tail into rapid and odd contortions. This is most observable in the Polytmus, from the effect that such motions have on the long feathers of the tail. That the object of these quick turns is the capture of insects, I am sure, having watched one thus engaged pretty close to me."

This is a tricky question because in the passage, the author never directly states his opinion about what hummingbirds eat; readers have to infer it based on the evidence he presents. The author begins the passage by stating that while old scientists used to think hummingbirds ate only flower nectar, modern writers think that they feed "largely, and in some cases wholly," on insects. He then presents evidence suggesting that hummingbirds eat insects, and in discussing the contents of hummingbirds' stomachs, says that scientists sometimes find both insects and honey. For the rest of the paragraph, he provides evidence suggesting that hummingbirds eat insects.

What can we infer from this? Well, we can tell that it's not likely that the author thinks hummingbirds eat only flower nectar, because he provides evidence supporting the idea that they eat insects. This means that we can also discard the answer choice "hummingbirds eat neither flower nectar nor insects." It's quite reasonable to think that the author thinks that "hummingbirds eat a mixture of flower nectar and insects" because he mentions that sometimes honey is found along with insects in

Sample Question #1

Based on what is said in the passage, the author most likely believes that _____.

- **A.** hummingbirds eat only flower nectar
- **B.** hummingbirds eat a mixture of flower nectar and insects, but mostly flower nectar
- **C.** hummingbirds eat a mixture of flower nectar and insects, but mostly insects
- **D.** hummingbirds eat neither flower nectar nor insects

hummingbirds' stomachs. So, we need to figure out whether he probably believes that they eat mostly insects or mostly flower nectar. Let's look at how the author phrases his description of the contents of hummingbirds' stomachs: "in almost every instance their stomachs have been found full of insects, sometimes, but not generally, mixed with a proportion of honey." So, if "in almost every instance" the hummingbird stomachs examined were "full of insects," but "sometimes, but not generally" honey was also found, the correct answer is C.

Sample Question #2

How does the quotation from Mr. Gosse relate to the evidence provided by other scientists earlier in the passage?

A. It contradicts the previous evidence and supports a different hypothesis.

B. It supports the same conclusions that the previous evidence supports.

C. It suggests that some of the previous evidence may be true, but some may be false.

D. It suggests that the earlier evidence applies not only to hummingbirds but to another type of bird as well.

In approaching Sample Question #2, let's consider what Mr. Gosse is saying. Paraphrasing this, Mr. Gosse is saying that he has seen hummingbirds contort themselves in the air and he's pretty sure they're doing this in order to catch insects. The evidence provided by scientists earlier in the passage supports the idea that hummingbirds eat insects, just like Mr. Gosse's does. We can't say that Gosse's evidence contradicts the earlier evidence or that it suggests that some of it may be false. It also doesn't suggest that the previous evidence can be applied to birds other than hummingbirds, because Mr. Gosse says that he is only discussing hummingbirds and we are to infer that the Polytmus is a hummingbird. So, B is correct.

Sample Question #3

What evidence does Mr. Gosse have to support the claim that hummingbirds eat insects?

A. He observed one flailing around in the air and concluded that it was eating insects.

B. He read in a reputable scientific journal that they eat insects.

C. He examined the contents of a hummingbird's stomach and found many insects in it.

D. A hummingbird got into his collection of live insects, and soon after, all of his insects were missing.

To answer Sample Question #3, we have to consider the quotation attributed to Mr. Gosse found at the end of the passage, the same quotation we considered in Sample Question #2. Mr. Gosse doesn't mention anything about having a collection of live insects, getting his information from a scientific journal, or dissecting a hummingbird's stomach, so we can ignore those answer choices. He actively observes a hummingbird and surmises that they eat insects based on that observation, so A is correct.

Word Usage in Context

Knowing the definitions of obscure words is certainly advisable on the GRE, but you are likely to come across situations arranged by the exam's Verbal section in which you still don't have the foggiest idea what a given word means. In situations like these, context—the information conveyed by the words and sentences surrounding the mystery word—is crucial, as it is the only clue that you have in tracing down the meanings of these terms. On the GRE, you will often be presented with reading passages often written about topics far from your own experience and expertise. Every discipline has its own technical vocabulary, so you can expect these passages to utilize words that are utterly foreign to you. Consider the following three examples, as well as the advice on how you might employ context in making sense of the underlined terms.

> The procession passed from the <u>narthex</u> of the church into its <u>nave</u>, bringing the
>
> long line into the sight of the people gathered into the main body of the building.

Even if you've never seen either of the underlined terms before, you can still make sense of this sentence. A good first step in analyzing words contextually is to take a step back from the word-level focus to a sentence-level focus. What's going on here? Well, the subject of this sentence is a "procession": a formal, solemn parade. We're told that it's "passing from" a church's "narthex" into its "nave." This tells us a lot: "narthex" and "nave" must each refer to locations at which a parade of people might be. These locations must be parts of buildings. If you had to take a guess and inferred that "narthex" and "nave" are likely specific parts of churches, you'd be correct. But what distinguishes them? The sentence can help us out in figuring this out, too. What we are told is the effect of the procession moving from the "narthex" to the "nave"? Doing so "bring[s] the long line into the sight of people gathered into the main body of the building." So, a "nave" must be an interior part of a church adjacent to the main body of the edifice, whereas a "narthex" is likely more exterior. These inferences are correct: the "narthex" is *the entrance area to a church*, while the "nave" is *the main congregational area of a church*.

Let's take a look at another example.

> Although many religions believe in some form of immortality, it is important to
>
> recall the prominent place of <u>metempsychosis</u> in many world religions, for Western
>
> thinkers can believe that all theories of immortality only have to do with the soul
>
> outlasting bodily existence.

It's not very likely that you're familiar with the word "metempsychosis," but the sentence gives

us enough clues to form a solid inference about what it means. Unlike in the last example, where movement from one unknown term to another was involved, this term is involved in the logic of the sentence. This allows us to use meaning-laden words, like conjunctions, to figure out where the term fits in the author's discussion. Take a look at the sentence broken into phrases and clauses with these terms bolded.

Although many religions believe in some form of immortality

it is important to recall the prominent place of _____ in many world religions

for Western thinkers can believe that all theories of immortality only have to do with the soul outlasting bodily existence.

You'll notice we've left out the actual word we're attempting to define. If staring at an imposing word you don't know is making you nervous, pretend it's a blank! After all, we're drawing our information from the surrounding sentence, not from the word's component parts.

The next step is to analyze the sentence logic: the introductory phrase is contrasting "some form of immortality" against "metempsychosis." This could imply that "metempsychosis" is either the opposite of immortality or a type of immortality. Let's keep reading to get some more detail.

The author also states that "metempsychosis" is an important part of many world religions, and that we should keep this in mind because the Western perspective can assume that all theories of immortality "only have to do with the soul outlasting bodily experience." There it is! Subtly, there's enough evidence for us to figure out what is meant by "metempsychosis." It has to be a theory of immortality that isn't the soul outlasting bodily experience, since we're told to keep it in mind and that most Western thinkers assume that every theory of immortality "[has] to do with the soul outlasting bodily experience." This conclusion may be less specific than our previous one, but the question may only require a general understanding of what is meant by the term in question—and our logic is correct: "metempsychosis" is the belief in reincarnation (or the "transmigration of souls") in contrast to the theory of everlasting life after death.

Let's tackle one more example sentence.

The acceptance of the heliocentric model of the solar system greatly shook up

religious sensibilities, which strongly held to the view of Earth as the center of the

universe.

Like the last one, this sentence makes meaning through contrast, and by looking at the grammatical "scaffolding," we can figure out what the meaning of "heliocentric" is, or at least specifically what it is not. Knowing that a word is some sort of opposite of a different term in a sentence is just as valuable as knowing what it means, at least when it comes to answering questions

that ask you to pick out its meaning from several options. Whether your approach results in positive or negative characterizations, it can still help you pick out the correct answer.

In this sentence, "heliocentric" is specifying a particular "model of the solar system." What did this particular model do? Acceptance of it "greatly shook up religious sensibilities." Why is that? We're not told why in a direct manner (e.g. "because . . . "), but those "religious sensibilities" that were shaken up by the heliocentric model are those that "strongly held to the view that the Earth is at the center of the entire universe." Aha! So, if the acceptance of another model shook up religious sensibilities that thought that the Earth was the center of the universe, it's logical to infer that whatever "heliocentric" means, the "heliocentric" model of the solar system is one in which the Earth is not the center of the universe. This is indeed the case: the heliocentric model of the universe posits that the sun is the center of the solar system instead of the earth.

Defining Creative Uses of Terms

Technical vocabulary aside, words can be used in a myriad of different ways, and knowing the dictionary definition of a term can offer little help when an author is using the term in a unique way, such as in a metaphor or other literary devices. For instance, although there is such a thing as a "red herring," this expression means vastly different things in the following two sentences:

> The old men had long harvested colonies of red herring to feed the indigent residents of the poor island.

> John was aware that Peter's objection was a mere red herring, attempting to mislead the people following his argument.

In the first sentence, the word merely indicates a kind of fish—nothing else! In the second case, however, it is a metaphorical expression for using an argument (or other rhetorical device) to trick your listeners into arriving at a wrong conclusion regarding the topic under discussion. You could figure out these meanings using the rest of each sentence, respectively: the "red herring" in the first sentence must be something edible, as the old men are collecting it "to feed" residents. In the second sentence, "red herring" can't be edible, as it's used to characterize a type of "objection" in that it "attempt[s] to mislead the people following [an] argument." Be on the lookout for terms being used creatively—especially if they seem like words with which you're already familiar! Why would the GRE test the meaning of a term that means exactly what most test-takers think it means? In situations like these, it's prudent to be suspicious and to take a bit of extra time examining the passage to make sure you're reading it correctly.

Passage-Level Context Clues

A word's relevant context isn't always found in the same sentence as the unknown term; it could be located elsewhere in the passage. Next, we'll illustrate the process of interpreting words based on their context in a full passage. The following passage is adapted from "Robespierre" in John Morley's *Critical Miscellanies* (1904). Its tone and style are byzantine, making it a difficult read.

"Of all the men of the Revolution, Robespierre has suffered most from the audacious idolatry of some writers, and the splenetic impatience of others. M. Louis Blanc and M. Ernest Hamel talk of him as an angel or a prophet, and **the Ninth Thermidor** is a **red** day indeed in their **martyrology**. Michelet and M. D'Héricault treat him as a mixture of Cagliostro and Caligula, both a charlatan and a miscreant. We are reminded of the commencement of an address of the French Senate to the first Bonaparte: 'Sire,' they began, 'the desire for perfection is one of the worst maladies that can afflict the human mind.' This bold aphorism touches one of the roots of the judgments we pass both upon men and events. It is because people so irrationally think fit to insist upon perfection, that Robespierre's admirers would fain deny that he ever had a fault, and the tacit adoption of the same impracticable standard makes it easier for Robespierre's wholesale detractors to deny that he had a single virtue or performed a single service. The point of view is essentially unfit for history. It is folly for the historian, as it is for the statesman, to strain after the imaginative unity of the dramatic creator. Social progress is an affair of many small pieces and slow accretions, and the interest of historic study lies in tracing, amid the immense turmoil of events and through the confusion of voices, the **devious** course of the sacred torch, as it shifts from bearer to bearer. And it is not the bearers who are most interesting, but **the torch**."

In this passage, there are five words and phrases to consider. Let's treat them in the order in which they appear—which brings up another good piece of logic that you don't want to forget when defining words based on context: the context on which a word depends for its meaning to the reader is far more likely to appear before the word in question than after it in the passage. When a reader encounters the word in question, he or she only has what the author has stated thus far to go on—the reader hasn't gotten to the rest of the passage yet. In the interest of being intelligible, then, it benefits the author if he or she can embed enough context clues in their work to make sure the reader understands what they're saying when they use a particularly difficult word or use a more familiar word creatively. This may seem like an exceedingly obvious point, but it can be easy to forget the third or fourth time you re-read a passage on test day, and thus warrants explicit attention.

The first underlined phrase we encounter, "Ninth Thermidor," is a good example of historical jargon. In order to interpret this word, it is easiest to use the next two bold words, namely "red" and "martyrology." The three appear in a single sentence, so its highly likely that their meanings are associated. The latter word contains "martyr," so perhaps you can begin to interpret its meaning. The suffix "-ology" might be of help, too. What other words end in "-ology"? "Geology," "paleontology," "biology," "sociology," "psychology"—and what do these words have in common? They're all academic fields that study certain things. So, "martyrology" likely has something to do with the academic study of martyrs. A martyr is a person who sacrifices him- or herself for his or her cause. That's quite a bit of information; let's see how it lines up with the other underlined terms in the sentence.

If you read "martyrology" in the context of "red day," you can begin to guess at its meaning. "Red" is being used in a negative way, underscored by that emphatic "indeed" and somewhat suggested by contrast with the overwhelmingly positive connotations of "talk of him as an angel or a prophet." The definition of "martyr" has to do with people dying, so it's likely that "red" is sharing in that meaning somehow here, indicating that the given day was a very bloody day. The use of red is thus likely metaphorical for indicating a day on which a martyr's death was very important and heinous in the eyes of his or her followers.

The meaning of "red" and "martyrology" combine to suggest that we're discussing a particular martyr, and there's only one logical conclusion of whom that might be: Robespierre. Was Robespierre a martyr in the religious sense of the word? Not at all. But the passage is calling him a martyr all the same. From all of these kinds of thoughts, we can begin to realize what is meant by the "Ninth Thermidor." This is the name of the day when Robespierre was overthrown. (Revolutionary France had created a new calendar, hence 'Thermidor' was the month in question.) The author is writing about how Robespierre garners various reactions from writers. In this sentence, Morley is stressing that for some people, the overthrow (ultimately leading to Robespierre's execution) was like the martyring of a prophet or an angel. Hence, this event would be seen as being very "red"—a dreadful martyring of the great Robespierre.

The final two underlined terms can be understood by looking at the sentences in which they are contained. The word "devious" does not only deal with being *wicked* or *crafty*, a meaning you might reflexively assume it is using. It can also be used to indicate something that is *indirect in its way of proceeding on a given course*—as when someone "deviates" from a path. Be careful that you don't fall into the trap of choosing an answer choice just because it has a familiar and valid meaning of the word listed. You're tasked with picking out the meaning of the word as it's used in the passage. Words might be used creatively by authors so that they convey meanings not listed in the dictionary definition at all, as in the next example.

In the passage's last few sentences, the "sacred torch" is the basic narrative of history and social progress. How do you figure that out? Well, we're told that "social progress is . . ." and then the author conveys that it is the result of lots of small events and actions—pieces of a puzzle, perhaps,

or the trees in a "forest and trees" perspective. Once you figure out that "the bearers" are associated with the small-scale actions ("the trees"), it becomes apparent that "the torch" is associated with the abstract general results ("the forest"). Because the sentence started with "social progress," it makes sense to infer that this is what is meant by "the torch." Hence, history is said to be most interesting for its study of tracing the "path" of the overall story of history, not the particular facts and persons along the way. Hence, "devious" just refers to the shifting path of history, and "the torch" refers to history itself as it progresses onward.

A Word on Question Types

Did you notice another example of word use in context? Directly before the passage citation above, the word "byzantine" was used. Did you know its meaning? As an adjective, it is not limited to describing the Eastern half of the Roman Empire. It can also mean *too complex or ornate*—describing not only buildings but really anything that is so complicated that it is overwhelming. At first glance, many of the terms and passages the GRE Verbal section presents you with may seem byzantine and beyond your mental reach. The mere suggestion that you can't answer a question because you don't know a word's meaning parallels unfortunately neatly with a common concern when answering GRE Verbal Sentence Completion and Sentence Equivalence questions; however, the test does not expect you to know the meaning of the words it calls your attention to in passages and then asks you to define. It does expect you to piece the meaning of a word together from the rest of the information around it. While this information can often be complex and subtle, the test has to give you some context clues to work from for these types of questions. If after the first read-through of a passage, you find that you still have no idea what an indicated term means, read it again, more slowly, paying particular attention to the words and sentences immediately surrounding the term in question. With practice, you'll be able to sift the relevant connotations, literary devices, and vocabulary associations from the passage and use them to define the indicated term.

Adapted from "Robespierre" by John Morley (1904)

Of all the men of the Revolution, Robespierre has suffered most from the audacious idolatry of some writers, and the splenetic impatience of others. M. Louis Blanc and M. Ernest Hamel talk of him as an angel or a prophet, and the Ninth Thermidor is a red day indeed in their martyrology. Michelet and M. D'Héricault treat him as a mixture of Cagliostro and Caligula, both a charlatan and a miscreant. We are reminded of the commencement of an address of the French Senate to the first Bonaparte: 'Sire,' they began, 'the desire for perfection is one of the worst maladies that can afflict the human mind.' This bold aphorism touches one of the roots of the judgments we pass both upon men and events. It is because people so irrationally think fit to insist upon perfection, that Robespierre's admirers would **fain** deny that he ever had a fault, and the tacit adoption of the same impracticable standard makes it easier for Robespierre's wholesale detractors to deny that he had a single virtue or performed a single service. The point of view is essentially unfit for history. It is folly for the historian, as it is for the statesman, to strain after the imaginative unity of the dramatic creator. Social progress is an affair of many small pieces and slow accretions, and the interest of historic study lies in tracing, amid the immense turmoil of events and through the confusion of voices, the **devious** course of the sacred torch, as it shifts from bearer to bearer. And it is not the bearers who are most interesting, but **the torch**.

Sample Question #1

Which of the following is closest to the meaning of the underlined word "fain" in context?

- **A.** Happily
- **B.** Begrudgingly
- **C.** Factually
- **D.** Despite that

When the author uses the word "fain," he is discussing two groups of people: those who admire Robespierre and those who dislike him. He's painting these groups as being single-minded in their perspectives: the people who like Robespierre "insist upon perfection," and those who don't like him "deny that he had a single virtue or performed a single service." "Fain" precedes "deny he ever had a fault," and our subject at this point is "Robespierre's admirers." They might realistically "deny that he ever had a fault." So, it doesn't make sense for "fain" to mean "begrudgingly" (B) or "despite that," (D), as each of these answers contrasts Robespierre's admirers against their actions in a way that just isn't present in the sentence. A few hints suggest that the perspective of Robespierre's admirers is unrealistic— "irrationally" describes people "insist[ing] upon perfection," and later, it's referred to as "an impracticable standard." So, it doesn't look likely that "fain" means "factually" (C). This leaves us with one remaining answer, "happily" (A). This makes sense: if someone were a big fan of Robespierre, he or she might be enthusiastic about denying that Robespierre ever had a fault.

"Wholesale" appears in the same sentence as "fain," near the end of it when the author is discussing those people who don't like Robespierre:

"It is because people so irrationally think fit to insist upon perfection, that Robespierre's admirers would fain deny that he ever had a fault, and the tacit adoption of the same impracticable standard makes it easier for Robespierre's wholesale detractors to deny that he had a single virtue or performed a single service."

Paraphrasing might be handy here: the author is saying that because some people insist on claiming that Robespierre was perfect and had no faults, that makes it easier for other people who don't like Robespierre (his "wholesale detractors") to "deny that he had a single virtue or performed a single service." Both groups discussed are viewing Robespierre in a binary fashion. Skimming the answer choices, you can notice that two of them (A and B) have to do with selling things—neither of those can be correct, as this passage isn't describing selling anything. "Cheap and low-quality" (D) doesn't make sense to describe the "detractors," especially as the author doesn't seem to be favoring one perspective above the other, but pointing out a flaw that both have. This leaves us with (C), "comprehensive and total." This is the best answer: any "detractor" of Robespierre who claims that Robespierre never "had a single virtue or performed a single service" could be described accurately as "comprehensive and total" in the sense that their detractions are "comprehensive and total"— metaphorically black and white, without any grey.

> ## Sample Question #2
>
> Which of the following is closest to the meaning of the underlined word "wholesale" in context?
>
> A. Sold in large quantities
> B. Sold to someone who will then sell the item again to the consumer
> C. Comprehensive and total
> D. Cheap and low-quality

> ## Sample Question #3
>
> Which of the following is closest to the meaning of the underlined word "service" in context?
>
> A. A positive, helpful action
> B. Action that relies on skill and is provided in exchange for money, as opposed to tangible good
> C. A scheduled religious gathering
> D. The military in general

"Service" appears at the end of the sentence on which the previous questions focused:

"It is because people so irrationally think fit to insist upon perfection, that Robespierre's admirers would fain deny that he ever had a fault, and the tacit adoption of the same impracticable standard makes it easier for Robespierre's wholesale detractors to deny that he had a single virtue or performed a single service."

There's some parallelism going on here! "Performed a single service" is placed in parallel with "had a single virtue." Thus, since "hav[ing] a . . . virtue" is a positive thing, "perform[ing] a . . .

service" must be a positive thing in this context too, too. None of the answer choices is particularly negative, though, so let's go over each. There's no mention of the military (D), any religions (C), or economic transactions (B). A is the best answer, and it's not surprising that it has the strongest positive connotation.

Working with a phrase opens up the author's opportunities for creative but comprehensible language use, so it's important that you parse each individual word. The phrase "imaginative unity" appears after the author discussed the "wholesale" positive and negative perspectives people have on Robespierre, afterward saying, "The point of view is essentially unfit for history. It is folly for the historian, as it is for the statesman, to strain after the imaginative unity of the dramatic creator." Where does this get us? The "imaginative unity" is specifically that "of the dramatic creator." "Dramatic" here is used for its association with the theatre ("drama"), so a "dramatic creator" is someone who creates a drama like a play—you can shorthand "dramatic creator" as "playwright." So, what kind of "imaginative unity" might a playwright have in writing a play that a politician ("statesman") or historian would lack? There's no mention of collaboration (A) or unlikely cooperation (B). Multiple authors (D) aren't being discussed either, just a hypothetical, general playwright, historian, and statesman. This leaves us with (C), the correct answer. The author is stating that it's absurd for a historian to try to write history like a playwright writes a play. In a play, scenes can be written so that they come together to form a whole that addresses particular themes or triggers a particular emotional reaction in the audience—the parts can be designed to function as a whole in some way. History is different. The author is saying that seeing a figure like Robespierre as wholly good or bad is inaccurate and a perspective not suitable when writing history, while it might work in a creative work.

Sample Question #4

Which of the following is closest to the meaning of the underlined phrase "imaginative unity" in context?

A. The collaboration of multiple people on a single creative endeavor

B. Highly unlikely cooperation between arguing factions

C. The constructed wholeness of a creative work designed to accomplish particular rhetorical goals

D. A state in which many authors concern themselves with a given idea or set of ideas particularly relevant to current events

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Summary and Paraphrase

Just as you need to process food in order to make it part of your body, so too must you process a passage in order to understand its meaning. One way of processing a passage is making sure you can restate its points in your own words (paraphrase it) or encapsulate the passage's entire message and argument in a short statement (summarize it). These skills are particularly useful at sketching out the underlying structure of a text. Certain GRE Verbal questions may test these skills directly, and each can be a great boon to employ as a self-check while reading a passage to interrogate your understanding of it. Practicing these two skills explicitly before test day can help you readily and immediately distill passages summaries and paraphrases from passages in an exam environment.

Parapharsing vs. Summarizing

Paraphrasing and summarizing are similar, though not exactly equivalent, tasks. The main difference is in the material that each skill treats as relevant: when paraphrasing, it is important that you retain the exact sense of the specific text, down to the level of wording. A paraphrase that leaves out or doesn't capture some key facet of the original sentence is a subpar paraphrase. In contrast, summarizing requires you to distill the most important points from a larger selection of text, omitting anything that is not key to its main point. A summary that conveys every point and detail made in a given selection is a subpar summary. Because of its focus on relatively short amounts of text, paraphrasing requires you to pay attention to retaining the specific content of the sentence(s) at hand, whereas in summarizing, you are working with lots of material and should instead make sure your statement refers to all of the major units (e.g. paragraphs, sentences) that you are being asked to encompass while not being too general in your approach.

Paraphrasing: A Play-by-Play

Passage Excerpt	Paraphrase
When in the Course of human events,	If
it becomes necessary	a group
for one people	must
to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them,	declare itself to be politically independent from another group, the acting group should
a decent respect to the opinions of mankind requires that they should	explain why this action is necessary.
declare the causes which impel them to the separation.	

The difference in the content relevant to each skill creates a natural division in the scope of questions testing them. The detail-oriented nature of paraphrasing means that you'll usually be asked to paraphrase a sentence or a short paragraph, while the broader focus of summarizing means that summarizing questions might span from a few sentences to a whole paragraph or even the entire passage.

Summarizing: A Play-by-Play

Passage Text	Main Point Summary	Passage Summary
When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.	A group declaring its political independence from another group should explain what motivated it to do so.	We the Colonies are cutting political ties with Britain because we believe government should uphold certain natural rights or be abandoned, and the British King has violated these rights repeatedly.
We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness.--That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, --That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their future security.--Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States.	We believe that the purpose of government is to uphold certain natural rights. If a government does not do this, it should be abolished and a new one set up that accompilshes this goal.	
	Government shouldn't be altered on a whim, and people tend to bear symptoms of a poor government instead of dealing with the root cause. But at a certain point, when too many rights are curtailed, people must change a bad government.	
	The preceding general description applies to the relationship between Britain's colonies and its king.	

Let's practice applying these skills to the following passage in which one sentence is underlined and another is in bold.

It is in business' best marketing interests to emphasize a similarity in values shared between corporation and consumer, so it comes of little surprise that a consumer trend toward sustainable, "green" products has been accompanied by an ugly shadow: "greenwashing." Originally coined in a 1986 environmental essay by Jay Westervelt, we define the term to indicate the practice of making specious claims about the environmentally responsible nature of certain of a company's policies, services, or products. <u>Due to the marked lack of regulation of advertising claims about the "greenness" of a product, greenwashing has diverted funds away from the pursuance of legitimate environmentally beneficial changes to advertising sometimes nonexistent and other times laughably insignificant practices, as a competitive market forces businesses to prioritize seeming green over actually being green.</u> **Consumer response to a glut of false claims forms another symptom of "greenwashing": buyers are less apt to put any stock in a product's claims at all, and as a result, have no way to reliably differentiate between "actually green" and "seeming green" without a great deal of research—research highly unlikely to be done in the aisles of a supermarket or department store, even with the ubiquity of mobile devices.** Regulating verifiable claims to a product's greenness can help us take a step in the right direction, but consumer trust in such claims may take much longer to recover after the problem begins to be addressed.

The underlined sentence is more easily summarized. In order to produce a summary, however, you need to note all of the pertinent details in the selection in question. To do this, quickly make a list (either on paper or in your mind):

- There is a lack of regulation regarding advertised claims of "greenness."
- This has led to "greenwashing."
- This diverts funds from useful environmental practices.
- It does so because it prioritizes merely seeming to be green.

You could summarize these details as follows: "A lack of regulation has led to the prioritization of merely seeming to be green over actually being so, thus diverting money from legitimate green practices." This is a kind of paraphrase, but the general tone is that of summarizing a position, not processing it into your own words.

For the bolded selection, a paraphrase is perhaps more helpful. Once again, you should process the details of the sentence first:

- "Greenwashing" leads to certain consumer responses.
- Buyers come to disbelieve all claims of "being green."
- They do not differentiate between true and false claims of "being green."

Based on these gathered details, you could paraphrase this lengthy sentence into your own words as follows: "Greenwashing also negatively affects the attitude of consumers, who become cynical and thus ignore all claims regarding all claims of 'being green,' which are difficult for them to verify when purchasing products." This paraphrase correctly emphasizes that it is "the consumers" who are negatively impacted.

What if we wanted to summarize the entire passage? Taking a step back to consider its main topics is a good place to start; after that, you can consider how the topics relate to one another. It's a business passage focused on "greenwashing"—it defines this process as a type of problem-causing behavior and then describes two negative effects it has had on the market and consumers. From that, we could summarize the passage thusly: "The false labeling of consumer products as environmentally friendly, "greenwashing" has created competition in which seeming green is more important than being green, leading to faltering consumer trust." When considering a summary statement, make sure that some aspect of it relates to each part of the passage; you don't want to leave any big concepts out.

Next, let's try some summarizing and paraphrasing practice questions based on the following passage, which is adapted from John Stuart Mill's *The Subjection of Women* (1869).

> The very words necessary to express the task I have undertaken show how arduous it is. But it would be a mistake to suppose that the difficulty of the case must lie in the insufficiency or obscurity of the grounds of reason on which my convictions. The difficulty is that which exists in all cases in which there is a mass of feeling to be contended against. So long as opinion is strongly rooted in the feelings, it gains rather than loses instability by having a preponderating weight of argument against it. For if it were accepted as a result of argument, the refutation of the argument might shake the solidity of the conviction; but when it rests solely on feeling, worse it fares in argumentative contest, the more persuaded adherents are that their feeling must have some deeper ground, which the arguments do not reach; and while the feeling remains, it is always throwing up fresh entrenchments of argument to repair any breach made in the old. And there are so many causes tending to make the feelings connected with this subject the most intense and most deeply-rooted of those which gather round and protect old institutions and custom, that we need not wonder to find them as yet less undermined and loosened than any of the rest by the progress the great modern spiritual and social transition; nor suppose that the barbarisms to which men cling longest must be less barbarisms than those which they earlier shake off.

Sample Question #1

Select the passage excerpt best paraphrased by the statement "people often refuse to see reason when arguing a point based on emotional evidence."

- **A.** "So long as opinion is strongly rooted in the feelings, it gains rather than loses instability by having a preponderating weight of argument against it."
- **B.** "And there are so many causes tending to make the feelings connected with this subject the most intense and most deeply-rooted of those which gather round and protect old institutions and custom . . ."
- **C.** "The difficulty is that which exists in all cases in which there is a mass of feeling to be contended against."
- **D.** "[I]f it were accepted as a result of argument, the refutation of the argument might shake the solidity of the conviction"

Always evaluate each option presented for a given question. Don't just jump to the correct answer.

A. This could be summarized as saying, "If an opinion is rooted in feelings, then it gains instability by having arguments against it." Now, in context, notice that the author continues by saying that if the opinion were held based upon an argument, a counter-argument would shake the conviction; however, the "instability" introduced actually leads the emotional person to think that there is a deeper (emotional) reason for the argument. So, the instability actually makes it stronger by making them ignore the counter argument, looking for a deeper emotional reason—and thus not listening to the logical reasons provided. This makes option A a very tempting correct answer. (As you will see below, it is correct, but we must continue on through the other options—just in case!).

B. This sentence could be summarized as saying, "Their feeling-based convictions have many causes, the strongest of which are old institutions and customs." This is focusing on the causes of the given emotion-based conviction.

C. This brief sentence is just stating a fact, namely that the difficulty of Mill's project in this passage is like any other case where there is much feeling attached to some idea or cause.

D. This sentence is saying that if someone has an opinion based on argument, it is likely to be refuted by argument. Be careful, notice that this is not speaking about how people who hold opinions based on emotions (though it is implying that this phenomenon exists).

Based on all of these options, A is the best. You needed more context, but the point was clear: people who have emotion-derived arguments don't listen to reason and actually become more entrenched in their emotional opinions in such cases!

Sample Question #2

Which of the following best summarizes the passage excerpt given below?

"For if it were accepted as a result of argument, the refutation of the argument might shake the solidity of the conviction; but when it rests solely on feeling, worse it fares in argumentative contest, the more persuaded adherents are that their feeling must have some deeper ground, which the arguments do not reach; and while the feeling remains, it is always throwing up fresh entrenchments of argument to repair any breach made in the old."

A. People who can distinguish their arguments from their emotions are generally more fervent in defending them.

B. We should only accept arguments that incorporate both emotion and logic.

C. While beliefs based on logical reasoning can be corrected, it is very difficult to change beliefs founded on emotion.

D. People who argue from an emotional basis rarely change the opinions of those who make logical arguments.

For this kind of question, begin by gathering a quick outline of details:

- If the opinion were based on argument, it would be shaken by counter-argument.
- When based on emotion, such counter-arguments seem to show that there is a deeper emotional reason for holding the conviction.

Now, consider each option:

A. The passage in question is not about those who can distinguish arguments from emotions. Indeed, it is about people who completely confuse the two and hold on to the emotional underpinning with great fervor.

B. The passage in question is only talking about the response of how a person holding an emotion-based argument responds to a challenge. It says nothing about why we should hold arguments based on both emotion and logic.

C. This option seems most promising. It is saying in brief what we noted above in our two points above.

D. This option is the complete opposite of the passage provided. The passage is talking about how logical arguments are basically useless against those who hold arguments on an emotional basis, not vice-versa.

Thus, as discussed above, option C is the best provided. Luckily, it is a rather clear choice, as should be evident from the discussion above!

Inferences, Conclusions, and Applications

A close reading of a passage will not only reveal what the passage says: it will also reveal what it does not say. That is, it will reveal the implications of the passage. Sometimes, a small word or expression such as "only now" or "formerly" will indicate that the state of affairs changed at some point in time. For example, a sentence may read, "Formerly, computer science had been seen as an area to be studied only by specialists." Even if the passage continues to discuss this former time when computer science was not taught in detail to non-specialists, the passage will have embedded in its narrative a hint that things are no longer like this. You can infer that, according to the passage, many courses of studies now include computer science studies. You likely can think of parallel examples for the expression "only now."

Likewise, a passage may wish simply to imply possible important second-order effects (or results) of whatever is being discussed. For instance, you may be presented with a lengthy discussion of the development of targeted advertising. The passage may list several examples of new advertising techniques and how they are implemented; however, in passing, it might state, "Such subtly influential techniques shape consumers' wants and desires." While such a remark is just a minor aside in light of the passage's main discussion, it implies further issues worthy of discussion. Indeed, perhaps the passage explains how social networks advertise based on one's network of friends, while search engines advertise based upon one's political affiliation. The passage may thus imply that the social networks influence your wishes in one way (i.e. by making them more like those of your friends), while the search engines influence your desires in a manipulative way based solely on the secret planning of advertising companies and corporate overlords. Of course, this example is not meant to implicitly support such theories; it is chosen merely to give you an example of how one little remark may draw your attention to subtle details that could carry forward further implications.

Likewise, a passage may discuss a historical figure, noting certain aspects of his or her character. Perhaps some events are mentioned in which he or she acted in a particular way, or, perhaps, his or her writings are mentioned. Based on these remarks, you may be asked to make inferences about what that figure would think about some other topic. For example, consider the following remark from J. S. Mill in his work *On Liberty* (1859): "It is not by wearing down into uniformity all that is individual in themselves, but by cultivating it and calling it forth, within the limits imposed by the rights and interests of others, that human beings become a noble and beautiful object of contemplation." You may then be asked what Mill would think about the proliferation of new kinds of music. From what he has said, you can likely guess that he would embrace such things as expressions of human individuality.

Let's work through a sample passage and an associated question, paying particular attention both to what the passage directly states and what it implies. You'll need to recognize both types of information in order to answer the sample question.

Scientists now hypothesize that the universe contains mostly dark matter. Studying matter not visible to the naked eye is inherently problematic, but dark matter actually absorbs light, presenting a significant problem to research: how do you study something invisible, much less something invisible millions of lightyears away?

Scientists can map out the hypothesized locations of dark matter by observing the gravitational lensing of galaxy clusters. Gravitational lensing is an effect of general relativity articulated by Einstein in 1936. If light is traveling from a distant star to your eyes observing the star on Earth, things can get in the way of the traveling light, and due to their mass, can act as a lens and bend the light. As a result of this bent light, the observer may see multiple images scattered around the light source. The number of images the observer sees and how complete they are depend on the relative orientation of the bodies involved: the closer the lens is to the source, the more complete the images the observer sees. Thus, once it is known where the observer is standing and where a light source is located, one can determine the location of the lens—whatever is making the light bend. By analyzing images of galaxy clusters in which no observable matter accounts for their light's bending, scientists can map out the hypothesized location of dark matter.

Sample Question #1

Astronomer A is studying a galaxy cluster. She sees three complete images of the cluster projected around a light source. Astronomer B, another scientist in the same lab, is studying a different galaxy cluster. She sees only one incomplete image of the cluster projected near a light source. Based on the passage, what can the scientists conclude?

A. Astronomer B's galaxy cluster is located nearer to a light source than Astronomer A's cluster is to a light source.

B. Astronomer A's galaxy cluster is closer to Earth than Astronomer B's galaxy cluster.

C. The astronomical object bending the light around Astronomer A's galaxy cluster is located nearer to a light source than is the astronomical object bending the light around Astronomer B's galaxy cluster is to a light source.

D. Nothing can be definitively concluded about the relative locations of the astronomers' galaxy clusters based on this observation.

In order to identify nuanced information in a passage, we will need to pass through the text with a "fine-toothed comb." Begin by noticing that the first sentence says that scientists have now formed their hypotheses about dark matter. This implies that they did not do so before. In the second sentence, the author notes that it is very difficult to see dark matter its distance and characteristic of light absorption. Thus, the implication is that researching dark matter is perhaps even more difficult than researching very tiny or very distant things, both of which would be invisible to the naked eye— but at least would not also absorb light!

The second paragraph focuses primarily on the use of gravitational lensing and what it means. It is not very important for understanding this passage that Einstein articulated one of the theories discussed. Notice the fact that the third sentence uses the word "lens" to help explain the phenomenon rather exactly. The point is that gravity can create a kind of bend in light traveling through space—as a lens does with light that is traveling to one's eyes. In the next sentence, the phenomenon is described. Even if you are not completely sure what is meant by the description, you can grasp the general idea.

Somehow, this bending light makes multiple images appear around whatever is the ultimate source of the light. Then, the author explains one aspect of how lensing creates this altered appearance. He or she focuses on the effect of the proximity of the "lens" (or, better stated, "lensing matter"). If the lens is close, then the multiple images will be more complete copies. This also implies that if the lens is less close, the images will be less complete. Also, notice that the author does not say anything about the "number" of such images. The implication is that there will be more images that are also more complete; however, we really can't be sure of that based on the passage. (In the question discussed below, however, this will become clearer.) Notice, of course, that this requires that one know the distance between the observer and the light source. Once these facts are established, then the location of the lens—e.g. of the dark matter—can be calculated. It is important to note that this will also tell one how far the dark matter is from the light source. (This is an obvious point, given what is discussed throughout the passage; however, you must pay attention to it in order to answer the question below correctly.) Hence, we can well understand why the passage has focused on the idea of lensing. It is the way by which otherwise "invisible" patches of matter can be detected.

Now, let's apply this close reading to an example question, like the one provided with this passage. Notice that this question requires an application of the information from the passage to a possible scenario. Having closely noted all the points in the passage, it will be easy to ascertain the correct answer. Notice that the two astronomers are in the same lab. Therefore, we know that they are observing their phenomena from the same position. This will help to simplify the question. Because Astronomer A sees more and complete images, the lensing body must be closer to her particular galaxy cluster. The less complete view seen by Astronomer B indicates that there is more distance between her galaxy cluster and the lensing body associated with it.

This scenario is only described by answer option C. Option A is not acceptable because the distance discussed in the passage is the distance between the lensing source and the light source, not

between the cluster (or whatever object) and the light source. Likewise, B is not acceptable because the lensing has nothing to do with the distance between the observer and the thing observed. (Once again, it is about the distance between whatever matter is bending the light and the light source—whether that is the galaxy cluster itself or some other light source.) Based on what the passage has said, the two clusters may be equally distant from the observers. The images, however, can differ based on the placement of the gravity-causing bodies that bend the light. Finally, D is not acceptable precisely because C is fine!

Let's try answering a few more sample questions about the same sample passage.

Remember the author's unusual use of the words "lens" as a verb? Time to pinpoint those uses and investigate them! Unique uses of terms or neologisms may require that an inference be made in order to figure out what the author means when he or she uses those words.

The author says that certain things can get in the way of traveling light and "due to their mass, can act as a lens and bend [it]." The "lens" is the matter doing the bending. Thus, we can conclude that in the phrase "gravitational lensing," "lensing" means "bending," so B is correct.

Sample Question 3 directs us to a sentence chock-full of inference clues—so full that it asks us not about what we can infer, but what we cannot. Let's break down its logical moves:

1. Investigating matter you can't see is difficult.

2. Dark matter absorbs light

3. This makes dark matter even harder to study than matter you merely can't see because:

a. Dark matter is invisible.

b. Dark matter is also very far away.

Sample Question #2

In the phrase "gravitational lensing," "lensing" most nearly means which of the following?

A. Orbiting
B. Bending
C. Mapping
D. Absorbing

Sample Question #3

Which of the following CANNOT be inferred from the following sentence?

"Studying matter not visible to the naked eye is inherently problematic, but dark matter actually absorbs light, presenting a significant problem to research: how do you study something invisible, much less something invisible millions of lightyears away?"

A. Dark matter is not visible to the naked eye.
B. Dark matter is invisible.
C. If something absorbs light, it is invisible.
D. If something absorbs light, it is dark matter.

Look at that—B certainly isn't the answer, as we came to that conclusion based on our initial para-phrasing. If we can infer B, we can also infer A, since anything that is invisible is not visible to the naked eye.

We just need to decide whether C or D is the option that isn't supported by the sentence. On the plus side, we know that the answer we don't choose *will* be supported by the text, so having narrowed down our options to two choices, we can approach the problem from either perspective. At this point, we need to take a very close look at the relationship the author posits between dark matter, invisible things, and things that absorb light. The relevant section of the passage is this selection:

> . . . but <u>dark matter</u> actually <u>absorbs light</u>, presenting a significant problem to
>
> research: how do you study <u>something invisible</u>, much less something invisible millions
>
> of lightyears away?

The author is using a complex sentence structure, but notice how dark matter's absorption of light is paralleled with "something invisible." The fact that dark matter absorbs light is presented as the cause of the significant problem to research, which, upon elaboration, is defined as dark matter being invisible and very far away. Well, dark matter's ability to absorb light has nothing to do with it being far away, so its ability to absorb light must be the cause of its being invisible. This means that we can infer that if something absorbs light, it must be invisible (C). If this answer choice can be supported, the correct answer—the one that can't be supported—must be D. Let's check to make sure. D states, "If something absorbs light, it is dark matter." This isn't necessarily true—the passage presents dark matter as an example of one thing that absorbs light, but nowhere does it claim that dark matter is the only thing in the universe that absorbs light. Regardless of the validity of this statement, it is not supported by the passage, so D is, indeed, the correct answer.

Authorial Choices: Text Structure

Linguistics considers language as a combination of "semantics," or what words mean, and "syntax," the order of words as presented in sentences and sentences as presented in paragraphs. While much of the GRE's Reading Comprehension is focused on semantics, some of its questions may ask you to take a critical look at a passage's syntax and comment on what exact effects it is having on readers. The order of the author's words can help underscore certain points and distract from others; rhetorical moves like parallelism and repetition can spotlight the author's arguments, whereas the exact moment at which an author presents an opposing argument might be designed to distract you from its merits. In this lesson, we'll learn how to take a critical look at passage structure and see how the simple order of words in a sentence can create subtle connotations.

Structural analysis is explicitly concerned with how each part of the passage, whether at word-, phrase-, sentence-, or paragraph-level, immediately follows the part preceding it. A metaphor may help: consider each part of the passage as part of a linear progression of elements, like beads on a string. What do you as the reader get from the fact that this exact phrase starts the sentence? That it is followed by this other phrase? As you try to answer these questions, you might notice that some of the rhetorical "beads" a little ways apart are related in an interesting fashion. Great! Text structure claims don't need to involve immediately adjacent elements; they just need to concern parts of the passage that are near enough for the reader to recognize—consciously or subconsciously—as related.

Just as not every sentence you read contains a main idea, not all your structural observations are going to unearth rhetoric with significant meaning, e.g. the fact that a sentence begins with "the." But think of the effect of structure as a graph that can dip into positive or negative values: if the author is starting a sentence with "the," he or she is *not* bringing out any rhetorical acrobatics to grab your attention. Is he or she trying to slip the sentence by you, perhaps to get you to pay more attention to his or her thesis instead of an opposing claim? Observations of a lack of attention-grabbing structure can be just as useful as noting the presence of such elements.

An example of complex text structure might involve inversions in which the author begins by presenting the argument he or she will eventually dismantle. For instance, the author might say, "At first glance, philosophy appears to be quite a waste of time." The opening expression, "At first glance" hints that something more is coming, as does "appears," which introduces some qualification and suggests that there is a different reality beyond this initial surface appearance. The author might next cite several points that support this outlook, such as, "We all know stories of crazy philosophy professors, ranting and raving in class," and, "In contrast with the important advances in the fields of medicine, there has been little in the way of progress in philosophical squabbling." Without the initial phrase "At first glance," you might read these statements as the author's earnest view on the subject. Still, given that the author said, "At first glance," you realize that these examples are actually quasi-counter examples. You rightly should expect the author to change course and express why it is that philosophy is *not*, in fact, a waste of time. Thus, he or she may go on to defend the usefulness of logic or the importance of political philosophy for civic life.

Here are some examples of structural features that may take on significance in a passage.

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Potentially Significant Structural Elements

- Parallelism

- Repetition

- Use of foreign terms

- Inversion of typical sentence syntax

- Repetition of particular syntax in consecutive or nearby sentences

- Use of hypothetical scenarios vs. real scenarios

- Rhetorical and/or leading questions

- Location of claim qualification vs. claim

- Order of key passage elements: thesis, evidence, counterclaims

- Use of interrupting phrases

- Placement of conjunctions or transitional words and phrases

- Inclusion of quotations

- Use of optional grammar structures/marks, e.g. exclamation marks

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Consider the structure of the following passage, adapted from *The Common Law* by Oliver Wendell Holmes, Jr. (1881). We'll next read it closely, step-by-step, for structural significance.

[1] If it were necessary to trench further upon the field of morals, it might be suggested that the dogma of equality applied even to individuals only within the limits of ordinary dealings in the common run of affairs. [2] You cannot argue with your neighbor, except on the admission for the moment that he is as wise as you, although you may by no means believe it. [3] In the same way, you cannot deal with him, where both are free to choose, except on the footing of equal treatment, and the same rules for both. [4] The ever-growing value set upon peace and the social relations tends to give the law of social being the appearance of the law of all being. [5] But it seems to me clear that the *ultima ratio*, not only *regum*, but of private persons, is force, and that at the bottom of all private relations, however tempered by sympathy and all the social feelings, is a justifiable self-preference. [6] If a man is on a plank in the deep sea that will only float one, and a stranger lays hold of it, he will thrust him off if he can. [7] When the state finds itself in a similar position, it does the same thing.

Qualification and Authortiy	**1** *If it were necessary to trench further upon the field of morals, it might be suggested*
Claim about Applicability and Scope	**2** that the dogma of equality applied even to individuals **only** within the limits of ordinary dealings in the common run of affairs.
Evidence for Applicability (Not Scope!)	**3** (a) You cannot argue with your neighbor, except on the admission for the moment that he is as wise as you, although you may by no means believe it. In the same way, (b) you cannot deal with him, where both are free to choose, except on the footing of equal treatment, and the same rules for both.

1 The phrase before the comma suggests the author's argument thus far is both comprehensive and decided. This makes him look authoritative and sets up the next claim as superfluous. The author couches the claim itself in not one "if," but two, as "if it were necessary . . .," it "might be suggested." Note that all this qualification witholds the author's opinion on the claim and does not necessarily imply that it is correct: yes, someone might suggest it, but that doesn't mean that person is correct. This prevents him from looking contradictory later in the passage when he develops the claim he introduces here and opposes the developed claim.

2 After all that reservation and distancing, the author presents a claim. This claim isn't his thesis: it's just a starting point to walk the audience through steps he wants them to see before we get to his thesis. All that reservation in the introductory phrases helps make this apparent.

3 The author then presents evidence in support of the claim that equality only applies in social scenarios. He only provides positive examples; stronger evidence would be anything demonstrating that the principle of equality does NOT apply in a scenario that is not a normal social situation. Note how the evidence is presented: the author takes an example from daily life and states it in ironclad terms: you can ONLY do this IF you make this assumption. Thus, the reader, who may not have considered the assumption but who has presumably at one point argued with someone, thinks, "Well, since I've argued with people before, I must have made this assumption." Perhaps the sneakiest part of the rhetoric is the phrase "although you may by no means believe it." Put a different way, you don't have to agree with this point for your past actions to support it! Note how this phrase appears after the statement in question, when the reader might be suspicious of it, rather than before it, where it would prime the reader to be suspicious of what they read next.

Point (b) reprises not only a similar point, but similar syntax and logic: you cannot do X, except if you do or have done Y. The repetition of syntax gives it more oomph, and this similarity is emphasized by the phrase "in the same way." Here, form works to support content.

| Focusing on Scope; Weakening Claim | **4** | The ever-growing value set upon peace and the social relations tends to give the law of social being the appearance of the law of all being. |

5   **But**

it seems to me clear

(1) that the *ultima ratio*,

    [not only *regum*, but of private persons,]

    is force,

6   **and**

(2) that at the bottom of all private relations,

    [however tempered by sympathy and all the social feelings,]

    is a justifiable self-preference.

Thesis: Parallel Claims with Inverted Syntax

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4 Here, the author suggests that the scope of the first claim is often broadened, and then attacks that scope as overly broad. Note how the author introduces some reservation in the phrase "tends to give . . . the appearance of." This tells us that the claim we've been considering isn't precisely the author's opinion; this sentence sets up for the author's main claim by transforming the first claim into one against which the author can clearly contrast his own.

5 "But" acts as a hinge between the previous sentence's "it may seem" claim and the thesis that follows. The author limits his thesis to his personal perspective ("it seems to me"); this helps differentiate his thesis from the impersonal and ultimately (in his view) incorrect claim we've seen so far.

6 The structure of this passage's thesis is particularly complex. It is presented as two claims that employ parallel syntax: note how each follows the form "that the [noun phrase] (interrupting phrase) is [second noun phrase]." This repetition makes it appear as if the two facets of the thesis might support one another because they're so similar, but really, they're distinct but related points. The parallel statements' repeated syntax is inverted, saving the most important and surprising elements ("force" and "justifiable self-preference") for the end of the sentence, where they gain more attention and emphasis than if they appeared in the middle.

There are a few other things to note about the thesis. It's in this particular sentence, the crux of the passage, that the author drops some Latin phrases to bolster his own authority and remind you he's an expert. The first interrupting phrase leads with "not only *regum*, but of regular people," focusing on a point that differentiates it from the earlier claim. Finally, note how the second interrupting phrase glosses over potential counterarguments as if the author has already considered and defeated them.

Readers aren't privy to this logic, though, so the author could be glossing over weak points in his argument by drawing attention to and then immediately away from them, because after all, who's going to pause to ponder counterarguments if they're presented before the thesis is even completely stated? Note also that in the phrase "a justifiable self-preference" that we run into "justifiable" first, so that by the time we hit "self-preference," its negative connotations of egoism have been mitigated.

Evidence for Thesis: Hypothetical	**6**	If a man is on a plank in the deep sea that <u>will</u> only float one, and <u>a stranger</u> lays hold of it, he <u>will</u> thrust him off if he can.
↓		
Parallel Scenario	**7**	<u>When</u> the state finds itself in a <u>similar</u> position, it does the <u>same</u> thing.

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6 The author's evidence for his thesis is introduced as a hypothetical situation. What does this do for the reader? Well, for one, it's vivid: suddenly we're not talking about abstract moral issues but a scene out of a disaster film where we're empathizing with a man fighting for his life. That empathy is carefully cultivated by structure: we start with "If a man is on a plank in the deep sea"—a bad situation—"that will only float one"—uh oh, that's worse—"and a stranger lays hold of it"—that's even worse. Note how it's "a stranger," though. Think about it: it's highly unlikely that somone swims up out of nowhere to this man and this plank, realistically. Was there a shipwreck? Were they both sailors? Passengers? Would the man try to save the other man at all? No—the author doesn't want you considering that because that weakens his thesis about "force" and "justifiable self-preference," so it's an anonymous, unfeeling "stranger." Note also that the author has abandoned all qualification here: the plank "will" (not might) only float one, and the man "will" (not might) thrust the other man off the plank if he can.

7 That empathy is put to use in getting you on the author's side. You feel bad for the man lost at sea, right? He'd be the main character if this were a film. So when the author parallels the hypothetical with a second claim, it transfers the actors and conditions of the hypothetical to the state, and "the state" is suddenly not just the abstract concept of "the state," it retains some of the tinges of a man lost at sea on a board making really tough moral decisions. Had the author reversed these, it would potentially be less effective, because you'd be applying the emotion retroactively to something about which you'd already read.

The author strengthens the comparison by using similar syntax for each example's sentence: the hypothetical "if" of the first sentence becomes the definitive "when" in the second, but the structure of cause and effect holds. This also allows the author to use both the word "similar" and the word "same," making the comparison even more explicit.

We can return to the beginning of the passage and see what was meant by the opening sentence. Holmes wishes to propose that the importance of equality as a moral principle extends only to the common and ordinary dealings that we have with one another. Yes, when people discuss something, equality is presupposed even for there to be discussion (or argumentation); however, the implication of the passage is that once we get beyond such common and ordinary things, considering all those places in which only one person can "win" or be successful (as in the example of the plank), then such equality no longer holds as a primary moral principle. Instead, competition, force, and self-preference are the guiding lights of all action, at least according to Holmes.

Let's now look at how our structural reading of a passage can help us answer questions about it. Sample Question #1 asks you to characterize the role of a particular sentence in the passage. The first step in addressing a question like this is imperative and consistent: locate the sentence in the passage. You will not be able to retain all of the details of the passage's structure on a sentence-by-sentence level after just one read-through. Take a peek back at the text and see what's going on.

At this point, we're midway between talking about equality as a limited social principle and the author's suggestion that "force" and "justifiable self-preference" are actually the more universal motivators. Indeed, the author's

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Sample Question #1

Which of the following best descibes the role of the given sentence in the passage as a whole?

"The ever-growing value set upon peace and the social relations tends to give the law of social being the appearance of the law of all being."

- **A.** A generalization and weakening of a previously supported claim
- **B.** A piece of evidence supporting the author's claim about the dogma of equality
- **C.** An explicit counterclaim that equality is not the principle governing idea of *all* interactions
- **D.** A statement about the scope of the author's claims about force as an underlying social principle

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main claim is given immediately after the quoted line, and it begins with "but." In our sentence, the author sets up the point he's been discussing as an argument against which to contrast his own.

Now let's take a look at the answer choices. The answer can't be C, since that describes the author's thesis, and that sentence follows the indicated one. It's not D, either—at this point in the passage, the author hasn't even mentioned force as an underlying social principle yet! A and B differ primarily in the way they suggest the author is manipulating the previous claim. Is he strengthening it (B) or weakening it (A)? Looking at our quick notes, we can see that the author is weakening the previous claim, broadening its scope and casting doubts on its universal applicability so that he can argue against it. A is the correct answer.

Sample Question #2

Select all answers that apply.

The location of the sentence presenting the hypotehtical scenario about the man on the plank allows it to do which of the following?

A. Preface the author's thoughts on the state's functions in a way that imbues them with emotional significance

B. Transition from a discussion of equality to one focused on force

C. Immediately follow a claim with a relevant example

Sample Question #2 could potentially have multiple correct answers, so we'll need to tread carefully. First, let's find that sentence about the man on the plank. Got it? Now we need to analyze its location in the passage. Well, it follows the author's thesis, and is followed by a sentence that uses the scene the author creates to explain how government functions. Answer choice A looks pretty accurate—because the "man and plank" sentence precedes the one about the state and compares the state to the stranded man, it does imbue the state with an emotional significance that it would lack as an abstract concept. Is our sentence transitioning from a discussion of equality to a discussion of force (B)? No, that happened earlier in the passage—the concept of force has been introduced and is the topic of discussion in our sentence. B is not correct. What about C? What sentence precedes ours? The author's thesis. The "man and plank" sentence is providing a relevant example of how force is a governing principle of social interactions, so C is correct. On test day, you'd need to mark both A and C to get this question correct!

Of all the topics covered by GRE Reading Comprehension questions, structure questions are perhaps most suited to the select-in-passage format, so let's consider one of those questions. A lot of sentences use the same structure as other sentences in the passage (2 and 3; 6 and 7), but we're looking for a single sentence that uses parallel structure. The only sentence that presents two claims using associated sentence structures is sentence 5, the author's thesis. Both claims are laid out using the same syntax, as we saw in our close reading of this sentence's structure.

Sample Question #3

Select the sentence tht employs parallel structure to associate the two claims it presents.

Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

Authorial Choices: Tone and Style

No passage is made up of mere facts. Your particular author's selection is attempting to make an argument about something. Like a conversation, tone and style will matter as you attempt to ascertain just what he or she is trying to convey in the selection given to you.

Broadly speaking, style can be easier to discern than can tone. As you initially skim your passage, get an overall "feel" for what the passage's way of communicating the information provided. Is it written in a scholarly and informative style like a piece of nonfiction from an encyclopedia? Is it like a scientific article, implying that it will express some new information? Does it mix styles? As you will see in the passage that we will analyze below, some passages can attempt to discuss deep matters of history and human experience while simultaneously writing in a style that is more familiar and, perhaps, conversational. When an author chooses to do this, ask yourself, "Why are these styles combined? What does he or she want to draw directly to my attention?"

Similarly, notice any interesting combinations of content and style. We tend to think of historical narratives as calling for a rather dry and direct style of nonfictional prose; however, if an author wishes to humanize a historical figure who is centuries dead, he or she might choose to write in a style that is almost like a conversation between friends. This would convey the fact that the figure is someone whom we can understand today. Therefore, you would be wise to pay attention to the kinds of familiarity that the author wishes to express in the passage.

Tone is more subtle, though it is not at all impossible to discern and judge while reading your passage. When you talk to a friend or colleague, you note many small things about how he or she discusses something. How does his or her voice sound? What does he or she imply with the way that he or she states something? If a friend has a sneering and dismissive tone, you rightly should interpret his or her words as being potentially critical. Thus, whatever he or she might say about a topic, you will understandably try to ascertain just how this or that remark is meant to express something negative about the subject being discussed. The tone would indicate such an interpretation is justified. Likewise, an optimistic tone implies that your interlocutor supports what he or she is discussing, indicating that you should interpret his or her words accordingly. Written tone functions a lot like spoken tone in this way. Thus, you should pay attention to how your particular author sets the general context of discussion, giving hints for how further remarks are altered by the tone chosen for the passage and its details.

Next, we'll walk through a passage that includes numerous shifts in tone and style to practice identifying the effects of these literary elements.

The following passage is adapted from *The Golden Bough* by James George Frazer (1922).

Who does not know Turner's picture of the Golden Bough? The scene, suffused with the golden glow of imagination, is a dream-like vision of the little woodland lake of Nemi— "Diana's Mirror," as it was called by the ancients. Diana herself might still linger by this lonely shore, still haunt these woodlands wild.

In antiquity this sylvan landscape was the scene of a strange and recurring tragedy. On the northern shore of the lake, in the sacred grove of Diana of the Wood, there grew a certain tree round which at any time a grim figure might be seen to prowl. He was a priest and a murderer; and the man for whom he looked was sooner or later to murder him and hold the priesthood in his stead. Such was the rule of the sanctuary. A candidate for the priesthood could only succeed to office by slaying the priest, and having slain him, he retained office till he was himself slain by a stronger or a craftier.

We picture to ourselves the scene as it may have been witnessed by a belated wayfarer on one of those wild autumn nights when the dead leaves are falling thick, and the winds seem to sing the dirge of the dying year: the background of forest showing black and jagged against a lowering and stormy sky, the lapping of the cold water on the shore, and pacing to and fro, now in twilight and now in gloom, a dark figure with a glitter of steel at the shoulder.

The strange rule of this priesthood has no parallel in classical antiquity, and cannot be explained from it. To find an explanation we must go farther afield. If we can show that a brutal custom, like that of the priesthood of Nemi, has existed elsewhere; if we can detect the motives which led to its institution; if we can prove that these motives have operated widely, perhaps universally, in human society, producing in varied circumstances a variety of institutions specifically different but generically alike; if we can show, lastly, that these very motives were actually at work in classical antiquity; then we may fairly infer that at a remoter age the same motives gave birth to the priesthood of Nemi. Such an inference, in default of direct evidence as to how the priesthood did actually arise, can never amount to demonstration. But it will be more or less probable according to the degree of completeness with which it fulfills the conditions I have indicated.

When you first begin to scan your passage, note any particular stylistic elements that stand out as indicating the author's tone clearly. For example, in the passage given above, the entire selection begins with a question. This signals immediately that the voice of the author (or the narrator, who functions to provide a glimpse of the author's voice) is shining through the prose. In this particular passage, it seems that the author wants to convey the idea that Turner's picture of the Golden Bough is well-known (or, at least, that it should be well-known to all).

The passage goes on to describe the scene. It is important that you keep in mind the initial familiarity that was indicated by the author at the start of the passage. For example, even in the first paragraph, the Roman goddess Diana is imagined to be present today—lingering on the shore depicted in the Golden Bough or haunting its woods. The author wants us to envision the ways that the image depicts something present and vibrant for us today. This background desire of the author will be important as you continue to read the passage.

In the second paragraph, some historical points are noted. The landscape in the image is described as being a place where tragedy occurred frequently. The author relates the activity of some kind of priesthood that included the killing of others in its "activity." Indeed, the grim description states that the priest would then be "replaced" (so to speak) by whomever might best him by slaying the current priest. The style so far is like that of romantic-era literature, vividly presenting scenes and images.

All of these details may be quite interesting, but what is the author's point in mentioning them? In the third paragraph, we get some indication of the "first-person" perspective that we discussed above. Once again, the author invites us to make the scene present to ourselves and to our imaginations. He wants us to imagine the tree on an autumn night, with vivid details regarding the sights and sounds of the murderous scene. The most important thing to keep in mind, however, is the familiarity and immediacy that the author wishes to convey. The particular details are serving this general tone of discussion that was set out from the very beginning of the passage.

The importance of keeping this tone in mind comes to the fore in the final paragraph. The author has invited us to imagine the passage with such vivid and immediate detail precisely so that we can "overcome" all of those details in order to see something beyond their immediate facticity, something beyond the particular details of Roman history and mythology. In the final paragraph, the author invites us to pass beyond the particular historical details that we might know about classical antiquity. He wants us to consider what deeper human motives might have created the brutal scene by the tree. That is, he is inviting us to consider that the scene was not merely a "historical blip on the radar" in classical Rome but, instead, that it represents something that springs from the universal depths of the human experience. It is something that is, perhaps, ubiquitous. Hence—and here, recall the tone from the start of the passage—it is something with which we are all quite familiar.

This is precisely what he is driving at when he remarks that the motives for the ghastly and murderous priesthood in question may have universal grounds in humanity. There may be many such particular institutions throughout history and various civilizations. These various manifestations,

however, are particular species in some generic reality that is ubiquitous to the human condition. That is why the author started from the very beginning of the passage with the idea of the familiarity of the scene. He was not attempting to have us think that we have all physically seen something like the scene depicted in the painting *The Golden Bough*. Yes, he portrays many details of the scene's physical appearance, but those physical depictions are subservient to a deeper meaning of "familiarity." He wants to suggest a deeper understanding of the priesthood of Nemi—i.e. the priesthood to Diana described in the passage (though you can interpret the passage without knowing what he means by the "priesthood of Nemi"). The true meaning is something almost as vivid and immediate as the very physical details of the image. It is something that lies in the roots of human nature, giving rise to the universal tendencies that create institutions like this ghastly and murderous priesthood.

Thus, the author wants to do something much more than art criticism or, even, an investigation of the depictions and stories of Roman myth. Instead, as he closes the passage, the author indicates that he wants to set the stage for an inference about just how this same universal motive could have once given rise to many instances of this kind of priesthood (and similar sorts of institutions) throughout history. His investigations are meant to be as broad as humanity itself—across all cultures and throughout all of history. This expansive perspective is initially contained in the small tonal hints that we gathered at the very start of the passage. Amazing!

Of course, remember that it took us some time to work through the passage and ascertain just what he wanted us to focus on. At first, it seemed plausible that he wanted us to imagine some physical or historical scene that had deeply familiar characteristics. As we read on, it became clear that the author was inviting us to see something even more familiar than that. Therefore, when you read the tone of a passage, take your time to ascertain just what the author wishes to convey. Sometimes, you will have to broaden or adjust what you think is indicated by his or her tone. This is just like any human conversation. As you listen (or, here, read), you must pay attention to the various clues you receive in the course of interpreting. Unpack these clues as you listen to (or "read from") your author's voice. As you can see, careful attention to such matters will help you to understand the complete sense of the passage.

Authorial Choices: Perspective and Bias

When reading a passage, you should always be aware of the author's perspective. A text is never just a jumble of words being presented to you robotically. Instead, it is a communication of ideas from one person to another. While the meaning is the most important aspect of the passage, you must always remember that the meaning is always someone's meaning—the author's. By keeping this perspective in mind, you will be able to interpret facts and reasoning in light of any biases that might distort your reading of the passage.

To begin, it is helpful for us to recall some basic ideas from courses in literature and interpretation. Each perspective on a subject is crucial to understanding its appearance and its overall "meaning." The most basic set of divisions of literary perspective are the basic modes of person used in writing. You likely recall from grammar (or, at least, foreign language courses) the major divisions of first, second, and third person.

When you write a sentence in the first person, you include yourself directly. Thus, the pronouns that focus on the first person of the writer are "I," "we," and "us." As we will note below, a passage in the first person allows you to become aware of the author's bias in a unique manner.

The "second person" is immediately addressed to another person. Thus, the most common second person pronoun is "you." In English, we do not have a proper plural form of "you," as is found in other languages. In America, however, you can find various forms of "you" that attempt to pluralize the pronoun when speaking directly to a group of people. Thus, in the South, someone might say to a group of students, "Y'all need to be quiet now." What is important for our purposes is to note that the second person is a voice of direct address—whether to one person or to group of people. As we will see, a passage in the second person has some unique manners of expressing the bias of an author.

The third person refers to a person or group of people not directly addressed. Thus, when you refer to someone as "he," "she," or to a thing as "it," you are speaking in the third person. The same holds for the plural pronoun "they." A narrative that is written in the third person has some unique features. An author can choose to limit a narrative to one person or one particular group. Thus, the perspective of such a passage will only take into account one person's or group's perspective. You will always follow the biases, emotions, knowledge, and general viewpoint of that one entity. Even in nonfiction, the third person limited can be used when the author attempts to detail some topic from the limited perspective of one person or group. This is very important when it comes to understanding the meaning and intention of the passage. By taking this one perspective, the author has chosen to "color" the passage in a particular, if limited way. Note, however, that the author does not write as though the person is speaking directly to the reader. That would be a lapse back into the first person. Instead, a limited third person perspective is one that addresses the reader by telling him or her about a particular person or group—and always limits the main narrative to the perspective of that group, its viewpoint, and its concerns.

Sometimes an author will choose to write in an "omniscient" third-person voice, which means that the third person perspective is not limited to any particular person or group. Instead, it is written from a kind of "all-knowing" view of the world described by the passage. Here, every possible character and viewpoint can be considered. Even if people and groups are continents away, the author can know of and write of both groups. When reading this kind of narrative, always remember that although the author and the reader know many details about the groups, those groups themselves are not omniscient! You have a view of the "big picture." They do not.

As you determine the passage's perspective, you need to note how this perspective relates to the passage's meaning as a whole. Above all, you need to pay attention to biases that might be at play because of the person (or persons) that are the direct focus enforced by the perspective. In the first person, an author will often expose his or her own feelings on a topic. Remember this when you notice various details. If the author seems to dislike a given theory, be sure to carefully read anything that he or she says that might be colored by this dislike.

When reading a passage written in the second person, always pay attention to how the author is addressing the party referred to as "you." Is the author trying to persuade his or her audience? Is it possible that information is being withheld, "doctored up" with rhetoric, etc? These are all important questions to ask yourself as you read a passage.

As for the third person, remember the issues discussed above for both limited and omniscient views. As we will see in the passage below, the third person is often coupled with a kind of "implicit second person." By speaking in the omniscient third person, the author may well be trying to convince you of something even though he or she never uses the word "you"!

Let's work through a sample passage adapted from John Muir's "Save the Redwoods" (1920) in an effort to consult the author's perspective and see what we can learn by paying attention to it, as well as to consider any biases the author might be demonstrating.

Forty-seven years ago one of these Calaveras King Sequoias was laboriously cut down, that the stump might be had for a dancing-floor. Another, one of the finest in the grove, was skinned alive to a height of one hundred and sixteen feet and the bark sent to London to show how fine and big that Calaveras tree was—as sensible a scheme as skinning our great men would be to prove their greatness. Now some millmen want to cut all the Calaveras trees into lumber and money. No doubt these trees would make good lumber after passing through a sawmill, as George Washington after passing through the hands of a French cook would have made good food. But both for Washington and the tree that bears his name higher uses have been found.

Could one of these Sequoia Kings come to town in all its godlike majesty so as to be strikingly seen and allowed to plead its own cause, there would never again be any lack of defenders. And the same may be said of all the other Sequoia groves and forests of the Sierra with their companions and the noble Sequoia sempervirens, or redwood, of the coast mountains.

In these noble groves and forests to the southward of the Calaveras Grove the axe and saw have long been busy, and thousands of the finest Sequoias have been felled, blasted into manageable dimensions, and sawed into lumber by methods destructive almost beyond belief, while fires have spread still wider and more lamentable ruin. In the course of my explorations twenty-five years ago, I found five sawmills located on or near the lower margin of the Sequoia belt, all of which were cutting more or less [Sequoia gigantea] lumber, which looks like the redwood of the coast, and was sold as redwood. One of the smallest of these mills in the season of 1874 sawed two million feet of Sequoia lumber. Since that time other mills have been built among the Sequoias, notably the large ones on Kings River and the head of the Fresno. The destruction of these grand trees is still going on. On the other hand, the Calaveras Grove for forty years has been faithfully protected by Mr. Sperry, and with the exception of the two trees mentioned above is still in primeval beauty. For the thousands of acres of Sequoia forest outside of reservations and national parks, and in the hands of lumbermen, no help is in sight.

Any fool can destroy trees. They cannot defend themselves or run away. And few destroyers of trees ever plant any; nor can planting avail much toward restoring our grand aboriginal giants. It took more than three thousand years to make some of the oldest of the Sequoias, trees that are still standing in perfect strength and beauty, waving and singing in the mighty forests of the Sierra.

The perspective taken by the author is a version of third-person omniscient. He is speaking about various events that pertained to sequoia trees, which he clearly thinks should not have been cut down. In a way, however, you should note that there is rhetoric implied in the passage attempting to draw you in as though he is addressing you directly. He never actually breaks into the proper second person by using "you." Nevertheless, there is a kind of directed argument implied just "beneath the

surface." The omniscient third person is being used to rally a number of facts to his side in defense of the redwood trees. Thus, we should note carefully just how he is using this omniscient view to express his particular bias and argue for conclusions that support his viewpoint. Let's consider this technique in each paragraph.

In the first paragraph, the author mentions how the various trees were used for sundry purposes. Clearly, by the end of the lengthy second sentence, you will note how vexed he is regarding these facts. Indeed, he compares the use of a tree sample to the use of human flesh as a proof of human greatness. He thus personifies the trees, making the act of cutting down these trees into something almost like murder (or at least human skinning)! Thus, be very careful at how he describes those who are cutting down the trees. They are portrayed as desiring only lumber and money. The author makes some pretty strong implications regarding how stupid a use this is for the wood in question. The author's points may well be completely accurate, but it is important to note the way in which his rhetoric is extremely charged.

In the second paragraph, the author claims that the immorality of harming trees would all be obvious if the trees could defend themselves. He really seems to overstate the obviousness of this position. He implies that if only the trees could speak then everyone would agree that they should be defended. Likely, his own bias on behalf of the trees is shining through this particular remark.

In the third paragraph, he makes claims about almost universal destruction of trees. It is claimed that even the smallest of mills has had massive output. The lumbermen are presented as voracious destroyers. Now, the reader has no immediate reason to think that Muir is lying in his article. The reader should, however, beware of taking his words too seriously without any critical attention. The reader has not been presented with exact figures, so the rate of deforestation is not well documented in this selection. Likewise, the reader should be very careful regarding claims regarding the lumbermen—as well as the protective measures being undertaken. Once again, one cannot say that Muir is lying. Nonetheless, the obvious bias and tone of the passage should lead the reader to exercise great caution in any extrapolation. He clearly wants to depict the universal destruction of the trees and to make the reader feel that all trees should be defended.

Indeed, the very last paragraph makes fine points regarding reforestation, especially with regard to the difficulty involved in regrowing the ancient trees in question. Nonetheless, it is crucial that you read this final point as a rhetorical emphasis on the main theme of the passage. The author is once again trying to cast his foe in a negative light and convince the reader of a particular need for defending the trees. Once again, this is not necessarily deceptive. Nonetheless, you will need to be very measured when interpreting the claims made, given the strength and obvious bias involved in Muir's rhetoric.

Let's next answer a few sample questions about the passage that test your ability to identify and analyze its biases and perspective.

The sentence indicated by Sample Question #1 appears after the author has provided two examples of Sequoia trees being destroyed for what he sees as wasteful purposes: to create a dancing floor and to show how impressive one is based on its bark. This sentence isn't making a concession (A) or praising trees for their usefulness (B); it's introducing a new topic in the context of previous examples. While we could say that it "begin[s] introducing the rest of the text from which the passage is excerpted," we can't infer that this text focuses on lumber instead of live trees. That's a bit of a stretch. The best answer is C: in the indicated sentence, the author sets up a weak formulation of an argument he will oppose. Note that there is no suggestion of any merits of using the trees for "lumber and money"; it's a one-sided portrayal, so calling it "a weak formulation" is a fair appraisal.

Sample Question #1

The purpose of the sentence "Now some millmen want to cut all the Calaveras trees into lumber and money" is _____.

    A. to make a concession and demonstrate that the author agrees with some of his opponent's views

    B. to praise the trees for their practical usefulness as well as their aesthetic beauty

    C. to provide a weak formulation of an opposing argument for the author to refute

    D. to begin introducing the rest of the text from which the passage is excerpted, which we can infer focuses on lumber instead of live trees

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In answering Sample Question #2, we need to identify how the passage might appear less biased. For the passage to seem less biased, it would need to incorporate more information about perspectives with which the author does not agree. As it stands, the passage is one-sided in its support of conserving trees; the author belittles using them for practical purposes, scientific investigation, or economic gain should such use result in the tree dying. Of the provided answer choices, A, B, and

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Sample Question #2

Brief discussion of which of the following would make the passage's perspective seem less biased?
Assume all statements provided are true.

    A. Detail on how the loss of its bark directly caused the death of the "skinned" Sequoia tree

    B. Deforestation of tropical rainforests

    C. Economic benefits of the lumber industry

    D. The fact that the dancing-floor for which a Sequoia was cut down was only used for one summer

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D would bolster the opinion that the author is already providing. Only C would help the passage appear less biased, because if the author discussed any benefits of the lumber industry, it would provide a more well-rounded perspective on the situation, even though he personally feels that using the trees for lumber is a big mistake.

The phrase "skinned alive" appears in the sentence, "Another [Sequoia tree], one of the finest in the grove, was skinned alive to a height of one hundred and sixteen feet and the bark sent to London to show how fine and big that Calaveras tree was—as sensible a

> **Sample Question #3**   **Select all answers that apply.**
>
> The author uses the underlined phrase "skinned alive" _____.
>
> **A.** to provide relevant detail that the tree in question was still alive when its bark was removed, and thus had not been chopped down
>
> **B.** to create empathy for the tree by inspiring horror and revlusion in the audience
>
> **C.** to introduce the comparison between great trees and great men that is developed later in the passage

scheme as skinning our great men would be to prove their greatness." The detail A describes is actually not as relevant as the answer choice makes it sound; A is not correct. One can accurately say that mentioning the concept of skinning a living thing alive works to create empathy for the tree by inspiring a strong visceral reaction in the audience, so B is correct. C is also correct, as trees are not usually thought of as having "skin," so this language works metaphorically and sets up for the comparison between trees and "great men" that ends the sentence.

Question Strategies: Reading Comprehension

Amongst the Verbal Reasoning section's question types, Reading Comprehension can seem comfortingly familiar. After all, while it may be the first exam on which you've had to face three-blank Text Completion questions and Sentence Equivalence questions, you've surely taken exams before which ask you to read a paragraph and answer questions about it. But don't let this perspective blind you to the challenges the GRE presents in its Reading Comprehension questions. It can be easy to give this question format the least attention of any of the Verbal Reasoning sections, but if you do this, you might find yourself surprised on test day with how difficult these questions can be.

Reading Comprehension questions on the GRE can take one of three forms: single-answer questions, in which one answer out of four is correct; multiple-answer questions, in which one or more of three answers is or are correct; and select-in-passage questions, which ask you to click on the sentence described. Single-Answer questions can still be challenging, so we'll devote an entire lesson to strategies specific to the Verbal section's most common Reading Comprehension question type. Multiple-Answer questions are often considered the most difficult, because you can't rely on process-of-elimination tactics to arrive at the correct answer and instead must consider each answer choice individually. Our lesson on this question format deals with this difficulty by supplementing additional strategies in place of process of elimination. Finally, Select-in-Passage questions can seem set apart from the other two formats, as this question format is often used to test your understanding of rhetoric, organization, and the structure of the argument at hand. We offer tactics to help you consider the passage as a compilation of component parts and select the one being described by the question stem.

Even though Reading Comprehension may be the most familiar part of the GRE Verbal Reasoning section, for many test-takers, it's also the most difficult part of the section. Prepare yourself for the test's particular challenges by brushing up on the idiosyncrasies of these question formats and adding some focused strategies to your arsenal.

Section Outline

Multiple-Choice with One Answer

Multiple-Choice with Multiple Answers

Text Selection

Multiple-Choice with One Answer

Many of the Reading Comprehension questions you'll encounter will be presented in a format you may find to be quite familiar. Four answer choices will be provided in response to a question or statement in the question stem, and to get the question right, you'll need to identify the one answer that is correct. Just because this question format is familiar doesn't mean you need to address every multiple-choice single-answer question in the same way. In this lesson, we'll present some strategies that you can use when you encounter this question format on test day. We will then outline how to apply some of these strategies in answering a few multiple-choice single-answer sample questions. Don't assume that these questions are going to be easy just because the format is perhaps the most typical of standardized tests. Practice approaching these questions as strategically as you approach other question formats and be ready with applicable strategies when you sit for your exam.

Process of Elimination

Each single-answer multiple-choice question presents four answer choices, and only one of them is correct, but we can look at this from a different perspective: three of them are wrong. Picking out the correct answer is the name of the game, but the more incorrect answer choices you can identify, the closer you get to that correct answer. In addition, if you find yourself having to guess on a question from which you've eliminated two of the potential answer choices, you have a 50/50 shot of getting it right picking at random instead of only a 25% chance!

Consider General Topics, then Details, to Orient Yourself in the Passage

This tactic works especially well with longer passages or those that move through lots of topics quickly. As you read the passage, consider the general topics it discusses, and maybe even make some quick notes or a simple flow chart. Begin answering a question by considering what general topics the question is asking about and matching the question's topics up with those in the passage to figure out where in the text the question's answer has to be located. Once you've done that, you can focus on details in the question stem to identify details in the passage and work your way to the correct answer.

Guess If You Skip Questions, But Eliminate Answers First

Your score on the Verbal Reasoning section is calculated based on the number of questions you answer correctly, not the number of questions you miss. Thus, it's in your best interest to guess on any questions about which you're uncertain. At worst, you'll miss it, but if you didn't know the answer, this isn't any worse than if you had submitted no answer at all. Also, you have at least a one-in-four chance of getting it correct by choosing at random. Make sure to try to identify some

incorrect answers to increase your odds if you do end up guessing on a single-answer multiple-choice question!

Read the Full Question Stem and Answer Before Answering

On test day, it can be all too easy to miss subtle details, but sometimes entire questions hinge on a single word in the passage or in a given answer choice. To combat the changes of missing such crucial details, read the full question stem and your selected answer choice to yourself before marking your answer. Isolating your view to just the answer you've selected as correct can help you identify any disqualifying subtleties.

Turn Answer Choices into Full Statements or Questions As Is Helpful

When reading and considering answer choices, you might find it helpful to consider them as either part of a statement of which the question stem forms the first half, or as an answer to the question stem formatted as a question. If you like true-or-false format questions, seeing each answer choice as a standalone statement can help you make use of this perspective, and forming a unique question from the question stem and each answer choice can help you focus on the task and hand and answer "yes" or "no" to each answer choice's claims.

As an example of what is meant by this, consider the following example, taken from Sample Question #3:

The author's statement "even a dog distinguishes between being stumbled over and being kicked" serves what purpose in the passage?

A. It supports the author's suggestion that human law is modeled on animal behavior

Encountering this on your exam, you could turn answer choice A into a statement:

"The author's statement "even a dog distinguishes between being stumbled over and being kicked" supports the author's suggestion that human law is modeled on animal behavior."

Every Claim Needs Evidence

For every claim that you assert on the Verbal Reasoning section through your answer choices, you need to be able to identify specific statements in the passage that back it up. If an answer does not pass this test, it is most likely incorrect.

Let's consider the following sample passage, adapted from *The Common Law* by Oliver Wendell Holmes (1881), and then practice answering a few single-answer multiple-choice questions about it.

It is commonly known that the early forms of legal procedure were grounded in vengeance. Modern writers have thought that the Roman law started from the blood feud, and all the authorities agree that the German law begun in that way. The feud led to the composition, at first optional, then compulsory, by which the feud was bought off. The gradual encroachment of the composition may be traced in the Anglo-Saxon laws, and the feud was pretty well broken up, though not extinguished, by the time of William the Conqueror. The killings and house-burnings of an earlier day became the appeals of mayhem and arson. The appeals de pace et plagis and of mayhem became, or rather were in substance, the action of trespass which is still familiar to lawyers. But as the compensation recovered in the appeal was the alternative of vengeance, we might expect to find its scope limited to the scope of vengeance. Vengeance imports a feeling of blame, and an opinion, however distorted by passion, that a wrong has been done. It can hardly go very far beyond the case of a harm intentionally inflicted: even a dog distinguishes between being stumbled over and being kicked.

Whether for this cause or another, the early English appeals for personal violence seem to have been confined to intentional wrongs. Glanvill mentions melees, blows, and wounds,—all forms of intentional violence. The cause of action in the cases of trespass reported in the earlier Year Books and in the Abbreviatio Plaeitorum is always an intentional wrong. It was only at a later day, and after argument, that trespass was extended so as to embrace harms which were foreseen, but which were not the intended consequence of the defendant's act. Thence again it extended to unforeseen injuries.

In answering Sample Question #1, let's take a look at the sentence in which this word appears:

The feud led to the <u>composition</u>, at first optional, then compulsory, by which the feud was bought off.

This sentence appears in the paragraph as the author is discussing how legal procedure is grounded in the history of dealing with blood feuds and people who want revenge. We can tell that whatever the "composition" is, it stopped the feud. Note that in the passage we're not talking about the composition of something: we're talking about "the composition," a thing in its own right. Thus, B doesn't make sense, because we'd need to be talking about the composition of something else.

Sample Question #1

Based on the context in which it is used, the underlined word "composition" in the first paragraph refers to which of the following?

A. A piece of prose writing

B. The makeup of a thing or group, such as a jury

C. A thing formed from various different parts

D. An official agreement between two parties

The word "composition" is used again soon after this sentence in the passage:

The gradual encroachment of the <u>composition</u> may be traced in the Anglo-Saxon laws, and the feud was pretty well broken up, though not extinguished, by the time of William the Conqueror.

A and C may be valid definitions of the term "composition," but nothing in the passage provides any evidence that these are the specific meanings being used in this particular circumstance. A may look particularly like a potentially correct answer if you're thinking of legal documents, but no evidence suggests that "the composition" is a written text. What do we know about "the composition"? Only that it stopped feuds. The specific language used is "by which the feud was bought off." It sounds like there are negotiations going on. Negotiations have to involve multiple parties, and this leads us to the correct answer, D: "composition" in this case refers to a type of official agreement between two parties. In this case, it would be the agreement between the two feuding parties as to what would need to be done to make the feud stop. Note that we didn't arrive directly at the correct answer, but got to it by way of identifying incorrect answers. Process of elimination can be a very handy tactic where it's applicable.

Sample Question #2

Which of the following does the author claim that appeals de pace et plagis and of mayhem most directly developed into?

A. Claims of trespass for foreseen but unintended harm
B. Claims of trespass for unintended and unexpected harm
C. Claims of trespass for purposeful harm
D. Compensations to offended parties that ended feuds

Sample Question #2 is testing your understanding of a sequence of development outlined in the passage. Specifically, it asks you to identify what concept directly developed from "appeals de pace et plagis and of mayhem." Where does the author talk about these type of appeals? In the first paragraph, when he's describing the history of law that emerged from the formal cessation of feuds. Skimming the paragraph, we find that the author states, "The appeals de pace et plagis and of mayhem became, or rather were in substance, the action of trespass which is still familiar to lawyers." Ok, this tells us that the answer is A, B, or C, but it doesn't yet allow us to differentiate between these. What else does the author say about trespass in this part of the passage? The author immediately states, "But as the compensation recovered in the appeal was the alternative of vengeance, we might expect to find its scope limited to the scope of vengeance," which has to be inspired by "a harm intentionally inflicted." He spends the first part of the second paragraph producing evidence that "early English appeals for personal violence seem to have been confined to intentional wrongs." Only at the close of the passage does the author mention trespass for foreseen and unintended harm and unforeseen and unintended harm: "It was only at a later day, and after argument, that trespass was extended so as to embrace harms which were foreseen, but which were not the intended consequence of the defendant's act. Thence again it extended to unforeseen injuries." So, given that these types of trespass, which correspond with answer choices A and B, are only mentioned at the passage' close as the types of trespass which eventually developed, "claims of trespass for purposeful harm," answer choice C, must be the correct answer, as these trespass claims eventually were broadened to include the other two kinds, but were initially only considered claims of purposeful harm.

Let's locate this statement in the passage and consider where it sits in the overall progression of topics. This phrase concludes the last sentence of the first paragraph: "It can hardly go very far beyond the case of a harm intentionally inflicted: even a dog distinguishes between being stumbled over and being kicked." What is "it" referring to? If we trace back another sentence, we find that the author is discussing the association between vengeance and compensation gained by appeals: "Vengeance imports a feeling of blame, and an opinion, however distorted by passion, that a wrong has been done." The author isn't talking about such disagreements starting feuds (answer choice C); he discussed feuds earlier in the passage and is now talking about the types of appeals that they led to and the constraints under which those appeals

Sample Question #3

The author's statement "even a dog distinguishes between being stumbled over and being kicked" serves what purpose in the passage?

A. It supports the author's suggestion that human law is modeled on animal behavior
B. It helps the author explain that intentionality is a prerequisite condition for inspiring vengeance after harm
C. It outlines the type of disagreements that the author claims likely resulted in many feuds
D. It provides an example equating intentional and unintentional harm

function. Furthermore, nowhere in the passage does the author suggest that human law is modeled on animal behavior (answer choice A); this is a wide-reaching claim that would require a lot of evidence to support. This leaves us with two answer choices that each state that the phrase is used as an example, but to different purposes. We just have to figure out which of these statements' purposes for the example matches the author's argument. Does he "equate intentional and unintentional harm" (D)? No, he's doing the opposite, drawing a sharp divide between them: the dog knowing when it's been stumbled over is it recognizing unintentional harm, and recognizing when it's been kicked is it recognizing intentional harm. D can't be the correct answer, so B must be, and we find that its reason for the example matches the author's argument. The author is claiming that for someone reasonably to seek vengeance, that person must have been intentionally—not unintentionally or accidentally—hurt by someone else. B is the correct answer.

Multiple-Choice with Multiple Answers

Perhaps the most challenging Reading Comprehension questions on the Verbal section of the GRE are those multiple choice questions that ask you to determine the correctness of not one but three distinct statements as responses to a single question prompt. Like in Text Completion problems, your decision about one particular answer choice does not affect your choice regarding the other two, and you have to identify all of the correct and incorrect responses to receive any credit for the question. Whereas the question stem in a Text Completion question often allows you to chain together various responses by way of context clues and the sentence's logical structure, you get no such help on multiple-answer Reading Comprehension questions. Tackling each answer choice one after another as independent true-or-false style questions is the necessary strategy here.

Read Each Answer Choice to Yourself, and Preface Each with the Question Stem

Part of the difficulty specific to this question type is differentiating each statement as a distinct claim to be analyzed. This can be difficult if you consider any answer choices labeled A-B-C as a single-answer multiple choice question out of habit. To counteract this tendency, read each answer choice to yourself mentally, starting with the question stem. Analyze each response and try your best to come to a conclusive answer about it before moving on to the next. If you need to return to a statement later, make sure your thoughts about it don't influence your view of other answer choices. Sometimes, it's easy to to combine the question stem and an answer choice into a question when reading them together. If you prefer this method, use it! You can just answer three distinct questions, and this will keep your thinking distinguished for each answer choice.

Consider (and Require!) Evidence in the Passage

Remember, even when discussing inferences and conclusions that can be drawn from the text, all of the information you need to solve the questions on the GRE's Reading Comprehension section is presented to you somewhere between the question stem, the answer choices, and the passage itself. On the other hand, we can invert this statement and make it a condition that you need to meet: you have to be able to point to specific evidence in the passage for whatever answer choices' claims you mark as true. Make sure that each of your responses passes this "evidence test." If you can't find evidence for a claim, it's most likely incorrect, no matter how factually correct it may sound. Watch out for questions about inferences that can or can't be made—even inferences need to be grounded in a concrete statement from the passage.

Skip Questions If Necessary . . .

Time is a limited resource on the GRE Verbal Reasoning section, so if you're unsure of one or more parts of a multiple-answer multiple choice question, you may want to skip the question entirely. After all, a correct multiple-answer multiple-choice question is worth the same as a correct Text Completion question or Single-Answer Reading Comprehension question, or any of the other types of questions on the section. Don't fall into the mental trap of assuming that multiple-answer multiple-choice questions are worth more to your final score just because they are more involved and likely more difficult than other questions in the section!

. . .but Remember to Invest the Time It Takes to Mark a Reasonable Answer!

With that said, if you do decide to skip a multiple-answer multiple-choice question, make sure to mark an answer for it. This requires a cursory consideration of each answer choice, but taking one minute to come to a conclusion about each answer choice you're uncertain about is much better than marking answers at random or spending five minutes only to arrive at answer choices you're no more confident in than if you spent only one minute or less on the question.

For the rest of the lesson, we'll practice using these tactics to work through a few multiple-answer questions that refer to the sample passage presented below, which is adapted from *Seven Discourses Delivered in the Royal Academy By the President* by Joshua Reynolds (1778).

All the objects which are exhibited to our view by nature, upon close examination will be found to have their blemishes and defects. The most beautiful forms have something about them like weakness, minuteness, or imperfection. But it is not every eye that perceives these blemishes. It must be an eye long used to the contemplation and comparison of these forms—and which, by a long habit of observing what any set of objects of the same kind have in common, that alone can acquire the power of discerning what each wants in particular. This long laborious comparison should be the first study of the painter who aims at the greatest style. By this means, he acquires a just idea of beautiful forms; he corrects nature by herself, her imperfect state by her more perfect. His eye being enabled to distinguish the accidental deficiencies, excrescences, and deformities of things from their general figures, he makes out an abstract idea of their forms more perfect than any one original—and what may seem

a paradox, he learns to design naturally by drawing his figures unlike to any one object. This idea of the perfect state of nature, which the artist calls the ideal beauty, is the great leading principle by which works of genius are conducted. By this, Phidias acquired his fame. He wrought upon a sober principle what has so much excited the enthusiasm of the world—and by this method you, who have courage to tread the same path, may acquire equal reputation.

First things first: in answering Sample Question #1, we need to locate this phrase in the passage and consider the sentence in which it is used and the point in the author's discussion at which that sentence appears.

"His eye being enabled to distinguish the accidental deficiencies, excrescences, and deformities of things from their general figures, he makes out an abstract idea of their forms more perfect than any one original—and what may seem a paradox, he learns to design naturally by drawing his figures unlike to any one object."

Whose eye are we talking about—to whom does the "his" that starts the sentence refer? Tracing back a few sentences in the passage, we find that the antecedent of this pronoun is "the painter who aims at the greatest style." So this sentence is stating that the painter, being able to see the flaws in objects and differentiate between these flaws and the general rest of the object, can consider them as they'd be without the flaws, and while this may seem illogical, learns to draw by drawing the figures without flaws instead of as they actually are. (Did you notice that we paraphrased the sentence there? Paraphrase is a useful tool for passages this rhetorically dense.) What in our paraphrase hinges on the phrase "what may seem a paradox"? "Seem" is the core of this phrase—what follows the phrase may look like a paradox, but the author implies that it is not actually a paradox. With all this in mind, let's go over the answer choices one by one. Remember, we're looking for any that are implied by the specified phrase.

By using the phrase "what may seem a paradox," does the author imply that most readers assume that learning to draw well involves drawing real objects accurately? Yes, it does, as we can prove based on the fact that this phrase precedes the phrase "he learns to design naturally by drawing his figures

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Sample Question #1

By using the underlined phrase "what may seem a paradox," the author implies that _____.

A. most readers assume that learning to draw well involves drawing real objects accurately

B. the statement that follows is logically sound

C. formal artistic training is necessary for those who wish to accurately depict flawed objects

unlike to any one object." This sets up the expectation that artists learn to draw by drawing the contrasting thing: the objects themselves. A is correct. Did you notice how we turned the question stem and answer choice into a question? This tactic can help to focus your attention on each answer choice individually without confusing them. We also identified evidence supporting our answer choice before we decided it was correct.

On to answer choice B. Let's phrase it as a question again: "By using the phrase "what may seem a paradox," does the author implies that the statement that follows is logically sound?" Yes, he does! Remember how we took apart the phrase "what may seem a paradox" and concluded that even though something may seem like a paradox, the author is actually implying that it is not a paradox? That would make it logically sound. B is also correct.

Answer choice C's question is as follows: "By using the phrase "what may seem a paradox," does the author imply that formal artistic training is necessary for those who wish to accurately depict flawed objects?" Hmm, this one doesn't inspire immediate conclusions like the previous ones did. It sounds like it could be correct, though, so let's investigate. Does the author talk about artistic training in the passage? Yes, but this occurs before he talks about how this training can be used to consider abstract perfect objects based on flawed real objects. He doesn't talk about using that training to accurately depict flawed objects—his focus is on how that training allows artists to paint the abstract, unreal, perfect objects. Furthermore, does this conclusion follow from the specific phrase "what may seem a paradox"? It does not. All of this leads to the conclusion that this answer choice is not correct.

On your exam, you would mark A and B as being correct and leave C blank, and you'd get credit for this question! Let's try answering another multiple-answer multiple-choice question.

Sample Question #2

The underlined phrase "the ideal beauty" refers to _____.

A. the beauty of the most perfect existing natural object of a kind
B. an abstract conception of beauty
C. the beauty of a natural, non-man-made object

Sample Question #2 asks us about the phrase "ideal beauty" as it's used in the passage. Let's locate this phrase in the passage and consider the surrounding material:

This idea of the perfect state of nature, which the artist calls the ideal beauty, is the great leading principle by which works of genius are conducted.

What is "this idea of the perfect state of nature"? To understand, we need to back up another sentence:

By this means, he acquires a just idea of beautiful forms; he corrects nature by herself, her imperfect state by her more perfect. His eye being enabled to distinguish the accidental deficiencies, excrescences,

and deformities of things from their general figures, he makes out an abstract idea of their forms more perfect than any one original—and what may seem a paradox, he learns to design naturally by drawing his figures unlike to any one object."

Hmm, it seems that by "the ideal beauty," the author is referring back to "an abstract idea of their forms more perfect than any one original." How does this fit in with the rest of the passage's claims? Well, the author began by claiming that any existing object has flaws, and that an artist's training allows him to see these flaws very clearly and distinguish between the general form of an object and its flaws. The "general form" is the perfect form (it doesn't actually exist), but the artist can draw it. That's "the ideal beauty."

Now that we've defined it for ourselves, let's see which answer choices match our conclusion. Does the underlined phrase "the ideal beauty" refer to the beauty of the most perfect existing natural object of a kind? Answer choice A sounds good—it's talking about the "most perfect" beauty of an object. But wait a minute: this answer choice is derailed by a single word: "existing." Remember how the author claims that all natural existing objects have flaws? Therefore, even the "most perfect existing object of a type will still have some flaws, and "the ideal beauty" is that of an ideal perfect object, not an existing one. A is incorrect.

Does the underlined phrase "the ideal beauty" refer to an abstract conception of beauty? Yes, it does! B is correct. Our work for A led us to this very conclusion: the perfect beauty of an object is necessarily abstract, since all natural, existing objects have flaws, according to the author.

On to answer choice C. Does the underlined phrase "the ideal beauty" refer to the beauty of a natural, non-man-made object? Nope! Our reasoning allows us to easily discount this answer choice, since again, this is an existing object, and "the ideal beauty" is abstract.

This question only has one correct answer: B. Remember, just because you can select multiple answer choices as correct doesn't mean that you have to do so!

Sample Question #3 is all about assumptions that the author makes in the passage. We'll have to be able to identify evidence of any assumptions we claim he is making; keep this in mind as we go over each answer choice.

Does the author assume that all real objects have flaws? Indeed he does, and he says as much in the first sentence: "All the objects which are exhibited to our view by nature,

Sample Question #3

Which of the following assumptions does the author make in the passage?

A. All natural real objects have flaws.

B. Only artists are able to notice subtle blemishes in natural objects.

C. The measure of an artist's ability is how well he or she is able to correct for natural flaws.

upon close examination will be found to have their blemishes and defects." That was easy! A is correct.

B is a bit more challenging: Does the author assume that only artists are able to notice subtle blemishes in natural objects? The word at the center of this answer choice is "only." It's a very strong word, so we should be able to find some evidence in favor of or against this assumption. Let's find out where the author talks about who can notice flaws in natural objects, and how he connects these people with artists:

> But it is not every eye that perceives these blemishes. It must be an eye long used to the contemplation and comparison of these forms—and which, by a long habit of observing what any set of objects of the same kind have in common, that alone can acquire the power of discerning what each wants in particular. This long laborious comparison should be the first study of the painter who aims at the greatest style.

The author never claims that only artists can perform this kind of comparison, merely that artists should get very good at it. B is not correct.

For C, we need to consider whether or not the author assumes that the measure of an artist's ability is how well he or she is able to correct for natural flaws. Where does the author talk about measuring artists' abilities? This could appear anywhere in the passage, really, but skimming over it, you'll find that at the end, the author claims, "This idea of the perfect state of nature, which the artist calls the ideal beauty, is the great leading principle by which works of genius are conducted. By this, Phidias acquired his fame." Here, the author claims that by following his advice and learning to sketch "the ideal beauty" of abstract, perfect objects, his audience can also create "works of genius" and become famous like Phidias. He concludes the passage by directly stating this point: "by this method you, who have courage to tread the same path, may acquire equal reputation." Thus, C is also correct. So, the best answers to this question are A and C.

Text Selection

It's easy to get stressed out when you find yourself facing a question with no answer choices on the GRE Verbal section. Instead of providing you with three or four options, Text Selection questions ask you to turn to the passage itself and consider each sentence as an answer choice. Shorter passages might only present you with four sentences from which to choose, but longer passages can present a great deal more options. This can be overwhelming, as suddenly the distinction between the question and the passage the question is about seems to collapse, and to review all of the answer choices, you need to reread the entire passage. Approached in a reflexive rereading rush, Text Selection questions can become time-consuming and more difficult than they need to be. Let's take a look at a different strategy you can use to home in on the sentence towards which the test is directing you.

1. Consider the question stem. Identify general topics that the correct sentence must mention.

First, look at the question stem. What does the sentence that the stem is directing you toward have to be about, in general terms? Don't focus on the details just yet; those will become relevant in a later step.

2. Skim the paragraph for topics and find the general part of the passage that concerns the general topics the sentence must discuss.

In looking over the paragraph again, don't try to keep all of the details of each sentence in your head—that will just stress you out, and you'll probably miss important information. Instead, skim over the passage and pay attention to the general topics that come and go as it progresses. What is the first part of the passage "about"? The next part? Keep your responses to this question simple—maybe a noun or two. You may want to jot down some notes. Figure out where that topic must appear in the passage based on your general reading. Identify a range of three or four sentences at most that has to contain the correct sentence.

3. Concentrate on the specifics of the question stem to narrow down to the correct sentence.

Now it's time to read in detail. Consider the specifics of the question stem: what the sentence in question has to be doing and saying in particular that differentiates it from other sentences about the same general topic. Question stems might describe a sentence in terms of how in functions rhetorically in the greater context of the passage (i.e. what it does) or simply describe or even paraphrase its content (i.e. what it says). Consider each of the sentences in the part of the passage that addresses the correct general topic, and select the one that best matches the prompt.

Let's try applying this strategy to a few Text Selection questions about the following sample passage, adapted from *Historical Materialism and the Economics of Karl Marx* by Bendetto Croce (1914; 1915 ed. trans. C. M. Meredith). Its language and subject matter are a bit complex, but don't let that make your nervous—all of the information you need to answer the questions correctly will be provided somewhere in the combination of the passage and each question stem.

[1] The possibility of a philosophy of history presupposes the possibility of reducing the sequence of history to general concepts. [2] Now, whilst it is possible to reduce to general concepts the particular factors of reality which appear in history and hence to construct a philosophy of morality or of law, of science or of art, and a general philosophy, it is not possible to work up into general concepts the single complex whole formed by these factors, i.e. the concrete fact, in which the historical sequence consists. [3] To divide it into its factors is to destroy it, to annihilate it. [4] In its complex totality, historical change is incapable of reduction except to one concept, that of development: a concept empty of everything that forms the peculiar content of history.

[5] The old philosophy of history regarded a conceptual working out of history as possible; either because by introducing the idea of a deity, it read into the facts the aims of a divine intelligence; or because it treated the formal concept of development as including within itself, logically, the contingent determinations. [6] The case of positivism is strange in that, being neither so boldly imaginative as to yield to the conceptions of teleology and rational philosophy, nor so strictly realistic and intellectually disciplined as to attack the error at its roots, it has halted halfway, i.e. at the actual concept of development and of evolution, and has announced the philosophy of evolution as the true philosophy of history: development itself—as the law which explains development! [7] Were this tautology only in question little harm would result; but the misfortune is that, by a too easy confusion, the concept of evolution often emerges, in the hands of the positivists, from the formal emptiness which belongs to it in truth, and acquires a meaning or rather a pretended meaning, very like the meanings of teleology and metaphysics. [8] The almost religious unction and reverence with which one hears the sacred mystery of evolution spoken of gives sufficient proof of this.

Sample Question #1

Select the sentence in which the author criticizes a view of history for its reliance on circular reasoning.

Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

Let's take apart this question stem. What are we looking for? The correct sentence has to be one in which the author is talking about historians, and it has to either mention or implicate circular reasoning. Let's skim over the passage looking for parts that could be about circular reasoning. The first paragraph is just about reducing the concept of history and all the obstacles inherent in doing that, so it doesn't seem like the sentence we're after will be found there. The second paragraph starts talking about historians (!) and other groups of thinkers who don't address these obstacles and come up with assumptions the author sees as incorrect. This paragraph looks more promising. Sentence 5 is talking about why the "old philosophy" didn't run into the problems outlined in the first paragraph, so it's not quite what we're looking for. But consider Sentence 6: here, the author is attacking "positivism"—a view of history. At the end of the sentence, the author states that this view "has announced the philosophy of evolution as the true philosophy of history: development itself—as the law which explains development!" If it's not clear that sentence is using circular reasoning, consider that Sentence 7 refers to this concept as "this tautology," a "tautology" being a logical fallacy of circular reasoning. Don't pick Sentence 7 as correct just because it contains the word "tautology," though—it's actually hearkening back to the concept set forward in Sentence 6, which is correct.

In Sample Question #2, the author discusses the reduction of history to certain core concepts in the first paragraph, not in the second, so we've already narrowed down our potential sentences to the four that make up that paragraph. How is the paragraph's discussion of reducing the concept of history structured? The author starts by telling us about the different obstacles that set apart history from other subjects when trying to isolate its core concepts. None of those sentences is the one we're looking for. Only in the last sentence, Sentence 4, does the author declare, "In its complex totality, historical change is incapable of reduction except to one concept, that of development: a concept empty of everything that forms the peculiar content of history." This is the correct answer!

Sample Question #2

Select the sentence in which the author tells readers to what concept historical progress can be reduced.

Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

Sample Question #3 involves the author criticizing other views, and this alone lets us narrow our search to the second paragraph. The author spends the first paragraph explaining the difficulties inherent in simplifying history to one or more core concepts; he doesn't argue against any other view specifically. He does that in the second paragraph, though. What exact flaw do we need to look for the author to address? Seeing a part of the concept of evolution as its source. This is a specific critique—so specific that we can skim the second paragraph looking for where this occurs and locate our sentence based on this factor alone. Reading over the second paragraph, we find that in Sentence 7, the author says, "the misfortune is that, by a too easy confusion, the concept of evolution often emerges, in the hands of the positivists, from the formal emptiness which belongs to it in truth." Aha! See how that lines up with our skeleton of the critique? The author claims that it's a "misfortune" that positivists see evolution as deriving from "the formal emptiness which belongs to it in truth." That's the critique we need to identify, so Sentence 7 is correct!

The unique factor of Sample Question #4's question stem and the sentence it describes is its mentioning of "other fields of study." Where does the author talk about other fields? The second paragraph is devoted to the author's specific discussion of (in his opinion) flawed views of history, but in the first paragraph, you can spot the phrase "to construct a philosophy of morality or of law, of science or of art, and a general philosophy." This phrase appears in Sentence 2, which states, as a whole, "Now, whilst it is possible to reduce to general concepts the particular factors of reality which appear in history and hence to construct a philosophy of morality or of law, of science or of art, and a general philosophy, it is not possible to work up into general concepts the single complex whole formed by these factors, i.e. the concrete fact, in which the historical sequence consists." Our analysis has to get a little subtle here to confirm

Sample Question #3

Select the sentence in which the author suggests that the author accuses certain thinkers of seeing a part of the concept of evolution as its source.

> Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

Sample Question #4

Select the sentence in which the author suggests that the field of history contains the concepts and objects of other fields of study.

> Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

this sentence as the correct one, but because it's the only sentence in the passage that mentions other fields, it would be a fantastic guess if you were running out of time and had to move on to other questions. Let's confirm our choice, though: the author says that it's possible to reduce the particulars of history to general concepts and then use the fields of law, science, art, etc. as examples of this kind of reduction. Thus, we can make the logical leap that the author sees these fields as examples of "the particulars of history," meaning that he sees them as being contained within history like our question stem says. Sentence 2 is correct!

Text Completion and Sentence Equivalence

At a glance, the two types of questions the Verbal Reasoning section uses that are not accompanied by prose passages can look extremely similar. They both involve a question stem with at least one blank and a relatively large number of words presented that might fit into that blank. Despite their apparent similarity, these are two distinct question types, and they ask you to perform two distinct tasks. Text Completion questions focus on your ability to interpret the context of a sentence and parse out its clues to determine the correct words for one, two, or three blanks. What goes in one blank can affect what is placed in another, and you have to get each part of the question right to get any credit. Sentence Equivalence questions, in contrast, test both context interpretation and synonym identification. There's only ever one blank involved in each Sentence Equivalence question, but multiple answer choices might make sense; however, only one will form a pair that makes the sentence mean close to the same thing.

In the next section, we examine each of these question types in turn, focusing on recognizing the specific challenges of each format and teaching particular strategies that cater to each one. In the "Context Clues and Agreement" lessons, we go over the common patterns that context clues often take. Familiarizing yourself with these particular constellations of meaning can help you identify them on test day and ground yourself in a particular approach to each one.

Section Outline

Context Clues and Agreement

Contextual Motifs: Positives and Negatives

Contextual Motifs: Comparisons

Contextual Motifs: Cause and Effect

Context Clues and Agreement

In GRE Sentence Completion and Sentence Equivalence questions, you are not looking for a meaning or synonym in a vacuum, sealed away from any other words. You are looking for a word that fits the context of a given sentence. Therefore, you will need to search through the sentence to discover the word or words that indicate the meaning of your blank or blanks. Sometimes, a question could really have two very different meanings for one (or more) of the blanks provided. For instance, a sentence might say, "The two girls had a _____ relationship, often strained because of their temperaments." In this sentence, the word "strained" provides the key for your blank, noting that the relationship is likely difficult. In another sentence, the context might indicate that you are looking for an opposed meaning of some sort. For example, consider the sentence, "While there were a plethora of reasons for entering the building, Herbert was dissuaded by a _____ counter-argument." In this sentence, the blank must be read in the context of the first half, where there are said to be many reasons for entering the building. Apparently, Herbert is dissuaded by just one proposed reason. Hence, a word like "single" or "solitary" would work well in this context.

Although context is very important for both types of discrete verbal questions, it is pivotal for certain ambiguous cases with regard to Sentence Equivalence questions. Normally, when you read through a Sentence Equivalence question, you ask yourself, "What two synonyms will both work for this sentence?" This can sometimes trick you, though, for three options will sometimes be acceptable, though only two of them are strict synonyms, while the third is only a partial synonym. For example, consider the sentence, "The two philosophers were quite _____, energetically flailing their hands as they discussed the topic, which brought great excitement to both of them." You might then be presented with three options, such as, "clamorous," "boisterous," and "spirited." It would be very tempting to pick the first two options, as they both describe someone or something that is loud; however, in the current sentence, you have the philosophers described in terms of being energetic and excited. Their volume is not stressed. Thus, "boisterous" and "spirited" are the best options, as they both indicate the idea of being excited. Even though "spirited" does not necessarily indicate the loudness expressed in "boisterous," we need two words that fit the context of the sentence. Since these two words are at least "partial" synonyms, they are fine for the purposes of Sentence Equivalence.

The word "agreement" can take on two different meanings when we talk about one part of a sentence agreeing with another. On the one hand, agreement indicates a grammatical concern. For instance, in a given sentence, there should be agreement in number between the subject and the verb, as when we write, "Mary and Jane eat," and not, "Mary and Jane eats." Likewise, there should be an agreement in noun-pronoun case when you use a predicate nominative, thus writing, "It is she," and not, "It is her." On the other hand, the word "agreement" can indicate that one word should accord with the wider content of a given sentence. Thus, consider the blank in a sentence such as, "The weighty _____ where not mere pamphlets but, instead, were full treatises on the topic of nature of logic." In the context of this sentence, you know that you need a noun that is not merely agreeing with the idea of being a book (or other general term for book). It must be a word that adequately matches the overall sentence context—"not pamphlets," "full treatises." Clearly, these

books are tomes of knowledge.

Note that these are two very different senses of context. The grammatical context will be less important for discrete Verbal text-completion questions. On the GRE, it is rare that finding the correct answer is a matter of a word not fitting the grammar of your question text. In contrast, the content-based context will always be of critical importance. You should look for a contextualizing word or words that provide some key for deciphering at least one of the answer blanks in the sentence. In multi-blank sentences, it is often the case that, once completed, this blank will then provide you with increased contextual clues with which you need the remaining blanks to agree.

Sample Questions

Sample Question #1: Sentence Completion

The museum curator gingerly placed the _____ on the wire display that had been carefully crafted to allow viewers a close look at its surface, which displayed a small detail of the painted scene that had once covered the complete object.

opus
shard
canvas
urn
tome

When reading this sentence, you need to gather every possible clue about the object that is placed on the wire. It is very tempting to think that the curator is merely placing a painting on the wall, though it is somehow incomplete. Notice several important facts, however. First, displaying this item required the careful crafting of a wire for its display. There is something about the item that differentiates it from other normal constituents of a museum collection (and especially from any painting, which would be a normal affair for hanging). Second, there is only a portion (a "small detail") of a former full painting. Third, the sentence notes that the full scene was at one time on the "complete object"—implying that the displayed object is incomplete. Indeed, it seems that the painting was on some other kind of object (like a jar, cup, or something else like that). Thus, your word must agree with all of these descriptors: uniquely sized, only a portion of a painting, and indeed only a part of some complete object The only word that reflects all of these is "shard," a word that describes just such a portion of a complete object. (Think, for instance, of a shard of glass.)

Sample Question #2: Sentence Completion

Scientists may often sound like they can only speak (i) _____, for their (ii) _____ expressions are all but unintelligible to the (iii) _____.

Blank (i)	Blank (ii)	Blank (iii)
jargon	equivocal	laity
slang	careful	pundits
parlance	cryptic	avant-garde

You really need some context to ascertain what is being said about how scientists speak. Otherwise, this short sentence is open to multiple interpretations. Luckily, you can note that scientists' expressions are described as being "all but unintelligible." Put another way, they are almost completely unintelligible. This implies that scientists speak in a way that only other scientists can understand. "Jargon" is the kind of technical vocabulary that develops in a given field of work or study, uniquely applying only to that field. This well matches the idea of being "unintelligible." An important followup question should arise in this context: "Unintelligible for whom?" The answer is obvious: "For anyone who isn't a scientist!" Put another way, you could say that it is "unintelligible" for those who are not adequately trained. Among the options provided, only the word "laity" describes someone as being untrained. Normally, we contrast "laity" to "clergy" as regards religions, but the term "laity" can describe anyone who is not part of a given profession. In this case, we are speaking of "laity" with respect to science. To such people, jargon would indeed seem very unintelligible! Notice an important factor as we continue through the question: the context is expanding! The final blank is very easy to answer. To such "lay" people, these jargon-filled expressions will not be understandable—a point quite obvious now that we have filled in much of the sentence. The only word that describes this kind of lack of intelligibility is "cryptic," a word meaning *obscure*.

Sample Question #3: Sentence Completion

The construction foreman reminded his workers to exercise caution around the worksite, as a recent _____ of workplace accidents had left him wary of another incident.

dearth
consideration
spate
promotion
omission
rash

Obviously, if the foreman wants workers to exercise caution, there must be some good reason for his injunctions. Notice also the clear point expressed at the end of the sentence that the foreman fears the occurrence of another incident. This all implies that there had been a number of accidents, at least in recent days. Therefore, you are looking for a word that would be synonymous to "set of occurrences" or something like that. You can eliminate "dearth," which means the opposite—namely, *a lacking*. Obviously, as well, "omission" is not acceptable. The context clearly indicates that the concern is not with omitting accidents! The foreman certainly doesn't want to "promote" accidents, and mere "considerations" of accidents would not likely be a source of wariness about recent incidents. Thus, the best two options are "spate" and "rash." Both of these words describe *a situation in which some set of incidents happen in a quick succession*—as when we say that there was a recent "run" of events or happenings. This does make sense in the context of the sentence clues, for such a spate or rash of events would likely leave someone quite fearful that more accidents could occur.

Contextual Motifs: Positives and Negatives

Sometimes when you are reading a GRE Sentence Completion question stem, you will have a set of two possible options that will both seem to fit the blank, of which you'll need to pick one. The sentence could take on almost two totally different meanings and still appear to be correct. At times, this vexing situation can be overcome by looking for positive and negative connotations elsewhere in the sentence. Perhaps somewhere before or after the answer blank(s), there is a word that actually expresses a positive or negative evaluation. The answer you are looking for will then have to match this connotation's sense. In the context of the connotation, you will have a sense of the meaning of the blank. Likewise, in multiple-blank sentences, you will then see how positives and negatives might be played off each other.

For example, a sentence might read, "Isidore was known for being crafty, therefore most people were very _____ to speak to him in detail about their lives." The word "crafty" is actually quite key for this sentence. It can seem to mean nothing more than *intelligent*; however, it actually connotes *a kind of deviousness and tendency to use one's intelligence to manipulate others*. Therefore, the blank in the sentence will be something that reflects this negativity. People will be "reticent" or "hesitant" to give such information.

Sometimes, you will need to adjust carefully as you read multi-blank sentences. For example, a sentence might read, "Despite being _____ by most of the world, the political figure was still _____ by those who lived closest to him and should have known well his many talents." In this sentence, it is expressed that the politician has many talents—a truly positive state of affairs. Since the sentence opens with "despite," you know that the two blanks will be different in connotation. The second will be a negative (or partially negative) expression. Apparently the people did not have a positive view of him (though they apparently should have, because the figure has many talents). The first blank is positive, for apparently the rest of the world understand his many talents. Thus, a word like "venerated" or "respected" would work very well for this blank. Notice how far you can get by noting the mere connotation of a word or phrase!

Sample Question #1: Sentence Completion

The oatmeal was so _____, she decided to offer it to a friend and wait for lunch instead.

hearty
nutritious
cloying
scalding
thick

You need to apply a complete attention span for this question. If the sentence were to end on the word "friend," you really would not know if the oatmeal was very good or very bad. If the person was a very kind friend, she might well share an excellent bowl with her friend. Thus, the word "so" would take on a kind of positive intensification of the word in the blank; however, "so" is really a blank intensifier—making the bad worse and the good better, so to speak. When you continue reading, even ending on "wait for lunch" would equally ambiguous to ending on "friend." It is only when you come to the word "instead" that you pick up on a very subtle negative connotation. Granted, this is very subtle, so you need to be careful. Apparently the person didn't want the oatmeal, implying that there is something wrong with it. Well, being "scalding" hot would go away with a bit of time, and being "thick" is not necessarily horrible. Indeed, it is not as negative in meaning as "cloying," which means *sickeningly sweet*. Thus, based on negative connotation, this is the best option.

Sample Question #2: Sentence Completion

She was taken aback when the interior decorator (i) _____ her house with (ii) _____ after she had expressly stated that she preferred a modern, uncluttered style.

Blank (i)	Blank (ii)
bedizened	ornaments
decorated	knick-knacks
beautified	souvenirs

For this sentence, the expression "taken aback" indicates that the person was *surprised and displeased* at the actions of the interior decorator. This connotes a negative attitude toward his or her activity. Now, at the end of the sentence, it is indicated that the person preferred a lack of clutter. This means that the decorator must have added something that caused clutter. While all three of the options for the second blank indicate things that could cause clutter if placed in public spaces, the word "knick-knack" best matches the tone of the sentence. A "knick-knack" is *something that is held to be worthless*—mere clutter and nothing else. Likewise, the person probably didn't think the decorator did a good job beautifying or decorating the house. Instead, given the negative tone, the homeowner likely thought that the decorator "bedizened" the house, a word meaning that he or she *decorated it in a gaudy / overdone manner*.

Sample Question #3: Sentence Completion

The group happily spent the entire day hiking through the (i) _____ forests, and she thought the experience (ii) _____ and was somewhat (iii) _____ to return to town.

Blank (i)	Blank (ii)	Blank (iii)
eerie	nightmarish	reluctant
forbidding	idyllic	eager
verdant	tolerable	prepared

For this question, it is easiest to start with the first blank, given the blatantly positive connotation of the word "happily." This indicates that you should look for something positive regarding the forests. Thus, the negative notions of being "eerie" or "forbidding" really do not describe well the woodland through which the people hiked. Instead, the word "verdant" is best, as it indicates something that is *green and lush with plant growth*. Likewise, using this positive connotation, you can plow through the second blank as well. Clearly, she couldn't have had such a happy hike and yet find the experience either "nightmarish" or even merely "tolerable." Instead, the word "idyllic" describes something that is *very happy*—a much better match to the positive connotation of the sentence clues. Finally, there is a kind of positive-to-negative turn in the final blank. If the person found the hike to be so ideal and wonderful, she likely did not want to return to town. Thus, she would not be either "eager" or "prepared" to do so. Instead, she would be hesitant or "reluctant" to do so.

We can also apply the logic of positive and negative connotations to Sentence Equivalence questions, though this requires us to change our tactics slightly. Sentence Equivalence questions already have you searching out a pair of words that, when placed in the blanks in the sentence, make for two sentences that are equivalent in meaning. So, if we figure out that a word with a positive connotation needs to go in the blank, we're going to need to find a pair of them.

Sample Question #4: Sentence Equivalence

She focused her critique on the painting's _____ colors, arguing that a different scheme would improve the work substantially.

lively
resplendent
garish
complementary
vivid
gaudy

The expression "would improve the work substantially" indicates that the critic had negative feelings about the painting. Her critique was not a positive affirmation of its character, for if it were, she would not suggest substantial improvements. Thus, you need to look for words that indicate something negative about the color scheme.

As always, it is best to eliminate answers. The word "lively" is positive in its connotation, indicating something that *expresses a lot of energy and brightness*. Likewise, when something is "resplendent," it has an *impressive* appearance *because of an excellent coloration*. This is overly positive as well. The word "garish" does have a negative connotation: it indicates something that is *overdone* or *has a showy appearance*. We would only describe something as "garish" if we wanted to insult its appearance. Similarly, "gaudy" refers to *showy* appearances as well and has a similarly negative connotation. The correct answers are "garish" and "gaudy."

Sample Question #5: Sentence Equivalence

The _____ of being the only redhead in a family of blondes made her particularly fond of her new hair color and apt to keep dying it regularly.

solidarity
appeal
alienation
draw
inconsistency
concern

For this sentence, you need to ascertain just what the person's thoughts are about being the only redhead in her family. It is stated in the second half of the sentence that this made her "particularly fond" of her new red hair. Apparently she has dyed her hair red and has had positive reactions from all of her blond relatives. This state of affairs is desirable—hence, she continues to dye her hair, keeping the color. Thus, we can understand in particular why "appeal" is correct. This expresses this positive notion of desirability. "Draw" can also express such a positive situation; when used as a noun, the word can mean *attraction, glamor*, or *allure*. These are our two correct answers. We can easily eliminate the other wrong answers, for none of them directly describe something as being a desirable state of affairs. Thus, "solidarity" indicates unity and agreement—but not being desirable. Certainly, "alienation" is too negative to be correct. "Inconsistency" and "concern" likewise do not adequately match the positive connotation of the expression "particularly fond."

Contextual Motifs: Comparisons

Sentence context will often provide two very helpful clues for ascertaining the word needed for a given blank: language that sets up comparisons or contrasts, and comparative or superlative adjectives. Contrasting language will be indicated by words and expressions like "Instead of X, Y," "Although X, in fact, Y," or "X seemed to be the case, but actually Y." When reading through question stems, you should note any such contrasting keywords. This will help you to establish the need for a pair of antonyms to express the overall idea of the sentence.

You will need to look for additional clues beyond this "contrast indicator." In single-blank sentences, you will look for one of the contrasting words in the sentence itself, and your blank will be the opposite of that second clue, which provides one half of the antonym pair. For example, consider the sentence, "Although Peter had been very famous for years, he was now almost completely _____ in the community that had once lauded him endlessly." In this sentence, you will gather two clues. The first is the "although," which indicates that there is a contrast being made; however, to understand the contrast in question, you also need to notice that Peter's fame is being contrasted to his current state—apparently one that lacks this fame, indicating that he is now a rather unknown figure.

In multi-blank sentences, you will sometimes have to look for partial secondary clues and then try pairs of answer options to see which ones express the contrasting relationship adequately. Terms of comparison are often coupled with adjectives of the comparative or superlative degree. (Comparative adjectives indicate "more"—as in, "faster" or "more illuminating." Superlative adjectives indicate "most"—as in, "fastest" and "most illuminating"). In such comparisons, you will notice that the sentence indicates an increase in intensity in some way. For example, the sentence might read, "For a long while, Paula was a decent swimmer, but with much practice, she managed to become quite _____." The sentence indicates that Paula has become a *better* swimmer. Thus, you will look for an adjective that expresses this increased ability. Hence, an option like "accomplished" would work very well, for it indicates a great deal of skill (in comparison with merely "decent" skills). Notice that comparisons are like contrasts in one sense: they each indicate different scenarios; however, whereas contrasts indicate a difference between two things, comparisons indicate that two things are alike, though one happens to be *to a greater degree* than the other.

Sample Question #1: Text Completion

She was infuriated by her classmates' _____; she wanted to do something to fix the problems that everyone else seemed to accept as inevitable.

rebelliousness
doggedness
culpability
quiescence
fervor

The context clues for this sentence exist on both sides of the semicolon. In the first half of the sentence, we are told that the person in question was infuriated by her classmates' behavior. This is a rather open clue, one that could be interpreted in many ways. (For example, an irascible person could be infuriated by someone's cheerfulness, and a kind person could be infuriated by someone's insensitivity).

In the second half of the sentence, notice what is said about the classmates (i.e. "everyone else"). The sentence states that the others are accepting things as being inevitable and are not willing to do anything to ameliorate (i.e. to *improve*) some situations. Therefore, all that we can say about these people is that they are inactive in the face of changes that need to be made. Luckily, this is enough for you to identify the character trait that the subject finds so infuriating. The word "quiescence" describes something that is inactive. It is related to "quiet," and comes from the Latin roots for to be *still or unchanging*.

Sample Question #2: Text Completion

The judge, despite occupying a (i) _____ role, demonstrated her (ii) _____ when she ruled in favor of the district lawyer.

Blank (i)	Blank (ii)
prominent	bias
disinterested	animosity
partisan	expertise

Begin by noticing the context clue provided by the word "despite." This indicates that the first blank is going to be contrasted to the second. The word indicates that what one might expect from the first blank is, in fact, not the case (as expressed in the second blank). Another example of how this could be used would be, "Despite having been a smoker all of his life, George's lungs were in perfect condition."

We need to go searching for more context clues. At this point, the actual facts being contrasted are unknown. This is where the closing part of the sentence comes in handy. Note that the judge is ruling in favor of the district lawyer—a fellow legal figure. This clue will help us with the second blank (and, in consequence, with the contrasted first blank). By ruling in favor of the lawyer, the judge certainly didn't express "animosity" toward that person. Furthermore, there are no other context clues that could justify this option for the second blanks. Likewise, no context clues indicate that we are discussing the judge's "expertise." The word "bias" makes a great deal of sense, though: by ruling in favor of a fellow legal professional, the implication is that the judge is acting in a biased manner toward a fellow lawyer. This provides enough context for explaining the first blank, which needs to be the opposite of "biased." The judge supposedly should be disinterested—that is, *un*biased, not expressing (or acting upon) personal interest in the case.

Sample Question #3: Text Completion

Many think that the medieval university was a(n) _____ environment with little _____ when, in fact, it was the locus of quite _____ disagreement both publicly and privately.

Blank (i)	Blank (ii)	Blank (iii)
inchoate	disputation	polite
tumultuous	homogeneity	fervid
tranquil	rigor	rare

For this sentence, notice the key expression "when, in fact." This indicates that there is a contrast being expressed between the first half of the sentence and the second half. Many people have one opinion of medieval university culture, when, in fact, the details are quite the opposite of that opinion. Notice that the first two blanks are related, both describing the wrong perception. (Be careful, though: the second blank is an antonym for "_____ environment", as the sentence states "*with little _____*" —i.e. without some trait).

The third blank is likely the best starting point. It is possible that the sentence could indicate that the university was a location of polite disagreement (though that is a strange way of speaking); however, if that were the correct answer, you would need something for the second blank like

"agreement." This is because the first blank would indicate that most people think the university was a site of debate, having little agreement—when, in fact, it was quite polite. We do not have the answer options for the second blank that would enable such an interpretation.

Therefore, return to your options and consider "fervid," meaning *intense*. This gives you everything you need to complete the context! If the university was, in fact, fervid in disagreement, this indicates that most people misunderstand its character by thinking that it was "tranquil"—*peaceful*. Furthermore, if people think that it was such a peaceful place, they would think that there was little "disputation" there. Thus, one answer helps to provide all of the contextual clues needed for completing the sentence!

Sample Question #4: Sentence Equivalence

Rather than increasing the opportunities to drill for natural gas, the bill intends to _____ oil production near national and state parks.

concede
provoke
inhibit
bolster
exonerate
discourage

There are two important context clues for this sentence. The first is the introductory word, "Rather." This indicates that the bill in question is going to do something other than increase drilling opportunities. This latter point is your second contextual clue. In contrast to increasing opportunity, the bill must intend to reduce oil production in someway. The only two options that indicate such a lessening of opportunities for drilling are "inhibit" and "discourage."

Sample Question #5: Sentence Equivalence

The politician played up a(n) _____ image, but was actually born into considerable wealth.

proletarian
impecunious
relatable
bourgeois
spendthrift
parochial

The second half of this sentence provides the main context clues needed for answering the question. The "but was actually" indicates that there is a contrast being expressed. The contrast is to someone who is considerably wealthy. Thus, the implication is that the politician played up an image of not having considerable wealth. That is, he acted as if he were poor. The only two options that match these contextual clues are "proletarian," meaning *of the working class*, and "impecunious," meaning *having little money*. (The latter word comes from the Latin for "cow," prefixed by the prefix "in- / im-", meaning *without*. In simpler times, cows and livestock would be a kind of money. The "cowless" man was a poor man).

Sample Question #6: Sentence Equivalence

Amongst the chattering patrons at the museum's fundraiser, the _____ artist was incongruous.

stolid
wry
jovial
reticent
laconic
gregarious

This sentence establishes a contrast between the chattering patrons and the artist, who must not have been a talkative person. (To be "incongruous" indicates not fitting into a given environment). Thus, only two options describe the artist in question, namely "reticent" and "laconic," both words that indicate a lack of willingness to speak.

Contextual Motifs: Cause and Effect

One of the easiest kinds of clues to spot in the GRE Verbal Section are cause-and-effect clues. The old maxim, "Every effect is in some manner like its cause" really comes in handy for questions like these. Often, there will be a close correlation of meanings between cause and effect. While not synonymous, these words often pair well. If you know the general meaning of a cause, it is easy to answer the question, "What is the effect of this cause like?" Similarly, for a given effect, it is easy to ask and answer, "What might the cause be like for this effect?"

For example, consider the sentence, "The powerful, muscular man amazed his neighbors with his ability to _____ wood with his bare hands." Here, you can easily identify the cause. The powerful, muscular man is a cause of some effect in the wood. You will want to look for an effect that "measures up" to the cause. He won't merely "bend" or "saw" the wood. Many kinds of men could do this—even ones that are not strong and muscular. It will be a much better option to choose something like "shatter." This effect "measures up" to the cause.

Sometimes, the language of cause and effect is a bit more muted in the sentence structure. For example, consider the sentence, "After Rula removed the pebbles from the path, the smooth surface was no longer as _____ to walkers who used to slip on the walkway." In this sentence, the "cause" involved here is the removal of the pebbles, which renders the surface smooth. Once upon a time, people slipped on the pebble-covered pathway. Now, it must not be as dangerous. The removal of the pebbles has at least provided the occasion of safety being a possibility.

Consider another indirect case of cause-and-effect language: "Given his irascible temperament, Peter inspired _____ in his younger siblings." In this case, it is Peter's temperament that is a cause with regard to his siblings. Likely, it is the cause of fear, for being "irascible" indicates that someone is easily angered.

Sample Question #1: Text Completion

Byron was known for his (i) _____ style of speaking, which could almost cause temporary deafness in some listeners based on their (ii) _____.

Blank (i)
equivocal
stentorian
compelling

Blank (ii)
propinquity
response
volume

Begin by identifying the cause-effect relation expressed in this sentence. Byron's speaking is the cause of deafness of listeners. Depending on the options for the first blank, you may or may not need to factor in possible values to be chosen for the second blank. The word "equivocal" is used to describe a word or term that has two possible meanings. For example, "bank" can be used either to describe the side of a river or a financial institution. Hence, without any further context, we say that the word "bank" is equivocal. Likely, this is not the cause of deafness, so we can set it aside. The word "compelling" really does not describe a style of speaking that is likely to cause deafness. Thus, even if you did not know that "stentorian" is an adjective meaning *loud* (especially describing a style of speaking), you could pretty easily eliminate your way to the correct answer!

Now, for the second blank, you can just consider the options. Do not be tempted by "volume." The second half of the sentence is describing the people—not Byron's voice. Thus, it is not due to the listeners' own volume that they are deafened. No, rather, it is Byron's volume that is the cause of deafness. Likewise, "response" does not make any sense in this regard. Only "propinquity" is appropriate, for it means *closeness*. The people's closeness to Byron—with all of his stentorian speaking—is indeed a cause of their temporary deafness.

Sample Question #2: Text Completion

Ronald had quite a jittery temperament and would (i) _____ at even muted (ii) _____ as long as they were sudden, so it was not surprising that he dropped his coffee when the tuba's music (iii) _____ through the large hall.

Blank (i)	Blank (ii)	Blank (iii)
balk	noises	resonated
startle	notes	blew
bid	suggestions	intoned

First, let's identify any cause-effect relationships mentioned in this sentence. An obvious cause-effect relationship is the one between the tuba's music and Ronald dropping his coffee. Furthermore, since he has a jittery temperament, we can guess that any sudden noise would cause a startled reaction. Since the last blank has the clearest cause-effect relationship, let's start there in considering our answers. The word "blew" is really informal. While the tuba player may blow into a tuba, it is more proper to say that the tuba then makes a sound that remains in the hall. Likewise, the player does "intone" a melody; however, you need to have some action that can be said to be "through the large hall." The only option that makes sense, therefore, is "resonated"—meaning *sounded through*.

We need a jittery action for the first blank and a synonym for "sound" for the second. Luckily, the only word that indicates a jittery action is "startle." (When someone is said "to startle", he or she is alarmed or frightened. Thus, we can say both, "The tuba startled Ronald," and, "Ronald startled at

the sound of the tuba"). For the second blank, only "noises" is a good synonym for "sound" (i.e. the general cause of Ronald being jittery and startled).

Sample Question #3: Sentence Equivalence

Surrounded by a band of marauders, the unarmed reconnoiterer only had one prudent recourse: to _____.

| retaliate |
| strike |
| spy |
| parley |
| collaborate |
| capitulate |

The word "reconnoiterer" may be unfamiliar to you. It is related to "reconnaissance"—the discovery of information undertaken before a particular activity occurs. (We often use such words to describe the activities of armies). The reconnoiterer is surrounded, so the band of marauders is the cause of whatever action pertains to the blank in the sentence. He is likely going to have to surrender in some way. In any case, he will no longer remain a secret agent, looking for information. Thus, it is not likely that the marauders' presence will cause him to "retaliate," "strike," or "spy." (Indeed, the marauders are causing the spying to stop). We do not know if he will collaborate with them, either; however, he will turn himself over to them. Thus, the remaining two options are the best, though they are not strict synonyms. To "parley" is to *discuss terms with another party* (as the spy will do vis-à-vis the marauders). To "capitulate" is to *give in to another party*. In a way, stopping one's spying is a kind of capitulation. This is almost certainly what the marauders would want.

Sample Question #4: Sentence Equivalence

Arriving home hours later than they had originally planned meant that the cleaning they had wanted to accomplish before their guests arrived had to be _____.

| exhaustive |
| delayed |
| expeditious |
| tabled |
| scheduled |
| unsanitary |

The late arrival of the people is the cause regarding their changed cleaning plans. Apparently, they had wanted to complete some cleaning of their house before the arrival of their guests; however, this has changed because of their lateness. The overall effect will likely be either that no cleaning will occur or that any cleaning that does occur will need to be done quickly.

It makes no sense for the cleaning to be called "exhaustive." That is quite the opposite of what is needed. Exhaustive cleaning would take far too much time! The word "scheduled" is too general. It could mean either that they are going to schedule the cleaning for later or that they are going to schedule it carefully to be done. In either case, the word is inexact and does not have a good matching word among the other options provided. Likewise, we have no justification for making the strange remark that the cleaning will be "unsanitary." The lateness may mean that the house will be unsanitary because it is not cleaned at all; however, if it is cleaned in some way, it doesn't make sense to say that such cleaning would be unsanitary. It is true that the cleaning should be "expeditious"— *quick and efficient*; however, the pair "tabled" and "delayed" are closer synonyms. When something is "tabled," it is *delayed*—as though you took something that was actively being considered and placed it down on a table to await a later review and/or discussion. Therefore, these last two options are the best description of the effect of the people's lateness.

Question Strategies: Text Completion

Text completion questions form an important part of the GRE's Verbal Reasoning sections. These questions combine a focus on context clues with a demand for mastery of a large number of advanced terms. In facing them, one of your main concerns might be the risk of encountering vocabulary with which you are unfamiliar. You also might be worried about your ability to work with context clues—what if you miss one or more? It's difficult to anticipate which parts of the sentence will be significant and which will not at first glance. In the following sections, we present methods geared toward helping you sort through the sentence step-by-step to identify every aspect of context required to determine the correct answer. We also walk you through other strategies you can use specifically when facing text completion questions. Some of these come in particularly handy in those hopefully rare cases where unfamiliar words make an appearance. Don't just study a list of words and conclude that you're ready to apply them in Text Completion questions: by going over this question type's idiosyncrasies, you can be even more prepared to demonstrate your mastery of GRE vocabulary on test day.

Section Outline

Single-Blank Text Completion

Multiple-Blank Text Completion

Single-Blank Text Completion

Many of the Text Completion questions asked of you on the GRE's Verbal Reasoning section will involve only one blank and require a tactical approach somewhat different from those that involve two or three blanks. In this lesson, we consider a few strategies specific to this particular question type. Try them out and find which tactic or combination of tactics works best for you.

Read the Question and Come Up With A Synonym

One strategy that works particularly well when approaching single-blank Text Completions is to read the sentence and contemplate the blank in it before reading the answer choices. What sort of word would make sense in the blank? Come up with a couple of terms that you think would make the sentence make sense. Then, read the answer choices and compare them to your ideas. Which are closest to your predictions? Focus on those as the potential correct answers. By asking you to come up with your own theory about what word goes in the blank, this strategy gets you to read actively for cues regarding connotation, relative strength, and thematic consistency without having to go down a list and make sure you consider each individually. Having a prediction against which to compare the answer choices can make it obvious which of the choices don't meet the sentence's requirements.

Eliminate Incorrect Answers

Text Completions' answer choices will each be the correct part of speech and be able to function syntactically (grammatically) in the blank. To facilitate this, some of them may include prepositions or other modifiers. Thus, don't expect to eliminate any answer options because you know a noun needs to go in the blank but one of your options is a verb.

You may be able to make some non-grammar-based inferences about the blank that allow you to cull multiple incorrect answer choices at once. For example, if you identify that the word that goes in the blank needs to be a word for a person, you might be able to knock out a few incorrect options.

If you skip a single-blank Text Completion, jot down some notes about any answer choices you've determined can't be correct. You don't want to have to "retrace your steps" later if you return to the problem!

Use Process of Elimination to Deal with Unknown Vocabulary

What if the worst should happen and you're faced with a word you've never seen before? Don't let it throw you off. For single-blank Text Completions, you're presented with four other words. If you know those, pretend that the question only gives you those four options and a "None of the Above" option—but in this case, the "None of the Above" is the unknown word. After all, one of the terms is the correct answer, so if you can determine that it is not any of the four words you know, you've just

determined that the correct answer has to be the word that you don't know.

If you're faced with two or more words you don't know in a single-blank text completion, it may be best to eliminate any incorrect answer choices that you can and pick from the remaining answers. You still have a chance of getting the question correct and can return to it later if you have time, but it's best not to spend too much time stewing over one particular question when you have the rest of the section with which to contend. Don't let your perspective get too bogged down by a single question: confidently continue.

Make Sure You Understand Words' Part-of-Speech-Specific Definitions

Some words can function as multiple parts of speech and take on different meanings based on the form in which they're used. Briefly pause to read your list of answer choices and consider if you've mistakenly assumed the wrong definition for such a term, especially if you've assumed a common definition is in play but the word has a more obscure meaning! "Rubric" is a great example of this. Used as a noun, it can refer to *a statement about purpose, a category, or the grading scale constructed for a particular assignment*. Used as an adjective, it can mean *red*. Consider the following lists, which present examples of words that can be used as different parts of speech. One list highlights those terms whose definitions don't drastically shift between different parts of speech—they mean pretty much what you'd expect them to mean. The other list is composed of terms that do subvert this expectation and mean very different things depending on the grammatical context in which they're used. "Precipitate" and "fell" illustrate this point below. For more examples, consider the words "plumb," "table," "stint," "tender," "pedestrian," and "flip."

Precipitate

Noun: a solid substance that falls out of solution and collects at the bottom of the vessel in specific chemical reactions
"We were surprised at how much precipitate we collected until we realized we'd used a different acid than we intended.

Verb: to encourage or cause to happen
"The website worried that social media would precipitate a fast and widespread reaction to the news story even though it had been taken down the day it was posted."

Adjective: hasty and not well-thought-out
"On Friday, the group decided to go camping that same weekend, an especially precipitate decision as none of the members had ever been camping before."

Noun: a highland moor
"Sheep dotted the <u>fell</u>, but the shepherd was nowhere to be seen."

Fell

Verb: past tense of "fall"; contacted the ground due to a temporary loss of balance
"When the vase <u>fell</u> off of the bookshelf, she led out an audible gasp before it shattered on the floor."

Adjective: ferocious and deadly
"The travelers in the forest were apprehensive about the prospect of encountering <u>fell</u> beasts, especially because they had no weapons."

Read the Complete Sentence for Each Potentially Correct Answer Choice

Don't simply look back and forth between the sentence and answer choices trying to figure out which word goes in the blank. Read the entire sentence or passage to yourself in your head and substitute each answer option in the blank. Sometimes, especially with longer texts, there might be a context clue at the end of the sentence that affects your understanding of what goes in the blank, and reading the entire sentence with each answer choice in it can help mismatches stick out more easily than just zeroing in on the blank and its immediate surroundings.

Always Mark an Answer, Even When Skipping the Question to Return to Later

If you're not 100% sure about a certain single-blank Text Completion, it can be helpful to return to the question later with a fresh perspective; however, this assumes that you will have time enough on the exam to do so. Always mark an answer down for a question even if you're skipping it with the intention to do more work on it later. There's no guessing penalty on the GRE; your scores on the Verbal section are calculated based on the number of correct answers you submit. Always selecting some answer means that even if you run out of time, you've still got a chance of getting the question correct.

Keep in mind that the more answer choices you can identify as incorrect, the better odds you'll be giving yourself if you end up needing to guess on a question. Every answer choice you omit from consideration in a single-blank Text Completion problem improves your chances of picking the correct one at random by 20%.

Let's next go over some sample single-blank Text Completion questions to practice using some of these strategies.

Sample Question #1

His peculiar _____ hiring art students in his business set him apart from his competitors who would never think of it.

aversion to
infrequency of
bent for
indifference to
disgust at

As his competitors would "never think of it," hiring art students is something peculiar and interesting for the subject of the sentence. The correct answer should mean *a particular fondness for or inclination toward*, the definition of "bent," when used as a noun.

Read the sentence and consider which word would make sense in the blank. The phrase "who would never think of it" is used to describe "his competitors"; this tells us that the subject does hire art students, while his competitors never do. Don't jump to the answer choices to narrow them down just yet, though: come up with a hypothesis about what goes in the blank. It has to be a positive word about the subject's specific pattern of hiring art students, which we're told is "peculiar." Words like "penchant," "habit," "wont," or "preference" would work, with proper prepositions.

Now look at the answer choices. "Aversion to" and "disgust at" can't work because we know that the subject likes hiring art students. "Indifference to" doesn't make much sense, because the subject presumably has opinions about hiring art students in his business, as he does so even though it is "peculiar." This leaves us with "infrequency of" and "bent for." "Infrequency of" doesn't make sense, as the sentence contrasts the subject's hiring of art students against his competitors, who wouldn't hire any. It's a binary: frequency has nothing to do with it, and if we were to speak in terms of frequency, the subject would more frequently hire art students than his competitors (because he hires any at all.)

Through process of elimination, we've arrived at the correct answer: "bent for." If you looked at the answer choices first, it'd be easy to assume that "bent" was being used as an adjective meaning *angled out of the expected (often straight) shape*. That's not how the term is being used, though! It's being used as a noun meaning *inclination*, which matches up well with our predicted definition.

Sample Question #2

Although it is widely accepted today, Wegener's theory of continental drift was hotly contested when it was first proposed, which made him a _____ in the scientific community of the time.

pariah
hero
firebrand
debutante
writ

Wegener's theory was rejected when it was first proposed, and thus the scientific community must have not thought highly of it; therefore, "pariah," which means *outcast*, makes the most sense in this sentence.

Reading the question stem in Sample Question #2 makes it apparent that the correct answer has to be a word for a person. We're told that Wegener's theory was initially "hotly contested," but is now "widely accepted." We then need to find a word describing what Wegener's theory made him at the time it was first proposed. The correct answer is going to need to have a definition or negative connotations of being rejected from a group, like "outsider."

Ok, let's look at the answer choices. "Writ" can't refer to a person, so we can ignore it. "Hero" is far too positive, so we can ignore it too. "Debutante" refers to *a rich young lady first going out in society*, which doesn't fit the context of the sentence at all.

This leaves us with "pariah" and "firebrand." Both could work in a negatively-connoted sentence, but are distinguished by their specific meanings: "firebrand" means *a fervent person igniting radical shifts*, whereas "pariah" means *social outcast*. While these answer choices both might seem good, "pariah" is the better one, as "firebrand" has the connotation of someone purposely trying to start arguments and debates, e.g. a rabble-rouser, whereas we're not clued in that Wegener had any such motive, merely that he was rejected by the scientific community. This makes "pariah" the better answer.

Multiple-Blank Text Completion

Text Completions on the GRE Verbal Reasoning section need not limit themselves to a single blank: two- and three-blank text completions increase both the complexity and the difficulty of the task of selecting the best words for their blanks. In this lesson, we outline some particular strategies you can use when answering questions in this format on test day.

Slow Down and Work Carefully

When coming across a multi-blank text completion question, especially after working on a problem of a different format, the first thing you need to do is ignore the urge to rush through it. The sight of multiple blanks and a question stem that is likely longer than those used in single-blank text completion problems can set your heart to pounding. Time required and problem difficulty may increase with the number of blanks in a text completion, but the test designers know that. Make the most of your testing time by focusing and working through these questions at a measured pace. Missing the import of even a tiny preposition can cause you to err in your analysis for one blank, and there's no partial credit given!

You Don't Have to Determine the Blanks in Order

Don't assume that you have to decide on your answers to the blanks in the same order in which they appear in the sentence. Oftentimes, a phrase or clause that concludes the sentence may provide vital information that allows you to determine a later blank before the first one. Take a moment to get the lay of the land for each question stem and decide which blank you want to attack first. It might not necessarily be the first one!

Analyze Sentence Logic Carefully

Accurately understanding the logical shifts that take place in the lengthy question stems often associated with multi-blank text completions is crucial to answering these questions correctly. While it's important to consider subtle details, remember to mentally "step back" from the question and consider any comparisons, contrasts, or logical shifts signaled by terms used to signal transitions or shed light on the way concepts interact in the text (e.g. "except," "however," "despite"). You may want to read the sentence a few times to make sure that your understanding of its content and of its logical underpinnings sync up.

Eliminate Incorrect Answers

Even though you're answering multiple distinct questions for a single multi-blank text completion question, make sure to eliminate any answers you can discern are incorrect. You're only choosing from three answer choices for each blank, so if you can narrow your options down from three words to two, this greatly improves your odds (by 16.7%) of getting that blank correct if you end up having to guess.

Use Process of Elimination to Deal with Unknown Vocabulary For Each Blank

Eliminating incorrect answers can also be a go-to tactic if you don't recognize one or more of the terms present in the answer options for a multi-blank text completion. If you can determine that the two words you do know for a particular blank's options don't make sense in context, the correct answer will be the third, unknown word. If you don't recognize multiple words for a particular multi-blank text completion, it may be best to flag that question and return to it later if you have time. But remember the next point!

Always Mark an Answer, Even When Skipping the Question to Complete Later

Even if you plan to return to a question later and have plenty of time to do so, always mark an answer down for each part of a multi-blank text completion question. If you're confident in some but not all of your responses, you may want to jot down a note indicating the blank you're not sure of to save time and avoid potential confusion later.

Read The Entire Sentence When Substituting in Each Word

Even when deciding on which word works best in one blank out of three, read the entire sentence to yourself mentally when trying out different words. This way, it's much harder to accidentally miss subtle cues located far from the blank in question. Reading the whole sentence in this way can help prepare you for determining which words go in the other blank(s), as once you've picked out a word for your first blank, you'll have a more complete picture of the sentence as a whole.

Read the Complete Sentence Once You've Picked Out All Your Answer Choices

Once you've selected each of your answer choices, read the completed sentence to yourself mentally before moving on to the section's next question. Don't rush through this, as any inconsistencies you catch might only become apparent once you've substitute in your three-word combination as a test.

Let's practice a few sample questions so you can try putting some of these strategies into action.

The professor was an utter (i) _____; he was not merely attentive to details, but was absolutely (ii) _____ in his attention to every individual point.

Blank (i)	Blank (ii)
pedant	proper
boor	fastidious
tyrant	spendthrift

Let's approach this question slowly and carefully. In which order should we treat its blanks? The first blank will contain a noun for the professor, and that noun is determined by the clause after the semicolon. That clause contains a blank that needs an adjective—an adjective that is a more intense version of "attentive to details." Here, it makes the most sense to start with the second blank, since that adjective should be thematically related to the noun that goes in the first blank.

The second blank indicates an intensification from the adjectival phrase "attentive to details." "Spendthrift" doesn't have anything to do with attention or details; it has to do with how one spends money and resources, so it's incorrect in this sentence. "Proper" doesn't make much sense, as it is somewhat mismatched to the concept of paying attention to details and thus doesn't convey the intensification we need it to in the phrase "absolutely proper." Thus, the best option here is "fastidious," which means *very attentive to details*—an appropriate option.

As for the first blank, we now need to pick out a noun that describes someone who is very attentive to details. A "boor" is *someone with bad manners* and a "tyrant" is *someone who abuses his or her position of power*, so neither of those work in the sentence. When a professor is so attentive to detail, it is likely that he will be called a "pedant," which describes just such an academic person.

Let's read our sentence with our answer choices substituted in to make sure our complete answer makes sense: "The professor was an utter pedant; he was not merely attentive to details, but was absolutely fastidious in his attention to every individual point. That sentence is coherent and each term we picked is appropriate to its context. "Pedant" and "fastidious" are the best answers.

Let's look at another two-blank sentence completion and practice using this lesson's strategies to answer it.

Sample Question #2

The man continuously (i) _____ the medicine's amazing ability to cure everything from the common cold to dry skin to hair loss; however, not many of his listeners believed that the it was the (ii) _____ he claimed it was.

Blank (i)
rescinded
extoled
criticized

Blank (ii)
odyssey
zenith
panacea

Let's start with the first blank this time. The word that we choose for the first clause will set up how the salesman describes the medicine, and the word that goes in the second blank will need to be a noun matching that description, since it is found in the phrase "the _____ he claimed it was."

Our three options for the first blank are "extoled" (*praised highly*), "rescinded" (*took back a decision, law, or opinion*), and "criticized" (*pointed out the flaws or shortcomings of*). "Rescinding" we can ignore because it doesn't make sense in the sentence's context. This leaves us to determine whether the salesman was saying positive or negative things about the medicine. We can't really determine this based on the rest of the clause, so let's hold off and look at the second blank's terms. It's ok if you narrow down your answer choices a bit and then consider the other blank if you run out of context clues that can help you distinguish between the remaining choices.

The answer choices for the second blank are "panacea," (*cure-all* or *thing too good to be true*), "zenith" (*the top of something either literally (e.g. a mountain) or figuratively (e.g. a career)*), and "odyssey" (*a long journey full of difficult obstacles*). "Zenith" and "odyssey" don't make any sense in the sentence, so the answer must be "panacea," which makes a lot of sense as it specifically can describe a medicine that supposedly cures everything. This means that the salesman must be claiming that the medicine can cure everything, so he must be "extoling its amazing ability to cure everything . . ." Let's read over the entire sentence with these answer choices substituted into the blanks: "The salesman continuously praised the medicine, extoling its amazing ability to cure everything from the common cold to dry skin to hair loss; however, not many of his listeners believed that the medicine was the panacea he claimed it was."

Let's go over one more example, one that contains three blanks. The same tactics and principles apply to three-blank text completion questions as two-blank text completions.

Sample Question #3

The editor reduced the introduction from ten (i) _____ pages to two (ii) _____ paragraphs by (iii) _____ all of the unnecessary verbal flourishes that riddled its sentences.

Blank (i)	Blank (ii)	Blank (iii)
officious	sedulous	inhibiting
turgid	trenchant	qualifying
equivocal	succinct	excising

It looks like the first two blanks in this sentence are contrasting against one another in a before-and-after situation, so let's look at the third blank. We know from the start of the sentence that the editor "reduced the introduction from ten . . . pages to two . . . paragraphs," so what would he or she have to have done to "the unnecessary verbal flourishes that riddled its sentences"? Those would have to go, so our verb needs to mean something like "omitted" or "got rid of." Our options are "inhibiting" (*preventing* or *hampering*), "qualifying" (*hedging one's claims with reservations so that they are conditional*) and "excising" (*removing*). "Excising" is the best answer as it lines up with our prediction and the other two options don't make sense in the sentence.

Now we need to pick out terms for the first two blanks. The first blank will need to describe the introduction while it still contained "unnecessary verbal flourishes," so we will need a word that means something like flowery, not concise, or using a lot of words. Our options are "turgid," (*swollen or stylistically affected and overblown*), "officious" (*overbearing and offering help where one is not needed*), and "equivocal" (*ambiguous, unclear as to which of multiple sides one's statement is supporting*). "Turgid" is the best option because of its second definition, which can describe overly fancy prose in a negative manner.

For the second blank, we need to pick out a term that describes the introduction after it has been edited down to two paragraphs. This might be a term like "pithy" or "direct." Our options are "succinct" (*concise*) "trenchant," (*biting and sarcastic*) and "sedulous" (*hard-working*) . "Succinct" fits the context of the sentence because it can describe prose that has been shortened, so it's the best answer.

Let's test out our answer choices in the sentence: "The editor reduced the introduction from ten turgid pages to two succinct paragraphs by excising all of the unnecessary verbal flourishes that riddled its sentences." That works! These are the correct answers.

Question Strategies: Sentence Equivalence

It's easy to get thrown off when initially encountering Sentence Completion questions on your first Verbal Reasoning section. No matter whether you were just working on Reading Comprehension or Text Completion questions, Sentence Completions ask you to demonstrate a different set of skills. Like Text Completion, this question format tests your ability to discern the context of a sentence and other subtle cues; however, there isn't simply one correct answer: there are two. You're directed to identify the pair of words that, when inserted into the blank, create two coherent sentences with the similar meanings.

By meshing the tasks of synonym identification with contextual consideration, Sentence Equivalence questions present a unique challenge; however, since this question format tests two distinct skills, you can use either one as an entry point into the correct answer. In the following sections, we outline two strategies that you can use, one focused on each skill the format tests. In "Deriving Meaning from Context," we focus on the question stem first and rely on reading comprehension skills heavily. In "Equivalent Vocabulary," we focus on the answer choices first and make use of a strong grasp of GRE-level vocabulary.

We recommend that you not jump to conclusions and study only one of these approaches in isolation. Even if you feel that your skill set will lead you to heavily prefer one over the other, it's worth learning and practicing both so that you have multiple paths you can take to the correct answer on test day. If one or the other isn't yielding the desired result for a specific question, you can try the other and perhaps make a breakthrough that will let you arrive at the correct answer.

Section Outline

Deriving Meaning from Context

Equivalent Vocabulary

Deriving Meaning from Context

One way to approach Sentence Equivalence questions is by beginning with the question stem in isolation and analyzing it fully before even reading the answer choices. This method works particularly well if you've developed strong reading comprehension skills, as it relies on your ability to pick up on subtle shades of meaning, connotation, context clues, and sentence logic. You won't have to memorize a list of points to consider as you read the sentence, though! This is all wrapped up in the process of asking you to come up with your own prediction of what word will go in the blank.

You might be thinking, "Wait, before I've even read the answer choices? What good will that do? It's not that likely the word I pick will be an answer choice!" The point of coming up with your word or words for the blank is not to try and pluck the correct answer out of thin air before you even read the choices—far from it. This part of the strategy aims to get you to consider what we know about what has to go in the blank, e.g. is it a positive or negatively connoted word? What's the sentence talking about? What fits thematically and uses the appropriate strength? In order to come up with a word that is likely to go in the blank or even could make sense in the blank, you have to consider all of these things. Coming up with the word is just a way to get yourself to consider all of the contextual pieces at work in the question stem. If you read the sentence closely enough to produce your own hypothesis, you can be confident that you understand it well enough to pick out the correct answers from those handed to you.

Let's work through one example together before trying a few more problems with less guidance. First, let's just consider the question stem:

Sample Question #1

The _____ from the back of the room was disquieting to the nervous boy, who hated being laughed at.

We need to find a noun that goes in the blank that "the nervous boy" finds "disquieting"—that is, it makes him uncomfortable. A lot of things could be disquieting, so we'll need more information. The sentence provides this at its conclusion: "who hated being laughed at." If the nervous boy hates being laughed at, the word that goes in the blank must logically be some synonym of "laugh." Specifically, it might be a mean sort of laugh, the type of laugh that would be used in a situation in which someone is laughing *at* someone else rather than *with* them. Specific words we might encounter could be "snicker" or "snigger," both of which have this particular connotation. Other words for laughter could work too, though: for example, "giggle" could work in this context too.

Now that we have a good reading of the question stem, let's consider our answer choices:

- [ ] guffaw
- [ ] candor
- [ ] ridicule
- [ ] jaunt
- [ ] snicker
- [ ] derision

We can immediately ignore "jaunt" and "candor," neither of which have anything to do with laughter. "Ridicule" and "derision" could each easily have to do with a situation in which someone is being laughed at, but note that neither of them specifically refer to kinds of laughter, and we need words with that specific shade of meaning. The only remaining two answer choices are the best answers: "guffaw" and "snicker," both of which refer to types of laughter.

Let's go over one more question to practice this approach again.

Sample Question #2

The online journal saw its best work as its _____ of political speeches, as it regularly posted long articles that cut through speeches' rhetoric and detailed the substantive policy claims.

This sentence is a bit more complex. The word that goes in the blank will be a noun related to political speeches by the preposition "of." The journal sees this noun as "its best work," so there's presumably some sort of writing involved, as "online journal[s]" are written publications. What are these writings about political speeches specifically, though? We're told that the reason the journal sees these writings as its best work is that it often publishes "long articles"—ok, that's referring to our ambiguous "writings" we know the word in the blank has to capture, but now we're given more detail: these articles "cut through speeches' rhetoric and [detail] the substantive policy claims." It sounds like interpretive work is being done here: the journal analyzes the political speech to clarify what exactly is being claimed. Words that work in the blank would capture this sense, and thus be terms along the lines of "clarification" or "explanation." The terms we're after could also be more specific to literature, since we know they have to refer to a written work explaining a speech, another kind of text. Both text-specific and general terms could work, though, so keep that in mind as you look over the answer choices.

- [ ] exegesis

☐ equivocality

☐ exculpation

☐ recantation

☐ eulogy

☐ interpretation

This time, our guesses didn't line up exactly with one or more of the answer choices, but it isn't far to jump from "clarification" and "explanation" to "interpretation," especially since we figured out that the sentence is discussing interpretive work. If we'd only noticed that we needed a term referring to a type of text, "eulogy" and "recantation" might look like viable options, but given our close reading of the sentence, we know they aren't correct. Similarly, political speeches might be accused of being "equivocal," but "equivocality" doesn't work in the sentence's specific context. "Exculpation" doesn't make much sense either, and it doesn't refer to a written work specifically. The best answers are "interpretation" and "exegesis." Even if you didn't know what "exegesis" meant, we eliminated all of the other answer options, so you'd be able to get the question correct without knowing its definition. "Exegesis" fits the context of the sentence well and is a distant synonym of "interpretation"; it refers to a written interpretation of a written work, specifically a religious one.

Try applying the approach we've practiced to this next question on your own. Remember the importance of focusing on the question stem before widening your scope to consider the answer choices.

Sample Question #3

Instead of having a master narrative, the book contained many _____ around the same theme.

☐ euphemisms

☐ vignettes

☐ metaphors

☐ eulogies

☐ verifications

☐ anecdotes

This sentence contrasts what the book does contain with "a master narrative." It doesn't have a

master narrative, but it does have "many" of something "around the same theme." We'll need a plural noun, one of which a book could reasonably contain many of, and that can be said to have themes. This noun has to contrast against "master narrative," too, so in order for the comparison to make sense, our noun must refer to the larger structure of the book. If book is made up of many short narratives about the same theme instead of one big narrative, we need a word that means "short narratives"—something like "stories."

When we look at the answer choices with this in mind, it becomes easy to ignore choices like "metaphors," even though the literary context of the sentence might have made this a choice that seemed potentially correct if you didn't read the sentence closely to begin with. The correct answers are those that are closest to our approximation of "stories": "vignettes" and "anecdotes," each of which are not only refer to types of stories, but to *short* stories.

Equivalent Vocabulary

After reviewing a great deal of vocabulary for the GRE, you might want to practice a strategy that relies on your wide-ranging knowledge of obscure terminology. If so, this lesson is for you! We're going to tackle Sentence Equivalence questions from the "bottom-up" by reading and analyzing the answer choices before even looking at the question stem. Our goal will be to pair terms that could potentially function as synonyms so that by the time we approach the rest of the question, we have groups of answer choices associated, which will make it easier to pick the correct pair based on context from amongst the other pairs we've identified. Let's try this strategy out on the following sample question! But wait—we're only going to consider the answer choices first, remember?

- [ ] milieu
- [ ] ambiance
- [ ] style
- [ ] environment
- [ ] medium
- [ ] locale

First things first: we need to consider what each word means. You don't need to write out definitions on your exam, but we provide them so you can get a sense of the connotations of each.

milieu	the general social environment providing context for someone or something
ambiance	the feel or mood of a certain location or event
style	flair or personal touches included in a creative work; a widely-recognized aesthetic technique, e.g. "Greco-Roman style"
environment	the context in which something exists; nature considered as a whole (e.g. "the environment")
medium	the concrete materials that an artist uses to create art
locale	the place in which something takes place

How could we pair off these words in ways that make sense? "Milieu," "environment," and "locale" all relate to place in varyingly abstract ways; "medium" and "style" both have to do with works of art, and "style" and "ambiance" both have to do with the details of events or (potentially) of creative works. If we take a more in-depth look at these terms, though, we find that only a few could potentially work as synonyms. "Medium" and "style" are actually vastly different concepts—not synonymous at all.

Keep in mind that this process need not take very long: you can skim over the answer choices and consider how they mesh together relatively quickly. Even sketching a diagram or jotting down potential answer combinations could be accomplished in a matter of seconds.

It appears that potential pairs of words might be the following:

milieu, environment: both mean "place" but apply abstractly / generally

environment, locale: both mean "place" but apply concretely

At this point, it looks like all we might have to determine is whether our sentence calls for an abstract term or a concrete one. Now that we've analyzed our answer choices, it's time to consider the question stem.

Sample Question #1

> Many art critics believe a painter cannot be separated from the _____ in which she worked, leading many critics to brush up on the cultural history of famous painters' eras.

A word synonymous with "environment" certainly makes sense here, but does an abstract term or a concrete term make more sense? The rest of the sentence discusses "cultural history of famous painters' eras," so claiming that a painter can't be separated from the milieu (*social and cultural context*) in which she worked makes more sense than claiming she can't be separated from the locale (*the physical location*) in which she worked. The correct answers are "milieu" and "environment."

Let's work through one more sample question from the bottom up.

- ☐ cynicism
- ☐ largesse
- ☐ altruism
- ☐ pragmatism
- ☐ parsimony
- ☐ munificence

All of these words have to deal with one of two things: general outlooks or money. Some of them have to do with both. Let's define each one.

cynicism	the general belief that people act only to help themselves and not others
parsimony	extreme dislike of spending money or using resources
altruism	selflessness
pragmatism	realism and consideration of things as they are actually accomplishable / applicable
largesse	generosity in giving gifts or money
magnanimity	large-heartedness, generosity

We can first break apart our answer options into positive words and negative or neutral words.

Positive Words	Negative or Neutral Words
altruism	cynicism
largesse	pragmatism
munificence	parsimony

Which words from each list might together function as near-synonyms?

largesse, munificence

cynicism, pragmatism

Time to consider the question stem and figure out whether it sends us in a positive or negative direction.

Sample Question #2

While the millionaire believed it best to announce his charitable giving, many critics thought he wanted to advertise his _____ too much.

This one is a bit tricky, as "critics" are presumably not approving of the millionaire's announcing his charitable giving. While we are surrounded by this negative perspective, it doesn't affect the word

in the blank, which needs to be a restatement of or at least associated with "charitable giving."

"Cynicism" and "pragmatism" aren't correct. "Largesse" and "munificence," which both mean generosity and can specifically describe types of financial generosity, are the correct answers.

Try applying this tactic to the next sample question on your own! Remember to consider how the answer choices relate to one another before incorporating the question stem into your answer-selection process.

Sample Question #3

The young associate had a peculiar tendency toward _____ at the most serious moments.

- [ ] asceticisim
- [ ] levity
- [ ] irascibility
- [ ] intractability
- [ ] mirth
- [ ] sobriety

Let's define each of the terms:

asceticisim	the practice of self-denial and abstaining from pleasure, most often for religious purposes
levity	jollity, joy
frowardness	stubbornness, unwillingness to cooperate
intractability	stubbornness, unwillingness to change one's plans or opinions
mirth	glee, high spirits
sobriety	seriousness and stoicism; the state of not being intoxicated

"Asceticism" and "sobriety" can certainly be paired together, as can "levity" and "mirth" and "intractability" and "frowardness."

Now let's check out the question stem! It tells us that the associate has a "peculiar" habit of acting in some way "at the most serious moments." For it to be "peculiar," it couldn't be expected, or it

would need to be the opposite of what we'd expect. This means that "asceticism" and "sobriety" aren't the correct answers: that's the kind of behavior we'd approximately expect the associate to demonstrate in serious situations, which we're told is not the case. Plus, the connotations of "asceticism" and "sobriety" have more to do with self-denial and self-abnegation than with just maintaining a serious demeanor. While "sobriety" can refer to *seriousness and stoicism*, "asceticism" has no such meaning.

Would we be more surprised if the associate showed too much glee ("levity" and "mirth") or too much stubbornness ("frowardness," "intractability") in a serious situation? Too much stubbornness doesn't make as much sense as too much glee does, because glee and seriousness are opposites. The best answer choices are thus "levity" and "mirth."

Quantitative Reasoning

The GRE Quantitative Reasoning section is imposing in the breadth of material it covers and the variety of ways in which it tests them. In the following chapter, we break our lessons apart into sections that focus on the four main areas of mathematical content tested on the exam, and specific strategies that one can use when answering each of the test's four distinct question types. While it may seem like the best plan to jump into reviewing the most difficult material first, we recommend beginning your review by studying basic principles that you may not have considered closely for a long time. The details of these elementary concepts form the building blocks that you need to grasp the more complex principles, so a step-by-step, ground-up approach is advisable. In addition, we recommend paying attention not only to the mathematical principles you need to understand, but the specific ways in which the exam tests these principles. Familiarizing yourself with each of the question formats can not only help you save time on test day by applying efficient strategies, but it can also help you remain calm when confronted by challenging question formats. Designing your review to cover both basic and complex ideas as well as form and content is a recipe for success on the Quantitative Reasoning section.

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Section Outline

GRE Quantitative Review

Arithmetic

Algebra

Geometry

Data Analysis

Question Strategies: Quantitative Reasoning

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GRE Quantitative Review

Before tackling the nuts and bolts of the GRE Quantitative section's tested topics, it's important to make sure that you've brushed up on your basic mathematical skills and that you understand the specific context in which you'll be asked to solve quantitative problems. In "Calcultor Use," we consider the specifics of the digital calculator you'll be provided on the Quantitative Reasoning section. Understanding exactly what you can and can't do with this tool can save you valuable time and prevent potential mistaken assumptions about how it can augment your computational abilities when you sit for your exam. In "Properites of Algebra," we review the most fundamental assumptions about algebraic notation and the interaction of different variables. In "Word Problems: Mathematical Modeling," we take a general look at how to approach any questions that are presented in the form of a paragraph with content that you need to model algebraically. Before diving in to the rest of your Quantitative Reasoning review, take the time to shore up your fundamental skills. Establishing your mastery of these topics early can ensure they don't distract from your later review of more complex content.

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Section Outline

Calculator Use

Properties of Algebra

Word Problems: Mathematical Modeling

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Calculator Use

An on-screen calculator will be provided during the Quantitative Reasoning section of the GRE. It will look similar to the one shown at right.

This tool is helpful in expediting time-consuming computations such as long division, imperfect square roots, and multiplications of large and/or non-round numbers. Keep in mind, though, that most questions do not require complex calculations; it is important to be able to do some quick math in your head to avoid unnecessary calculator usage and save time. Below are some examples of calculations you should be able to perform quickly in your head during the exam.

	X

MR	MC	M+	(	)
7	8	9	√	C
4	5	6	×	CE
1	2	3	−	√
±	0	.	+	=

Transfer Display
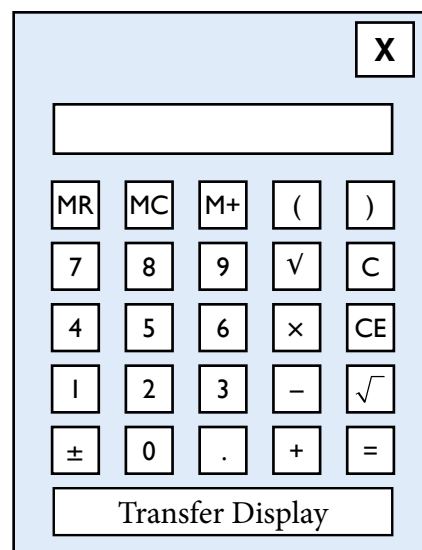
- $\sqrt{225}$
- 12×8
- $\dfrac{4250}{2}$
- 4^3
- 100×33

On the other hand, you should make use of the calculator to answer questions quickly that require more complex calculations such as the following:

- $\dfrac{240}{15}$
- $\sqrt{961}$
- 6^5
- $6 + \dfrac{9.77}{2}$

The last example involves two operations: addition and division. The on-screen calculator adheres to the algebraic rules that govern correct order of operations. Recall that the proper order of operations is as follows: parenthesis, exponents (including square roots), multiplication, division, addition, subtraction; the acronym "PEMDAS" can be used to help you remember the order of these operations. Let's look at an example to illustrate the importance of performing operations in the correct order.

$2 + 5 \times 2$

The correct answer to this expression is 12, since, per order of operations, we must perform the

multiplication first. This gives $2 + 10 = 12$. The on-screen calculator respects order of operations and will compute the correct answer regardless of the order in which the operands are entered, which is unlike most basic calculators. If we mentally perform the operations in the order in which they appear in the problem (addition before subtraction), we get $7 \times 2 = 14$, which is not the correct answer. Most basic calculators perform operations in the order in which they are entered and are oblivious to proper order of operations, so note this difference when using the on-screen calculator.

Three buttons found on the top left of the on-screen calculator allow usage of the calculator's memory. To add a number to the calculator's memory, use the "M+" button. This saved number can be recalled using the "MR" button. To clear the number from the calculator's memory, use the "MC" button. Left and right parentheses may be used at your option to separate the operands in an expression to ensure correct prioritization of computations. You may choose to use parentheses in the calculator, or perform operations separately to ensure that you reach the correct answer.

$$-\frac{6.3 + 2.8}{1.7}$$

Since division takes precedence over addition, we need to override this in order to get the correct answer. One way to do this is to use the parentheses keys so that the addition in the numerator is prioritized. To do this, we enter "(" 6.32 "+" 2.8 ")" "√" 1.7 "=" "±". Our final answer is: –5.3529412.

The other way to prioritize the addition in the numerator is to do the addition first by entering 6.3 "+" 2.8 "=" "√" 1.7 "=" "±". This also gives us the correct answer shown above.

A common mistake might be to enter the operands as shown: 6.3 "+" 2.8 "√" 1.7 "=" "±"

Note that this incorrectly computes the value of $6.32 + \left(-\dfrac{2.8}{1.7}\right)$.

Like most basic calculators, the "CE" button clears only the last entry on the calculator and the "C" button clears the entire display. The calculator will display "Error" if you attempt to enter computations that are undefined for real numbers. For example, entering 2 "√" 0 will yield an error message. "Error" will also be displayed if a calculation results in a number that contains more than eight digits, such as 10000 "×" 10000, as the on-screen calculator can display a maximum of eight digits.

Last, the "Transfer Display" button at the bottom of the on-screen calculator is used for Numeric Entry questions that contain only a single box into which the answer is entered. Clicking this button transfers the contents of the calculator's display into the answer box. Ensure that the transferred answer is in the proper format as indicated in the question (e.g. rounding, percents, etc.).

Properties of Algebra

Three simple properties form a major part of the core of algebra: the commutative, associative, and distributive. If you don't understand them well, then you may be confused by the more advanced principles covered later in this book. By taking the time to reexamine these properties, you can brush up on your knowledge of the key mathematical principles that lay the groundwork required to understand advanced algebra topics.

This review lesson examines the three main properties of algebra in detail: the commutative property, the associative property, and the distributive property. While the GRE Quantitative section won't directly ask you to identify demonstrations of each individual property, they form the bedrock of algebra, upon which the rest of its topics are built. You have to understand these basics before you can master the more complex topics, so let's review them now.

Commutative Property

The commutative property states that the order of the terms in addition and multiplication operations does not change the outcome of the operation. Note that the commutative property only applies for addition and multiplication, not for subtraction or division. General mathematical definitions of this property are shown at right.

$$a + b = c$$
$$b + a = c$$

$$a \times b = c$$
$$b \times a = c$$

Example 1
$$12 + 7 = 19$$
$$7 + 12 = 19$$

Example 2
$$1 + 2 + 3 = 6$$
$$1 + 3 + 2 = 6$$
$$2 + 1 + 3 = 6$$
$$2 + 3 + 1 = 6$$
$$3 + 1 + 2 = 6$$
$$3 + 2 + 1 = 6$$

To illustrate this, let's look at some applications of the commutative property of addition. Example 1 demonstrates the commutative property; the order of the operands 12 and 7 has no effect on the final answer of the summation. Notice that in Example 2, the order of the operands 1, 2, and 3 does not affect the final answer.

Now, let's look at some examples of applying the commutative property to multiplication operations. Like with addition, the order of the operands does not change the final answer. To further illustrate this point, let's consider Example 4; here, it is impossible to tell the order of the operands. No matter their order, the product of three twos is always 8.

Example 3
$$3 \times 4 = 12$$
$$4 \times 3 = 12$$

Example 4
$$2 \times 2 \times 2 = 8$$

Examples 5 and 6 show that the commutative property does not apply to subtraction and division operations. As we can see, changing the order of the operands when performing subtraction and/or division does change the final outcome. Be careful with the order of the operands in these calculations, because the commutative property does not apply.

Example 5
$$10 - 2 = 8$$
$$2 - 10 = -8$$

Example 6
$$20 \div 5 = 4$$
$$5 \div 20 = \frac{1}{4}$$

Associative Property

The associative property states that when adding or multiplying numbers, we can group them ("associate" them) in different ways without changing the final answer. Like the commutative property, the associative property does not apply to subtraction or division. It's shown written out mathematically to the right.

$$(a + b) + c = d$$
$$a + (b + c) = d$$
$$(a \times b) \times c = d$$
$$a \times (b \times c) = d$$

Note that the order of the operands does not change; these are not examples of the commutative property. The difference lies in the groupings, or "associations," of the operands with respect to each other. In all cases, we perform the operation that is in the parentheses first.

Example 1

$(9 + 5) + 2 = ?$	$9 + (5 + 2) = ?$
$(14) + 2 = ?$	$9 + (7) = ?$
$14 + 2 = 16$	$9 + 7 = 16$

Let's take a look at Example 1 in order to see the associative property in action. No matter in which location the parentheses are placed (i.e. whether we add the 9 and 5 together first or if we add the 5 and 2 together first), we get the same answer, 16.

Example 2

$(5 \times 6) \times 3 = ?$	$5 \times (6 \times 3) = ?$
$(30) \times 3 = ?$	$5 \times (18) = ?$
$30 \times 3 = 90$	$5 \times 18 = 90$

Example 2 demonstrates how the associative property works when multiplication is involved in parentheses. The principle remains the same: you can either begin by multplying the first and second values together or by multiplying the second and third values together. You'll get the same answer either way, which in this case is 90.

Distributive Property

The distributive property is useful when multiplying a group of terms in parenthesis. The distributive property is shown in general mathematical terms at right. Let's look at an example of this property, shown below.

$$a(b + c) = ab + bc$$

Example 1

$2(3 + 4 + 5) = ?$ $2(3 + 4 + 5) = ?$
$(2 \times 3) + (2 \times 4) + (2 \times 5) = ?$ $2(12) = ?$
$6 + 8 + 10 = 24$ $2 \times 12 = 24$

Example 1 demonstrates that if you distribute the 2 to each of the numbers in parentheses and then sum the products, you get 24 as your answer. If you instead sum the values in parentheses first and then multiply the product by 2, you get the same answer, 24. The distributive property expresses the fact that each of these "routes" can get you to the correct answer.

Example 2 shows another instance of the distributive property in action. Whether you choose to distribute the –3 first and then subtract the resulting products or to do the subtraction in parentheses first and then multiply the resulting value by –3, you get the same answer: –18.

Example 2

$-3(10 - 4) = ?$ $-3(10 - 4) = ?$
$(-3 \times 10) - (-3 \times 4) = ?$ $-3(6) = ?$
$(-30) - (-12) = ?$ $-3 \times 6 = -18$
$-30 + 12 = -18$

The distributive property is especially useful when dealing with expressions in which not all terms in the parenthesis are able to be summed. For example:

$$2(4x^2 + 3x^3)$$

$4x^2$ and $3x^3$ are not like terms and thus cannot be summed. We must use the distributive property to get the most simplified answer.

$$2(4x^2 + 3x^3)$$
$$(2 \times 4x^2) + (2 \times 3x^3)$$
$$8x^2 + 6x^3$$

Word Problems: Mathematical Modeling

If your worst GRE nightmare is facing Quantitative problems with no numbers in them, then you've arrived at the lesson that can help you face this fear. Many GRE Quantitative problems present concepts exclusively in verbal form, making the problem focus (sometimes exclusively!) on measuring your skill in translating from words to numbers and variables. In this lesson, we'll take a look at a few general problem types that require you to model described situations mathematically: problems in which the answer choices are the model equations themselves; problems in which you must necessary model the scenario as an equation in order to solve for the correct numerical answer; and problems in which you must model the scenario as the equation and the manipulate the equation to isolate a variable or for some other purpose.

Warm-Up: Interpreting Equations and Variables

Before we jump into writing equations ourselves, let's look at the practice problems to the right. The question stem for these questions provides an equation; all we have to do is associate variables with concepts. In order to identify one variable, we're going to have to identify the rest of them. How might we mathematically manipulate the number of pounds of strawberries a customer purchases? Well, if we're trying to find the total cost of the order, our expression eventually has to spit out a dollar value. To go from pounds of strawberries to the cost of those strawberries in dollars, we need to multiply pounds of strawberries by

A strawberry patch ships berries across the country. The total cost of each transaction is calculated using four variables: the price per pound of strawberries in dollars, the number of pounds of strawberries purchased, any coupon reductions applies as dollar values to the total cost, and shipping cost in dollars per pound of box weight. The weight of the box in pounds is multiplied by a constant to calculate the shipping cost. The equation $ab+bc-d$ is used to model this scenario.

Sample Question #1

Which variable in the expression must represent the pounds of strawberries a customer purchases?

- **A.** a
- **B.** b
- **C.** c
- **D.** d

Sample Question #2

Assume that the strawberry patch changes its coupons so that customers save a set amount of money per pound of strawberries they purchase. Which of the following equations, using the same variables, would now represent the total cost of an order of strawberries?

- **A.** a
- **B.** b
- **C.** c
- **D.** d

the cost of strawberries per pound. That gets us somewhere: we have to multiply. We know now that our variable isn't *d*; that's the coupon reduction, since it's the only part of the expression involving subtraction.

Variables *a*, *b*, and *c* each appear in the context of multiplication in our expression. This might seem nerve-wracking; how are we to tell them apart now? Well, what else would we use the information "pounds of strawberries" for? There's another part of the calculation we haven't considered: we need to know pounds of strawberries in order to calculate shipping cost, as "the weight of the box in pounds is multiplied by a constant to calculate the shipping cost." That's all we need to know: in the abstract, the "pounds of strawberries" number is involved in not one, but two products in our calculation. What's the only variable located in both products? Only the variable *b* is located in both products; therefore, the answer is B!

When you consider Sample Question #2, don't panic: this problem isn't nearly as tough as it appears at first glance. Consider what we already know: variable *d* has to represent the coupon, because it's the only one subtracted from the rest of the expression, which is calculating the cost of the order. What does our question say needs to change in the expression? Well, the coupon is now going to be calculated per pound. What does that mean? We need to multiply the coupon value by the number of pounds of strawberries a customer purchases. Which equation shows that? Remember, *b* represents the number of pounds of strawberries a customer purchases; we figured that out in the last question. We need to see "*bd*" in the expression, but not change anything else. A is correct.

Problems that Require You to Write an Equation

After dealing with those last problems, you should be confident that you can associate concepts with their respective variables. Writing out an equation for a scenario on your own should seem much less daunting now. If anything, you might even find it easier, as the process of equation-writing forces you to slow down and figure out which variables you need and how they're related.

There's only one variable in Sample Question #3: *m*, the number of minutes it takes to run a race. We know that *m* can be equal to or greater than 30 and less than or equal to 40. The fact that we're given four similar answer choices may seem intimidating, but it's actually quite helpful: all we have to do is narrow them down to the correct one. We just need to check the lowest and highest values that need to be accepted: 30 and 40. Doing

Sample Question #3

It takes no more than 40 minutes to run a race, but at least 30 minutes. What equation willing model this in *m* minutes?

A. $|m - 35| \le 5$

B. $|m + 35| < 5$

C. $|m - 35| > 5$

D. $|m + 35 \ge 5|$

this, we find that A is the correct answer, because if you subtract 35 from 30, you get -5, the absolute value of which is 5, which holds, and if you subtract 35 from 40, you get five, which also holds, but no values greater than 40 or less than 30 work in the expression.

Now, let's have a look at and solve Sample Question #4. This question asks us to come up with an equation to model the described situation. To show that c is of the profit of the transaction, we must represent the profit as the difference between the price and the value of the car, or "$(p - v)$"

To show that Abby's commission in dollars is a percentage of the profit, we use $0.01 \times c$ to convert the commission she earns to a percent.

Sample Question #4

Abby works at a car dealership and receives a commission, c. Her commission is a percent of the profit the dealership makes from the sale, which is the difference between the price of the car, p, and the value of the car, v. How much, in dollars, does the dealership earn per transaction?

    A. $p(v - 0.01c)$

    B. $(p - v)(0.01c)$

    C. $(p - v)(1 - c)$

    D. $(p - v)(1 - 0.01c)$

To shift the earnings from Abby to the dealership (which is what the question requires of us), we must take $1 - 0.01c$ since this will accommodate for the remaining percentage. For example, it shifts 75% (0.75) to 25% (1 – 0.75 or 0.25).

Putting this all together, we get a final expression of:

$(p - v)(1 - 0.01c)$ = dealership earnings

The correct answer is D. You can check this answer by substituting arbitrary values into the equation: letting $p = 300$, $v = 200$, and $c = 20$, we get a value of 80, which makes sense as the $100 profit must be distributed evenly between Abby ($20) and the dealership ($80).

Problems that Require You to Write and Solve an Equation

Some questions may require you not only to write an equation, but also to solve it. This may sound like many of the questions on the GRE Quantitative section, but these questions in particular demand the composition of an equation; however, they never ask for that equation. If you don't recognize this "secret" equation-writing step between you and solving for the correct answer, then these questions can become much more confusing and stressful than they need to be. We'll look at a few of these types of questions next so that you can test your skills and make sure that this style of question doesn't throw you off if you encounter it on test day.

Sample Question #5

A vegetable cart sells potatoes for $0.24 and tomatoes for $0.76. Fred bought 12 vegetables in total. He only bought potatoes and tomatoes. If Fred paid $6.52 total, how many potatoes did he buy?

A. 4

B. 5

C. 6

D. 7

There are a few common "types" of these questions to be on the lookout for. In one of these, a person spends a certain amount of money on products that each have a certain cost, and you are asked to determine how many of each product he or she purchased. This can seem like an impossible premise if you don't approach the question algebraically.

Sample Question #5 is one such problem. The first step in a problem like this is to set up your equation. Let's represent the total cost in cents so we don't have to deal with decimals or fractions, and let's call number of potatoes P and number of tomatoes T.

The total amount spent will be the sum of the prices paid for potatoes and for tomatoes. We know how much each vegetable costs, so we can use these price-per-vegetable costs as multipliers of our variables. After all, multiplying the number of tomatoes purchased by the price per tomato will yield price in dollars, and summing those values yields the total cost:

$$24P + 76T = 652$$

Uh oh: this looks bad. We have two variables but only one equation, so we can't use substitution or elimination to define them. We need to rewrite this equation so that it only has one variable. Is there a way we can do that using information in the question, perhaps information we haven't yet applied? Yes: we know that Fred bought 12 vegetables total, so the number of potatoes and the number of tomatoes have to sum to 12.

$$P + T = 12$$

There's our second equation. The easiest way to proceed here is to use substitution. Since we're asked for the number of potatoes, let's rearrange that equation to define P algebraically.

$$P = 12 - T$$

Substituting that into our first equation, we get the following:

$$24P + 76(12 - P) = 652$$

To find our answer, we just need to solve for P. Fred bought five potatoes, so B is correct.

$$24P + 76(12 - P) = 652$$
$$24P + 912 - 76P = 652$$
$$-52P + 912 = 652$$
$$-52P = -260$$
$$P = 5$$

There are quite a few similarities between this sample question and the last one, but this one ups the ante by not directly pricing the different sizes of boxes. Instead, it relates them to one another and requires you to work with these abstract terms. That won't be a problem: we can still model the situation using variables, though we're now working with three instead of two.

Let's use L to represent the price of a large box, M to represent the price of a medium box, and S to represent the price of a small box. Since one large box equals four medium boxes or eight small boxes in price, we can write out the following:

$$L = 4M$$
$$L = 8S$$

Sample Question #6

At a store, pasta is sold in three sizes. A large box costs the same as four medium boxes or eight small boxes. If James buys an equal amount of large and medium boxes of pasta for the price needed to buy one hundred small boxes, how many medium boxes of pasta does he buy?

   A. 8
   B. 9
   C. 10
   D. 11

See how both equations define L a certain way? That means that we can set them equal to one another. From this, we can say that

$$4M = 8S$$

or, reducing, that

$$M = 2S$$

We're told that James buys an equal number of large and medium boxes, and that the total price is equal to that of 100 small boxes. Now we need to incorporate another variable: the number of boxes of each that James buys. Let's call this x. James buys the same number of medium and large boxes, so we'll use the x multiplier in front of each of those variables, find their sum, and set it equal to the price of 100 small boxes.

$$xL + xM = 100S$$

Since x appears in front of each variable on the left-hand side, we can bring it outside of a pair of parentheses to simplify our equation. (This is the opposite action of distributing the x.)

$$x(L + M) = 100S$$

Ok, there are a lot of variables still in this equation. Just like in our last equation, we need to figure out how to write out our equation using only one variable. Let's get rid of M and L by rewriting them in terms of S. We've already defined L in terms of M and M in terms of S: $L = 8S$ and $M = 2S$, so let's substitute those expressions into our equation.

$$x(8S + 2S) = 100S$$

Aha! Since s appears on each side of the equation, it's eventually going to cancel out, and we can solve for x!

$$x(10S) = 100S$$

$$x = \frac{100S}{10S}$$

$$x = 10$$

James buys ten medium and ten large boxes of pasta, making the correct answer C.

One more sample question: in this one, we're going to put our skills to use in modeling a situation that might seem familiar if you ever took a general chemistry class. We need to combine two solutions that are made up of different percents of alcohol in order to come up with a solution that is a specific percent alcohol between the two we're combining. Even if this seems extremely unfamiliar, just rely on your mathematical modeling skills and don't let the unfamiliar context of the question throw you off!

Let x represent the number of liters of the 40% solution. Then it follows that x liters of the 40% solution plus 4 liters of the 10% solution will equal $(x+4)$ liters of a 25% solution. This can be represented by the following equation:

$$0.4(x) + 0.1(4) = 0.25(x + 4)$$

Now solve for x:

$$0.4x + 0.4 = 0.25x + 1$$

$$0.15x = 0.6$$

$$x = 4$$

Sample Question #7

A solution made up of 40% alcohol by volume is mixed with 4 liters of solution that is 10% alcohol by volume. How much, in liters, of the 40% alcoholic solution is needed to make a mixture that is 25% alcohol by volume?

    A. 3.2 liters
    B. 3.5 liters
    C. 3.8 liters
    D. 4.0 liters

You will need 4 liters of the 40% solution in order to make a mixture that is 25% alcohol by volume, so D is correct. Once you're confident in your ability to translate between prose and algebra, you can model complex situations with relative ease. If you get stuck on a word problem on test day and don't know how to start solving it, try modeling the situation using algebra. You may just figure out that you need to construct some equations to solve to get to the correct answer!

Arithmetic

Each of the lessons in this section examines some of the fundamental premises of math upon which the content of other sections relies. We begin by covering the most elementary premises of mathematical convention: integers, factors, and basic operations, including the standard order in which these operations are performed. We then expand upon our review of factors to consider least common multiple (LCM) and greatest common factor (GCF). Afterward, a trio of lessons investigates the numerous ways in which partial quantities can be conveyed: as fractions, decimals, ratios, proportions, or percentages. Finally, we introduce the concepts of exponents and roots. The topics covered in this section can easily appear so fundamental as to be simple and not worth reviewing, but no matter how confident you are in your mathematical skills, it doesn't hurt to make sure that even your well-tested mathematical knowledge covers everything the Quantitative Reasoning section does. Any gaps in your understanding of this section's principles can compound and cause difficulties for you later in your review, so it's best to start your review here and work toward more complex math only after you've established a solid foundation of basics.

Section Outline

Integers, Factors, and Basic Operations

Number Theory: Least Common Multiple and Greatest Common Factor

Fractions

Decimals, Ratios, and Proportions

Percentages

Solving Exponents and Roots

Integers, Factors, and Basic Operations

In this section, we will address the various classifications and applications of number theory, as well as a few simple operations. Number theory is used to help define numeric relationships, while operations are designed as a shorthand to apply relationships between numbers. Together, these two concepts form the core of arithmetic.

Numbers are fundamentally classified into a few different types. In arithmetic and algebra, it is important to pay attention to the type of number being addressed in the questions and equations you are dealing with. Certain numbers have specific properties that make them act differently mathematically, and they may require close attention in order to avoid miscalculations. The following table provides a comprehensive breakdown of number classifications, starting with the most general class (real numbers) and ending with the most specific (natural numbers).

Category	Description	Included Numbers	Excluded Numbers
Real Numbers	Real numbers include all numbers that can be represented on a number line. This includes all rational and irrational numbers, but excludes all imaginary numbers.	$\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7, 0, 5, 36$	$i, 4i, \sqrt{-1}, \sqrt{-4}, i\sqrt{3}$
Imaginary Numbers	Imaginary numbers involve the term "i," which is indicative of the square root of -1 ($\sqrt{-1}$).	$i, 4i, \sqrt{-1}, \sqrt{-4}, i\sqrt{3}$	$\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7, 0, 5, 36$
Rational Numbers	Rational numbers can be represented by a finite decimal. In other words, they can be represented as a ratio of two other numbers. This includes all integers, as well as all fractional numbers and all repeating decimals.	$\frac{3}{4}, -7, 0, 5, 36$	$\sqrt{3}, \neq$
Irrational Numbers	Irrational numbers are numbers that cannot be expressed as a fraction of integers.	$\sqrt{3}, \neq$	$\frac{3}{4}, -7, 0, 5, 36$
Integers	Integers are non-fractional numbers on the number line, including negative numbers, positive numbers, and zero. All whole numbers and all natural numbers are integers.	$-7, 0, 5, 36$	$\sqrt{3}, \neq, \frac{2}{3}, \frac{3}{4}$
Whole Numbers	Whole numbers are non-fractional numbers on the number line, extending from zero upward. Whole numbers include all natural numbers and exclude all negative numbers.	$0, 5, 36$	$\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7$
Natural Numbers	Natural numbers include all non-fractional positive numbers and exclude zero.	$5, 36$	$\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7, 0$

The figure below gives a visual representation of these classifications.
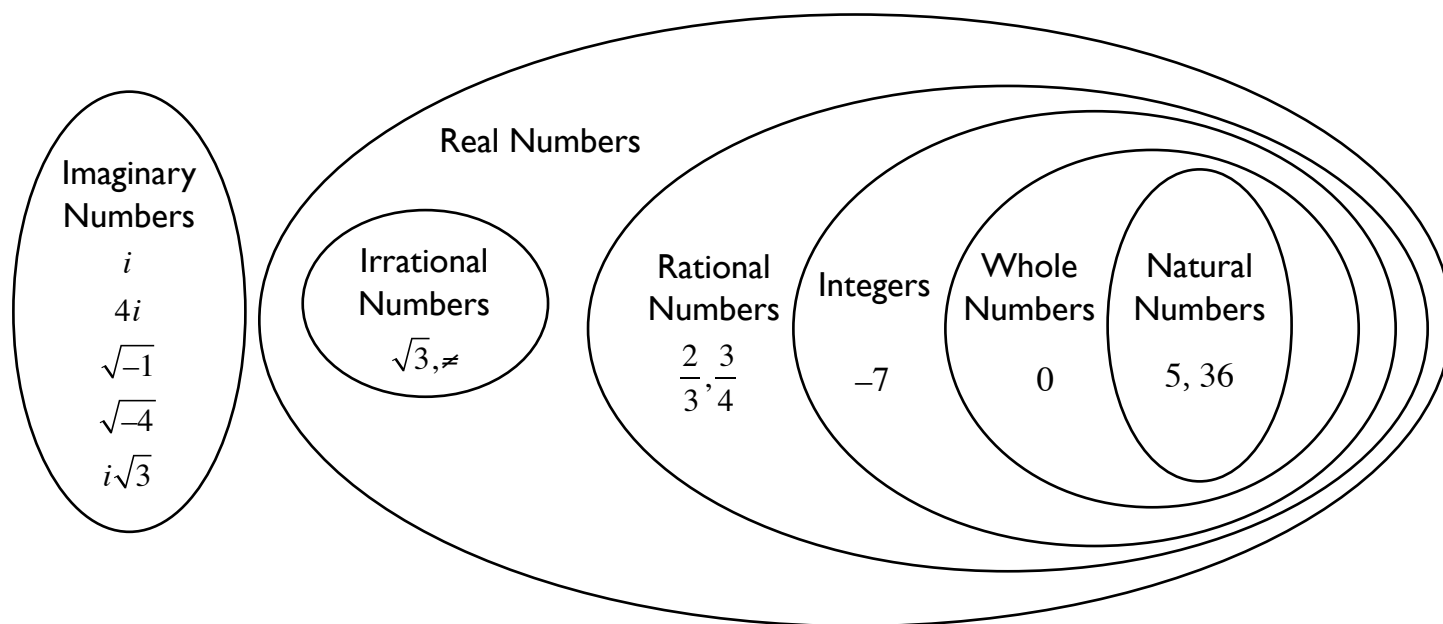
Sample Question #1 requires you to apply your understanding of different types of numbers. These questions can seem challenging at first glance, but if you understand what distinguishes the different categories of numbers from one another, it shouldn't provide too difficult.

Consider what we're told. What does it mean that n is a real number? Well, imaginary numbers are not real numbers by definition, so we can knock out answer choice C. Now to consider the rest of the problem: what do we learn when we're told that $n + \frac{6}{11}$ is irrational? $\frac{6}{11}$ isn't irrational—not by a long shot. It's rational. So, that means that for the expression to be irrational, n has to be the source of the irrationality. The correct answer is D.

Sample Question #1

n is a real number. If $n + \frac{6}{11}$ is irrational, what can we also say about n?

A. n must be a rational number, but not an integer.

B. n must be an integer, but not a whole number.

C. n must be an imaginary number.

D. n must be an irrational number.

Sample Question #2 is a bit more challenging than Sample Question #1, though it only focuses on one type of number: integers. To solve it, you'll need to recall that the category of "integers" encompasses positive whole numbers, negative whole numbers, and zero. A glance at the values presented shows that fractions are the main concern here. Fractions, by definition, can't be integers. There's the crux of the problem. Once you've realized these basic precepts, you can tackle the rest of the question: you just have to figure out which answer choice can result in a partial value between two whole numbers. If it can, it need not necessarily be an integer. Breaking down each answer choice and thinking about how you could get it from the provided integers can shed a bit of light onto the situation:

$\dfrac{a}{b}$ and $\dfrac{b}{c}$ are integers. Which of the following need not be an integer?

A. $\dfrac{a}{c}$

B. $\dfrac{a+c}{b}$

C. $\dfrac{a+b}{b}$

D. $\dfrac{b-c}{b}$

A. $\dfrac{a}{b} \times \dfrac{b}{c} = \dfrac{a}{c}$. As the product of integers, $\dfrac{a}{c}$ must be an integer.

C. $\dfrac{a+b}{b} = \dfrac{a}{b} + \dfrac{b}{b} = \dfrac{a}{b} + 1$, making $\dfrac{a+b}{b}$ the sum of integers, and, consequently, an integer.

D. $\dfrac{b-c}{b} = \dfrac{b}{c} - \dfrac{c}{c} = \dfrac{b}{c} - 1$, making $\dfrac{b-c}{b}$ the difference of integers, and, consequently, an integer.

We can demonstrate that $\dfrac{a+c}{b}$ need not be an integer through a counterexample. Let $a = 20$, $b = 10$, and $c = 5$.

$\dfrac{a}{b} = \dfrac{20}{10} = 2$ and $\dfrac{b}{c} = \dfrac{10}{5} = 2$, so the conditions of the problem are met; however,

$\dfrac{a+c}{b} = \dfrac{20+5}{10} = \dfrac{25}{10} = 2.5$, which is not an integer. This makes B, $\dfrac{a+c}{b}$, the correct response.

Order of Operations

Operations are the means of manipulating relationships between numbers. On the most fundamental level, there are four primary operations: addition, subtraction, multiplication, and division.

Addition is used to find the sum; the sum is the total of two or more numbers, when one value adds to another. Subtraction is used to find the difference; the difference is the numerical distance between two numbers. Addition and subtraction are inverse functions. This means that adding and subtracting the same number can cancel out. Example: $12 - 8 + 8 = 12$.

Multiplication is used to find the product; the product is essentially the result of compound addition—adding a number to itself a predetermined number of times. Division is used to find the quotient; the quotient is the number of times a given number can be added to itself to result in an original term. Multiplication and division are inverse functions, so multiplying and dividing by the same number can cancel out. Example: $4 \times 6 \div 6 = 4$.

There are various additional operations, such as exponents and logarithms, that are introduced in higher-level math, but they can always be reduced and explained in terms of addition, subtraction, multiplication, and division.

Order of operations refers to the rules governing the application of these relationships to a complex equation. Evaluating $5 \cdot 6$ involves only one step: multiplication. Things can get more complex when more operations become involved.

Order of operations dictates that when more than one operation is present in an expression or equation, they are evaluated in the sequence shown at right. This sequence gives the acronym PEMDAS, which is commonly taught alongside the mnemonic "Please Excuse My Dear Aunt Sally."

- **P**arentheses
- **E**xponents
- **M**ultiplication/**D**ivision
- **A**ddition/**S**ubtraction

Parentheses and exponents are always evaluated first. Multiplication and division can be evaluated simultaneously, followed by simultaneous evaluation of addition and subtraction.

Example: $(5 \cdot 6) - 2^3 + 3 \cdot 4 - 1$
Parenthesis: $30 - 2^3 + 3 \cdot 4 - 1$
Exponent: $30 - 8 + 3 \cdot 4 - 1$
Multiplication/Division: $30 - 8 + 12 - 1$
Addition/Subtraction: 33

When there are multiple sets of parentheses, always start from the innermost set and work outward. Example: $(5 \cdot 4(1 - 3)) + 4 = (5 \cdot 4(-2)) + 4 = (5 \cdot -8) + 4 = -40 + 4 = -36$

Sample Question #3 is a bit more difficult than the previous examples because it involves multiple layers of parentheses. Just treat interior parentheses before you deal with exterior parentheses and you can solve this problem easily:

$$7((5 + 2) - (5 \times 8))^2$$

First, we want to simplify the expressions contained in the inner parentheses.

$$7((7) - (40))^2$$

With that done, we can proceed to simplifying the expression contained by the exterior parentheses:

$$7(-33)^2$$
$$7(1089)$$
$$7623$$

Finally, we just need to deal with the exponent. The correct answer is 7623, D.

If the GRE Quantitative section asks you to find the sum of a given range of consecutive numbers, don't panic: there's a handy formula you can use instead of calculating the sum by adding each number individually. The formula is shown below as if you start at 1, and *n* is the highest number in the sequence.

$$\frac{n(n + 1)}{2}$$

Sample Question #3

Evaluate the following expression.

$$7((5 + 2) - (5 \times 8))^2$$

A. -87
B. 16
C. 5242
D. 7623

Factors

Factors refer to the various combinations of integers that can be multiplied together to result in a particular term. For example, 6 and 2 are factors of 12 because the product of 6 and 2 is 12.

Every integer will have the factors of 1 and itself, but certain terms can have numerous factors. Let's look at 100 as an example. Factors of 100 include: 1, 2, 4, 5, 10, 20, 25, 50, and 100.

$1 \cdot 100 = 100$
$2 \cdot 50 = 100$
$4 \cdot 25 = 100$
$5 \cdot 20 = 100$
$10 \cdot 10 = 100$

A term that is greater than one and has factors of only one and itself is known as a prime number. The prime numbers between 1 and 20 include 2, 3, 5, 7, 11, 13, 17, and 19. 2 is the only even prime number.

Every number can be reduced down to its prime factors, which, when multiplied together, result in the desired product.

For example, the prime factors of 100 are 2, 2, 5, and 5.

$2 \cdot 2 \cdot 5 \cdot 5 = 100$

To find the factors of a given term, it is usually best to build a factor tree. This will identify the prime factors of the number in question. A factor tree begins with any integer and visually deconstructs various factors of that number. A single term can have multiple variations of its factor tree (depending on what factors you choose for the highest level), but will always result in the same final prime factors.
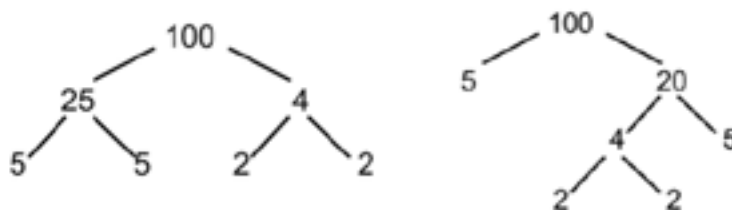
The product of any combination of prime factors will also be a factor of the original number. For example, if the prime factors of 100 are 2, 2, 5, and 5, then we know that the product of any combination of these will also be a factor of 100. For example, 10 is the product of 2 and 5 and must, therefore, be a factor of 100.

Let's tackle another sample problem, and let's consider Quantity B first. What will the remainder be when p is divided by 33?

4, 6 and 11 are factors of p which means that $2 \times 2 \times 2 \times 3 \times 11 \times n$ will equal p. We can group the 3 and 11 to see that 33 will always be a factor of p and will have no remainder. Thus Quantity B will always equal 0, no matter the value of n.

Now consider Quantity A. Let's consider first the values for p when n equals 1 through 5. If $n = 1$, $p = 264$, and the remainder is $\frac{4}{5}$, or 0.8.

If $n = 2$, $p = 528$, and the remainder is $\frac{3}{5}$, or 0.6.

If $n = 3$, $p = 792$, and the remainder is $\frac{2}{5}$, or 0.4.

If $n = 4$, $p = 1056$, and the remainder is $\frac{1}{5}$, or 0.2.

If $n = 5$, $p = 1320$, and the remainder is 0 (because when $n = 5$, 5 becomes a factor of p, and thus there is no remainder.

Because Quantity A can be equal to or greater than Quantity B, there is not enough information given to determine the relationship, and D is the correct answer.

Sample Question #4

n is a positive integer.

$$p = 4 \times 6 \times 11 \times n$$

Quantity A	Quantity B
The remainder when p is divided by 5	The remainder when p is divided by 33

A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information given.

Sample Question #5

If the product of two distinct integers is 143, which of the following could not represent the sum of those two integers?

A. 11
B. 144
C. –144
D. 24

Let's look at one more factor-based sample question: Sample Question #5. Since we're dealing with a product that comes out to a positive value, 143, it could be the product of two positives or two negatives. That being said, consider the ways we could factor: 1×143, -1×-143, 11×13, and -11×-13.

For each of these four possible factors, there are four possible sums: 144, –144, 24, and –24. Answer choice A, 11, is the only one not included, so A is the correct answer.

Number Theory:
Least Common Multiple and Greatest Common Factor

Comparing the factors of two different terms can provide useful information about the relationship between them. Most importantly, we can find the **greatest common factor**. When dealing with two terms, the greatest common factor is the largest number that divides into both terms with no remainder. To find the greatest common factor, you will need to start with two terms. Find the prime factors of each term individually, and then select all prime factors that the two terms have in common. Then, find the product of this combination of prime factors; this will be the greatest common factor. Note that sometimes the greatest common factor will be a prime factor if the terms in question have only one prime factor in common.

Let's find the greatest common factor of 126 and 336. To do this, we might draw factor trees for each number breaking each into a product of factors until we reach prime factors that cannot be reduced any further:
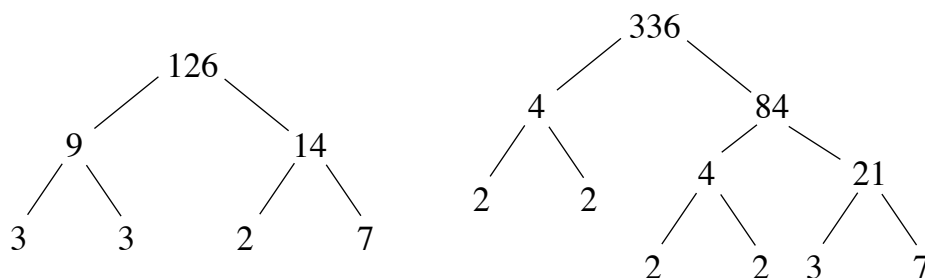
The prime factors of 126 are 2, 3, 3, and 7. The prime factors of 336 are 2, 2, 2, 2, 3, and 7. The prime factors they share are 2, 3, and 7. So, the greatest common factor of 126 and 336 is $2 \times 3 \times 7 = 42$.

Once the greatest common factor has been identified, it can be factored out of the terms in question: $126 + 336 = 42(3 + 8)$. This is especially useful when simplifying fractions.

The **least common multiple** is also particularly useful for manipulating fractions. When given two terms, their least common multiple is the smallest number that has both given terms as factors. For example, the least common multiple of 3 and 4 is 12. There are larger numbers that have both 3 and 4 as factors (such as 24); however, the least common multiple is the most applicable.

Finding the least common multiple can be a little bit complex; it is equal to the product of the given terms divided by the greatest common factor. Let's look at 126 and 336. Their greatest common factor is 42. The least common multiple will be:

$$\frac{126 \times 336}{42} = \frac{42336}{42} = 1008$$

The greatest common factor is essential for simplifying fractions, while the least common multiple is essential for addition and subtraction of fractions.

The greatest common factor and the least common multiple may be core concepts in questions that appear to be much more difficult than straightforward fraction problems. Like many topics on the GRE Quantitative section, least common multiple and greatest common factor will most probably be tested through the usage of word problems. Let's take a look at few calculation-heavy examples, starting with Sample Question #1.

In order to find all of the numbers that are multiples of both 8 and 18, we need to find the least common mutliple of these two numbers. One way to do this would be to list out the multiples of 8 and 18 and determine the smallest one that is common to both.

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88 . . .

Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144 . . .

The smallest multiple that 8 and 18 share—their least common multiple—is 72.

Alternatively, we could approach this problem a different way, finding the greatest common factor of 8 and 18 and dividing their product by it to find their least common multiple. To do this, we'd first need to identify shared factors. Factor trees facilitate this:
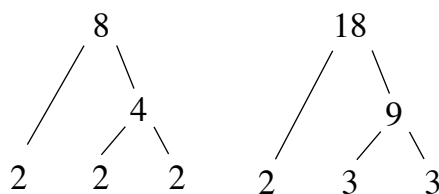
8 and 18 only share one factor, 2. So, the least common multiple of these numbers is:

$$\frac{8 \times 18}{2} = \frac{144}{2} = 72$$

Whichever method is used, we get the same answer for the least common multiple: 72. This means that every multiple of 72 is also a multiple of both 8 and 18. So, in order to find all of the multiples less than 10000 that are multiples of both 8 and 18, we simply need to find how many multiples of 72 are less than 10000, and to do this, all we have to do is to divide 10000 by 72.

When we divide 10000 by 72, we get 138 with a remainder of 64; therefore, 72 will go into ten thousand 138 times before it exceeds ten thousand. In other words, there are 138 numbers less than 10000 that are multiples of 72 and, by extension, also multiples of both 8 and 18. The answer is C, 138.

As we approach Sample Question #2, let's first distill all of the information we can from the question stem itself.

The greatest common factor of x and y is 10. This means that the prime factorizations of x and y must each contain a 2 and a 5.

The greatest common factor of y and z is 9. This means that the prime factorizations of y and z must both contain two 3's.

The greatest common divisor of z and x is 8. This means that the prime factorizations of z and x must both contain three 2's.

Thus, writing these conclusions out algebraically, we get the following:

$$x = 2^3 \times 5 \times A = 40A$$
$$y = 2 \times 3^2 \times 5 \times B = 90B$$
$$z = 2^3 \times 3^2 \times C = 72C$$

x, y, and z are positive two-digit integers. The greatest common factor of x and y is 10. The greatest common factor of y and z is 9. The greatest common factor of z and x is 8.

If $\dfrac{yz}{x}$ is an integer, which of the following could it equal?

- **A.** 96
- **B.** 119
- **C.** 145
- **D.** 162

We can substitute these equalities into the given expression and simplify.

$$\frac{yz}{x} = \frac{90B \times 72C}{40A} = \frac{6480BC}{40A} = \frac{162BC}{A}$$

Since y and z are two-digit integers (equal to $90B$ and $72C$ respectively), we must have $B = 1$ and $C = 1$. Any other factor values for B or C will produce three-digit integers (or greater). x is equal to $40A$, so A could be either 1 or 2. Therefore:

$$\frac{yz}{x} = \frac{162(1)(1)}{1} = 162$$

or

$$\frac{yz}{x} = \frac{162(1)(1)}{2} = \frac{162}{2} = 81$$

D is the correct answer.

Sample Question #3 asks us to apply our knowledge of least common denominators to fractions. Let's begin by calculating the total amount of pie that Elizabeth eats.

$$\frac{1}{9} + \frac{1}{10}$$

By glancing at the answer choices, we know that we want to stick to fraction notation, not decimal notation. To add these fractions, we need to get a common denominator. The best common denominator is the least common multiple, since it avoids introducing any need to simplify products later. The factors of 9 are 3 and 3, and the factors of 10 are 2 and 5, so their greatest common factor is 1. Using our formula for finding LCM by dividing the product of two numbers by their GCF, we find that the LCM of 9 and 10 is 90. Now we can make that our common denominator:

$$\left(\frac{1}{9} \cdot \frac{10}{10}\right) + \left(\frac{1}{10} \cdot \frac{9}{9}\right)$$

$$\frac{10}{90} + \frac{9}{90} = \frac{19}{90}$$

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Sample Question #3

Elizabeth serves herself $\frac{1}{9}$ of a boysenberry pie and $\frac{1}{10}$ of an apple pie. After Elizabeth takes her pie servings, Matt serves himself $\frac{1}{5}$ of the leftover boysenberry pie and $\frac{1}{15}$ of the leftover apple pie. Who eats more pie in total, and by how much?

    A. Matt eats more by $\frac{2}{75}$ of a pie

    B. Elizabeth eats more by $\frac{2}{75}$ of a pie

    C. Matt eats more by $\frac{1}{18}$ of a pie

    D. Elizabeth eats more by $\frac{1}{18}$ of a pie

</div>

This cannot be reduced since 19 is a prime number. Thus we know that in total, Elizabeth eats $\frac{19}{90}$ of a pie. Next, we must find out how much pie Matt eats. We are told that Matt eats $\frac{1}{15}$ of the leftover boysenberry pie and $\frac{1}{15}$ of the leftover apple pie. To illustrate this mathematically, we need to find out how much of the pies remain after Elizabeth has taken her portions, which is less than a whole pie in both situations.

Boysenberry pie: $1 - \frac{1}{9} = \frac{8}{9}$

Apple pie: $1 - \frac{1}{10} = \frac{9}{10}$

Next, we multiply the fraction of each pie that Matt eats by the total of each pie that is left. Then, we add these two fractions together to calculate the total amount of pie that Matt eats.

$$\left(\frac{1}{5}\times\frac{8}{9}\right)+\left(\frac{1}{15}\times\frac{9}{10}\right)=\frac{8}{45}+\frac{9}{150}$$

Again, we must find a common denominator.

$$\left(\frac{8}{45}\times\frac{10}{10}\right)+\left(\frac{9}{150}\times\frac{3}{3}\right)=\frac{80}{450}+\frac{27}{450}=\frac{107}{450}$$

This is the total amount of pie that Matt eats. To answer the question, we must compare this fraction to that which represents the total amount of pie that Elizabeth eats, $\frac{19}{90}$. To do this accurately, we need to express both fractions with a common denominator (450).

$$\frac{19}{90}\cdot\frac{5}{5}=\frac{95}{450}$$

Finally, we have comparable terms for the total pie each person eats:

Elizabeth: $\frac{95}{450}$ of a pie

Matt: $\frac{107}{450}$ of a pie

Here we can easily see that Matt eats $\frac{12}{450}$ more pie by finding the difference of these two fractions; however, this is not one of our answer choices. Reduce this fraction by a factor of 6, which is the greatest common factor of 12 and 450, in both the numerator and denominator.

$$\frac{12\div6}{450\div6}=\frac{2}{75}$$

Thus, the correct answer is A.

Fractions

Fractions are used to convey partial values. A partial value is a value that is part of a whole. More specifically, fractions are often used to express values found between 0 and 1.

Fractions consist of two parts: the "numerator" (the top part of the fraction) and the "denominator" (the bottom part of the fraction). These two parts are separated by a bar. You can read a fraction as conveying a mini division problem: each fraction's value is that of its numerator divided by its denominator.

Simplifying Fractions

Sometimes, fractions are made up of numbers that have *factors* in common. In the fraction below, both the numerator and the denominator are divisible by three. This means that the fraction is not in "lowest terms." Put a different way, it can be reduced so that it represents the same thing using smaller numbers.

When using fractions in math or when coming up with an answer that is a fraction, it is common to hear "reduce to lowest terms" as the last thing in a question's instructions. How do you reduce to lowest terms? You can start by analyzing the numerator and the denominator and identifying anything they have in common. Let's reduce the following fraction:

$$\frac{9}{36}$$

The factors of 9 are 1, 3, and 9. The factors of 36 are 1, 3, 9, 4, 12, and 36. It looks like the numerator and the denominator have three factors in common: 1, 3, and 9. Let's divide both the numerator and the denominator by their greatest common factor, 9:

$$\frac{9 \div 9 = 1}{36 \div 9 = 4} = \frac{1}{4}$$

This fraction is now in lowest terms. We can tell because the numerator is 1, and any fraction with 1 in the numerator can't be reduced any more, since the only factor of 1 is 1.

Simplifying fractions is essential in algebra. Being able to factor out multipliers or factors can allow you to reduce fractions into their most basic components.

For example:

$$\frac{3x + 6}{5(x + 2)} = \frac{3(x + 2)}{5(x + 2)} = \frac{3}{5}$$

Improper Fractions and Mixed Numbers

An **improper fraction** is a fraction in which the numerator is larger than the denominator. As a result, the fraction does not convey a value between 0 and 1, but instead conveys a whole-number value or a value between two other whole numbers. $\frac{3}{2}$ and $\frac{4}{2}$ are examples of improper fractions.

Mixed numbers are whole numbers accompanied by a fractions to convey a value in between two whole numbers. An example of a mixed number is $3\frac{1}{2}$. Fundamentally, a mixed number is an abbreviated form of addition.

$$3\frac{1}{2} = 3 + \frac{1}{2}$$

To convert a a mixed number to an improper fraction, you need to convert the whole number to a fraction and add it to the fractional term in the mixed number. Converting the whole number to a fraction requires multiplying it by 1, using the format $\frac{x}{x}$, where x is equal to the denominator of the fractional term of the mixed number.

$$3\frac{1}{2} = 3\left(\frac{2}{2}\right) + \frac{1}{2} = \frac{3 \times 2}{2} + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2}$$

When working with improper fractions, it is often easiest to rewrite them as decimals. In a similar vein, you can convert any mixed numbers you encounter to decimals in order to make them easier to work with.

To convert a mixed fraction to a decimal, simply divide the numerator of the fraction by its denominator and add the result to the given whole number.

$$3\frac{1}{2} - 3 + (1 \div 2) - 3 + 0.5 = 3.5$$

Check out the chart on the following page for an overview of how to convert between various types of formats for partial values. If you feel comfortable moving from one format to the other with ease, problems that try to trip you up by requiring you to display a value in a different format will have less chance of tripping you up when you take your exam. After the chart, we'll walk through Sample Problem #1. See if you can figure it out on your own first!

Sample Question #1

Quantity A	Quantity B
1.375	$\frac{14}{11}$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Decimal

1.75

Divide the numerator by the denominator.

$7 \div 4 = 1.75$

Take the entire decimal, including the whole number value, and place it over 1, then reduce to lowest terms.

$$\frac{1.75}{1} = \frac{175 \div 25}{100 \div 25} = \frac{7}{4}$$

$1\frac{3}{4} = 1 + \frac{3}{4}$

$\frac{3}{4} = 3 \div 4 = 0.75$

$1 + 0.75 = 1.75$

Take the part of the decimal to the right of the decimal point and convert it into a fraction by placing it over 1 and reducing to lowest terms.

$1.75 = 1 + 0.75$

$$\frac{0.75}{1} = \frac{75}{100}$$

$$\frac{75 + 25}{100 + 25} = \frac{3}{4}$$

$1.75 = 1\frac{3}{4}$

Divide the numerator of the small fraction by its denominator and add this decimal value to the whole number.

Improper Fraction

$$\frac{7}{4}$$

Numerator − Denominator = Numerator of Small Fraction

$$\frac{7}{4} \rightarrow 7 - 4 = 3 \rightarrow \frac{7}{4} = \frac{4}{4} + \frac{3}{4} = 1\frac{3}{4}$$

Mixed Number

$$1\frac{3}{4}$$

$1 \begin{array}{c} + 3 \\ \times 4 \end{array}$ Denominator × Whole Number + Numerator = Numerator of Improper Fraction

To solve Sample Problem #1, you'll need to make a few of the conversions outlined above. But which conversions? Deciding how to approach solving this problem is half the battle. You have two approaches you can take in this instance: you can either change 1.375 into a mixed number with a fractional component with a denominator of 11, or you can convert $\frac{14}{11}$ into a decimal. Pause to consider these two choices, and it should become apparent that one is much faster and simpler than the other. If you choose to convert 1.375 into a fraction, you could pretty easily convert it to a fraction in lowest terms, but those lowest terms might not necessarily involve a denominator of 11. Getting the fraction into that specific form would likely require extra work. Instead, you can just change the fraction you're given into a decimal. 14 divided by 11 and rounded is 1.273, and since 1.375 is bigger than 1.273, the correct answer is A.

Operations with Fractions

As with whole numbers, fractions can be added, subtracted, multiplied, and divided, but certain manipulations must be applied before the operations can be completed. Fractions behave a little differently from whole numbers due to the presence of the denominator. Fractions with the same denominator represent equal divisions. $\frac{3}{11}$ is the same as $\frac{1}{11}$ three times. Put slightly differently, if you have $\frac{3}{11}$ of a pie, you have the same amount of pie who has three pieces that are each $\frac{1}{11}$ of the entire pie.

To multiply fractions, simply "multiply across": apply the operation separately to the numerator terms and to the denominator terms. Remember to simplify the fraction afterward if possible. Keep in mind that if you multiply a fraction by another fraction and both fractions are less than 1, you're going to end up with a smaller fraction than either of those you multiplied. This may be fairly counterintuitive, but is worth remembering as a pattern that you can use to check your work.

Example: $\frac{2}{3} \cdot \frac{8}{9} = \frac{2 \cdot 8}{3 \cdot 9} = \frac{16}{27}$

In this example, the fraction cannot be simplified, so the final answer is $\frac{16}{27}$.

When adding or subtracting fractions, you need to make sure that all fractions involved have a common denominator. To create a common denominator, you may need to multiply one or both fractions by 1 in the form of the fraction $\frac{x}{x}$. This changes the way a fraction portrays its value, but not the value itself.

Let's add the following two fractions together: $\frac{3}{4} + \frac{4}{5}$

We have two different denominators and need them both to be the same before we can add the fractions. One shortcut that we can use is to multiply the first fraction by $\frac{x}{x}$ where x is the value of the denominator in the second fraction and multiply the second fraction by $\frac{x}{x}$ where x is the value of the denominator in the first fraction. While the resulting fractions may not be in lowest terms, each denominator will have a factor of the original denominator and the other fraction's denominator.

$$\frac{3}{4}\left(\frac{5}{5}\right) + \frac{4}{5}\left(\frac{4}{4}\right) = \frac{15}{20} + \frac{16}{20}$$

Now that both of our fractions have the same denominator, we can add together their numerators. Don't mistakenly add the denominators together, too!

$$\frac{15}{20} + \frac{16}{20} = \frac{31}{20}$$

Let's try working through Sample Question #2, a sample problem that involves adding fractions as well as making sure others are in lowest terms.

It's very important to consider which part of the cake the given number represents. Since $\frac{3}{7}$ of the cake is eaten, that means that the remaining amount of cake must be represented by the expression $1 - \frac{3}{7}$. To solve this expression, we need to put 1 in the same terms as the fraction. In other words, we need to make sure both values are expressed in a way in which they have the same denominator. To do this, we need to write 1 as $\frac{7}{7}$. At this point, we can subtract the numerators to find how much cake is left:

$$\frac{7}{7} - \frac{3}{7} = \frac{4}{7}$$

> ### Sample Question #2
>
> $\frac{3}{7}$ of a birthday cake is eaten at the recipient's party. Which of the following represents how much cake is left over?
>
> **A.** $\frac{10}{12}$
>
> **B.** $\frac{10}{18}$
>
> **C.** $\frac{12}{21}$
>
> **D.** $\frac{12}{28}$

Ok, now we know that $\frac{4}{7}$ of the cake remained after the party. To finish solving the problem, we have to figure out which of the listed answer choices is equal to $\frac{4}{7}$. $\frac{4}{7}$ is in lowest terms, so if we put each of the answer choice fractions into lowest terms, we should be able to spot the one that matches our math.

When working with all of the given answer choices at once, it helps to list them out so you don't confuse them.

A. $\frac{10 \div 2}{12 \div 2} = \frac{5}{6}$ **B.** $\frac{10 \div 2}{18 \div 2} = \frac{5}{9}$ **C.** $\frac{12 \div 3}{21 \div 3} = \frac{4}{7}$ **D.** $\frac{12 \div 4}{28 \div 4} = \frac{3}{7}$

Answer choice C matches our calculation of how much cake is left over after the party, so C is the correct answer. Notice how if you weren't paying attention to the question stem, you may have assumed that $\frac{3}{7}$ represents the amount of cake left over, not the amount of cake that was eaten. If you misunderstood this point, your math would have resulted in answer choice D looking like it was the correct option! Take your time when reading question stems so you don't rush and make errors.

One common situation in which you'll need to multiply fractions is when you have two fractions with different denominators and want to create a common denominator between them so as to compare their relative values in the same terms without estimating.

For instance, consider the sample problem shown at left. To solve it, you'll need to put the provided fractions in the same terms, then subtract one from the other. It doesn't actually matter which fraction we choose to subtract from the other; the problem instructs us to look for "the difference in value" between the two fractions, so we actually are looking for an *absolute value*, and the sign on the answer won't matter, as it will be considered to be a positive value anyway.

Sample Question #3

Which of the following represents the difference in value between $\dfrac{8}{17}$ and $\dfrac{2}{3}$?

A. $\dfrac{9}{51}$

B. $\dfrac{10}{51}$

C. $\dfrac{9}{53}$

D. $\dfrac{10}{53}$

To quickly and easily put two fractions in the same terms, remember that you can multiply each fraction's numerator and denominator by the denominator of the other fraction over itself. This will change the format in which the fraction is displayed, but not the fraction's value.

$$\frac{8}{17}\left(\frac{3}{3}\right)-\frac{2}{3}\left(\frac{17}{17}\right)$$

$$\frac{24}{51}-\frac{34}{51}=-\frac{10}{51}$$

The difference in value between the two fractions is thus $\dfrac{10}{51}$, making B the correct answer.

Dividing Fractions

To divide fractions, multiply the first term by the **reciprocal** of the second term. The reciprocal of a fraction is the fraction flipped so that its numerator and denominator terms are switched. For example, the reciprocal of $\frac{7}{8}$ is $\frac{8}{7}$.

Example:

$$\frac{6}{7} \div \frac{3}{4} = \frac{6}{7} \cdot \frac{4}{3} = \frac{6 \cdot 4}{7 \cdot 3} = \frac{24}{21}$$

Simplify.

$$\frac{24}{21} = \frac{8}{7}$$

To solve Sample Question #4, we need to divide $\frac{3}{8}$ by $\frac{1}{5}$ as shown in Quantity A.

$$\frac{3}{8} \div \frac{1}{5} = \frac{3}{8} \times \frac{5}{1} = \frac{15}{8}$$

Sample Question #4

Quantity A	Quantity B
$\frac{3}{8} \vee \frac{1}{5}$	$1\frac{7}{8}$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

At this point, we can turn the resulting improper fraction $\frac{15}{8}$ into a mixed number to compare it with $1\frac{7}{8}$:

$$\frac{15}{8} - \frac{8}{8} = \frac{7}{8}$$
$$\frac{15}{8} = 1\frac{7}{8}$$

It turns out that $\frac{3}{8} + \frac{1}{5} = 1\frac{7}{8}$, so C is the correct answer.

Compound or Complex Fractions

Compound fractions, also called complex fractions, are fractions in which the numerator or the denominator itself contains a fraction.

Example: $\dfrac{\dfrac{1}{2}}{4}$

For compound fractions, it is important to remember that a fraction is, fundamentally, an application of division.

$$\frac{\dfrac{1}{2}}{4} = \frac{1}{2} \div 4$$

Convert the denominator to a fraction.

$$\frac{1}{2} \sqrt{\frac{4}{1}}$$

Divide the fractions by multiplying by the reciprocal.

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Let's look at a more complex example.

$$\frac{\dfrac{7}{8} - \dfrac{3}{4}}{\dfrac{1}{2} \cdot \dfrac{4}{9}}$$

First, evaluate the numerator and the denominator.

$$\frac{\dfrac{7}{8} - \dfrac{6}{8}}{\dfrac{4}{18}}$$

$$\frac{\dfrac{1}{8}}{\dfrac{4}{18}}$$

Next, evaluate the compound fraction as a division problem.

$$\frac{1}{8} \div \frac{4}{18}$$

$$\frac{1}{8} \cdot \frac{18}{4}$$

$$\frac{18}{32}$$

Simplify the result.

$$\frac{18}{32} = \frac{9}{16}$$

Let's take a look at a sample problem involving compound fractions. The first thing we need to do is translate this story problem into an algebraic expression, and with a story problem this complex, it helps to have a game plan for approaching it in steps. First, we need to quantify the rates at which two leaks release water. Then, we'll be able to sum the two rates to find the total amount of water leaked per hour. At that point, we can use the information about how much water the tank initially holds (200 gallons) to figure out how long it will take that amount of water to leak out of the tank.

Let's get started. We're told that the first leak releases water at a rate of $\frac{1}{5}$ gallon per half hour. That can be mathematically expressed as a compound fraction:

$$\frac{\frac{1}{5} \text{ gallon}}{\frac{1}{2} \text{ gallon}}$$

That compound fraction can then be divided out to create a more typical fraction.

$$\frac{1}{5} \div \frac{1}{2} = \frac{1}{5} \times \frac{2}{1} = \frac{2}{5} \frac{\text{gallons}}{\text{hour}}$$

Sample Question #5

A cistern containing two hundred gallons of water springs two leaks simultaneously. One of the leaks releases water at a rate of $\frac{1}{5}$ of a gallon every half hour. The second one leaks at a rate of $\frac{2}{3}$ a gallon every fifth of an hour. Assuming neither of the leaks are fixed, after how many hours will the cistern be empty?

A. $\frac{375}{7}$ hours

B. $\frac{350}{8}$ hours

C. $\frac{155}{14}$ hours

D. $\frac{167}{14}$ hours

We can do the same thing for the measures of the second leak:

$$\frac{\frac{2}{3} \text{ gallon}}{\frac{1}{3} \text{ hour}}$$

$$\frac{2}{3} \div \frac{1}{5} = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3} \frac{\text{gallons}}{\text{hour}}$$

At this point, we can add together the two leaks' rates to get the total rate of water leaking from the cistern.

$$\frac{2}{5} + \frac{10}{3}$$

$$\frac{2}{5}\left(\frac{3}{3}\right) + \frac{10}{3}\left(\frac{5}{5}\right) = \frac{6}{15} + \frac{50}{15} = \frac{56}{15} \frac{\text{gallons}}{\text{hours}}$$

At this point, we know how much water is in the cistern to begin with (200 gallons) and at what rate water leaks out of the cistern $\left(\frac{56}{15} \text{ gallons per hour}\right)$. We need to figure out the time it will take for all that water to leak out of the cistern at that rate, so we need to set up an equation to model the situation in which the only missing variable is time, t. Since our rate uses the units of $\frac{\text{gallons}}{\text{hour}}$, if we multiply our rate by a time in hours, we'll end up with a number of gallons leaked in that amount of time. If we set that equal to 200 gallons, we'll end up with the amount of time it takes for 200 gallons to leak out of the cistern, the correct answer.

$$200 = t\frac{56}{15}$$

Alternatively, you could set up the equation using the logic that total water minus water leaked has to equal zero:

$$200 - t\frac{56}{5} = 0$$

Note that our first equation is simply this equation in a different form. Let's solve for t.

$$200 = t\frac{56}{15}$$

$$(200)15 = \left(t\frac{56}{15}\right)15$$

$$3000 = 56t$$

$$\frac{3000}{56} = \frac{56t}{56}$$

$$t = \frac{3000}{56}$$

Now that we've defined t as a fraction, we can reduce it to lowest terms to figure out which of the answer choics it matches. Doing this, we find that the correct answer is A.

$$t = \frac{3000}{56} = \frac{3000 + 8}{56 + 8} = \frac{375}{7} \text{ hours}$$

Decimals, Ratios, and Proportions

Decimals

A decimal measures the partial distance between two numbers as it would be read on a number line. Each decimal place represents an incremental division by ten.

Tenths: $0.1 = \dfrac{1}{100}$

Hundredths: $0.01 = \dfrac{1}{100}$

Thousandths: $0.001 = \dfrac{1}{1000}$

Ten thousandths: $0.0001 = \dfrac{1}{10000}$

In some cases, a decimal may be a rational number that contains a repeating pattern that is signified by a horizontal bar over the repeating pattern.

Example: $0.33\overline{3} = 0.33333333333\ldots$

Since the above decimal can indeed be written as a fraction $\left(\dfrac{1}{3}\right)$, it is a rational number.

Any fraction can be converted into a decimal by dividing the numerator by the denominator using a calculator. Some common fraction-to-decimal conversions are given below:

$\dfrac{1}{8} = 0.125$

$\dfrac{1}{5} = 0.2$

$\dfrac{1}{4} = 0.25$

$\dfrac{1}{2} = 0.5$

It is often helpful to glance at the answer choices before you start answering a question, because seeing the format of your final answer may influence your approach. Keep in mind that any fraction can be written as a decimal, but not every decimal can be written as a fraction.

Example: $\pi = 3.14159265359\ldots$

Since π is an irrational number (has endless non-repeating digits after the decimal point), it cannot be written as a fraction.

In addressing Sample Question #1, we must recognize that if x and y are both between 0 and 1, then they must be fractions or decimals. To determine their relationships, we can substitute values for x and y. Let's have $x=0.25$ and $y=0.50$ (because y must be greater than x). If x and y are both between 0 and 1, then xy is also between 0 and 1 (though smaller than either x or y individually).

$$xy = 0.25 \times 0.50 = 0.125$$

The value of $(xy)^2$ will be less than (xy), since squaring any number between 0 and 1 gives a smaller result.

$$(xy)^2 = \left(0.25 \times 0.50\right)^2 = (0.125)^2 = 0.015625$$

Finally, $\dfrac{y}{x}$ is largest, since dividing a larger number by a smaller one gives a result greater than 1.

$$\frac{y}{x} = \frac{0.50}{0.25} = 2$$

Sample Question #1

Given that $0 < x < y < 1$, which best describes the relationship between xy, $\dfrac{y}{x}$, and $(xy)^2$?

A. $\dfrac{y}{x} > xy > (xy)^2$

B. $(xy)^2 > \dfrac{y}{x} > xy$

C. $(xy)^2 > xy > \dfrac{y}{x}$

D. $\dfrac{y}{x} > (xy)^2 > xy$

The correct order is $\dfrac{y}{x} > xy > (xy)^2$, so the correct answer is A. Note that the substituted values for x and y were purely hypothetical; any values that satisfy the given inequality could be used.

Sample Question #2 asks us to identify digits in two decimals and compare their values. If you're not sure how to name a place, consider a value of one preceded by just zeros in it and convert that to a fraction. You can also do this in reverse: since we're looking for the thousandths and hundred-thousandths places, we could identify them this way:

$$\frac{1}{1000} = 0.001 \qquad \frac{1}{100000} = 0.00001$$

The thousandsths place is the third one from zero. By the same logic, the hundred thousandths place is the fifth one from zero. So, both Quantity A and Quantity B equal 2, making C correct.

Sample Question #2

Quantity A	Quantity B
The value of the digit in the thousandths place in 1.41247	The value of the digit in the hundred thousandths place in 2.94062

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

Ratios and Proportions

Ratios are mathematical relationships between two (or more) numbers that indicate how many times the first number is contained in the second (or third, fourth, etc.) number(s). Ratios may be represented using colons and/or fractions. They do not say anything about the quantities of the numbers involved other than their relative abundances. Since ratios can be written as either fractions or numbers separated by colons, they are usually written using whole numbers. It may be helpful to use the word "to" in ratios as a place marker for where to put the colon, or where to draw the line between the numerator and denominator.

For example, if for every egg used in a certain recipe the recipe includes three cups of flour, we could write the ratio of eggs to flour as $1:3$ or as $\frac{1}{3}$. Order is extremely significant when writing ratios and proportions: if we wrote $3:1$ or $\frac{3}{1}$, this would be the ratio of flour to eggs, not of eggs to flour. Ratios don't say anything about quantities actually used, just their *relative* abundance. For instance, if we wanted to double our recipe, the ratio would not change. We would use two eggs and six cups of flour, which would result in a ratio of eggs to flour of $2:6$ or $\frac{2}{6}$. These numbers are expression the same *relative* amounts as $1:3$ and $\frac{1}{3}$—for every egg used, we're still using three cups of flour. We're just using more of it. The amounts may have changed, but the ratio did not. This is supported by the math, as we can reduce our ratio of $2:6$ by dividing both sides by two to get $1:3$.

Proportions are similar to ratios in that they compare two values. For example, what proportion of a 10 inch sandwich have you eaten if you've eaten 5 inches of it? You can simply find a fraction to express this. Note how the units cancel and we're left with an abstract value:

$$\frac{\text{Amount of sandwich eaten in inches}}{\text{Total amount of sandwich in inches}} = \frac{5 \ \cancel{\text{in}}}{10 \ \cancel{\text{in}}} = \frac{1}{2}$$

This can also be expressed as a ratio of eaten sandwich to total sandwich, 5 in : 10 in, or in reduced form, 1 in : 2 in. Since the same units appear on both sides of the ratio, we could drop them at this point and state the ratio as $1:2$. We can interpret this reduced form by saying for every two units of sandwich you have, you've eaten one unit of the sandwich.

Proportions and ratios do not necessarily need to be different; for example, if there are 3 people in a room, all wearing baseball caps, the proportion of people wearing caps is 1. This comes from reducing the raw number ratio of $3:3$ or $\frac{3}{3}$.

Complex proportion and ratio problems can involve chains of ratios, so let's investigate how to solve these quickly and easily without getting bogged down in confusing scratchwork. Consider the following: you are making a cake that requires, by volume, three times as much flour as sugar, half as much milk as sugar, eight times more milk than baking powder and twice as much baking powder as salt. If you start with a teaspoon of salt, how many cups of flour do you need?

That's quite a lot of ingredients, and we're going to need to work through their proportional relationships in order to calculate the amount of each ingredient that is needed given the amount of salt in the cake. This becomes apparent as we identify the "chain": we're told about the proportion of flour to sugar, sugar to milk, milk to baking powder, and baking powder to salt. Now we just need to figure out how to organize these ratios' values so that we can do one long calculation to move from salt to flour. Let's use dimensional analysis, an organized approach to dealing with unit conversions in which you multiply by conversion factors and cancel units. In this case, instead of dealing with units like inches and feet, we're going to deal with amount of flour to amount of sugar, etc. We can do this because the problem defines a consistent ratio between flour and sugar that stays consistent even if we're working with different quantities. We're using the ratio to identify the quantities much like we use the conversion factor of, say, twelve inches in one foot to identify the number of inches in five feet.

Now, let's get to writing our equation. Careful labeling is essential, especially in a testing environment! Start by writing the quantity of salt that we're given: one teaspoon (tsp). Then, figure out what ratio involves salt and another ingredient. We're told that we need "twice as much" baking powder as salt, so that's a $2:1$ or $\frac{2}{1}$ ratio of baking powder to salt if we want the units of "salt" to cancel, we need to make salt the denominator of the fraction. It already is, so we're good to go.

$$1 \; \cancel{\text{tsp salt}} \times \frac{2 \; \cancel{\text{tsp baking powder}}}{1 \; \cancel{\text{tsp salt}}}$$

The units of "salt" cancel, leaving us with the necessary quantity of baking powder. Now, we just need to repeat this as necessary until we get to flour. We can write out one long equation and cancel lots of units at once to save time. Note that we don't use the units of teaspoons for the various conversion factors—we don't have to. The problem tells us that we use "twice as much" baking powder as salt, so this $2:1$ ratio would hold true if we started with teaspoons, tablespoons, cups, or gallons. We start with the unit of teaspoons and it's never canceled out, so it is also our answer's unit.

$$1 \text{ tsp} \cancel{\text{salt}} \times \frac{2 \; \cancel{\text{baking powder}}}{1 \; \cancel{\text{salt}}} \times \frac{8 \; \cancel{\text{milk}}}{1 \; \cancel{\text{baking powder}}} \times \frac{1 \; \cancel{\text{sugar}}}{2 \; \cancel{\text{milk}}} \times \frac{3 \text{ flour}}{1 \; \cancel{\text{sugar}}} =$$

$$1 \text{ tsp} \times \frac{2}{1} \times \frac{8}{1} \times \frac{1}{2} \times \frac{3}{2} = \frac{1 \times 2 \times 8 \times 1 \times 3}{1 \times 1 \times 2 \times 2} = \frac{48}{4} = 12 \text{ tsp}$$

If we use 1 tsp of salt, we'll need to use 12 tsp of flour. The ratio of salt to flour is $1:12$ or $\frac{1}{12}$.

Let's start answering Sample Question #3 by writing out what we're told in algebraic form. We're going to use the same ratio skills we've been practicing thus far to solve this problem, even though we're working with abstract variables instead of concrete things like teaspoons of salt and flour.

$$a + b + c = 400$$

$$a = \frac{1}{3}b$$

$$b = \frac{1}{4}c$$

Because a is defined in terms of b and b is defined in terms of c, we can define a in terms of c using substitution.

Sample Question #3

The sum of a, b, and c is 400. a is $\frac{1}{3}$ of b and b is $\frac{1}{4}$ of c. What is the value of a?

    A. 20
    B. 25
    C. 36
    D. 64

$$a = \frac{1}{3}\left(\frac{1}{4}c\right) = \frac{1}{12}c$$

We have all of our variables defined in terms of c, so we only have to work with one variable. Let's substitute a and b in terms of c into the addition equation.

$$a + b + c = 400$$

$$\left(\frac{1}{12}c\right) + \left(\frac{1}{4}c\right) + c = 400$$

At this point, we need to solve for c. We can do so by creating a common denominator and finding the sum of the terms on the left side of the equation as an improper fraction. After that, we can isolate the variable by multiplying by the reciprocal and simplifying.

$$\frac{1}{12}c + \frac{1}{4}c\left(\frac{3}{3}\right) + c\left(\frac{12}{12}\right) = 400$$

$$\frac{1}{12}c + \frac{3}{12}c + \frac{12}{12}c = 400$$

$$\frac{16}{12}c = 400$$

$$\left(\frac{12}{16}\right)\frac{16}{12}c = 400\left(\frac{12}{16}\right)$$

$$c = \frac{4800}{16} = 300$$

Now that we've defined c, we can define a since we defined a in terms of c. B is the correct answer.

$$a = \frac{1}{12}c = \frac{1}{12}(300) = \frac{300}{12} = 25$$

Ratio and proportion problems can also give you a ratio and the quantity of one value in the ratio ask you to solve for the quantity of the other value. Sample Question #4 does this and adds a bit more complexity in that you have to account for other members of the group who are not students and thus don't fall into either of the ratio's categories.

Begin by eliminating people in the class who are not students. Subtracting 1 teacher, 1 administrator, and 30 evaluators from the 224 total people in the class leaves 192 students.

$$224 - (1 + 1 + 30) = 224 - 32 = 192$$

We also need to determine the ratio of juniors to seniors from the information in the question. We must add together the ratio of male students to the ratio of female students to find the total number of students in the ratio: 32. Thus, out of every thirty-two students, thirteen are male.

$$\frac{13 \text{ male students}}{19 \text{ female students}}$$

$$\frac{13 \text{ male students}}{13 + 19 \text{ total students}} = \frac{13 \text{ male}}{32 \text{ total}}$$

Now we can set up a proportion and solve for the number of male students.

$$\frac{13 \text{ male}}{32 \text{ total}} = \frac{x}{192}$$

$$\frac{13}{32} \times 192 = x$$

$$78 = x$$

The correct answer is A.

Sample Question #4

The ratio of male students to female students in a class is 13 to 19. If there are 224 people in the class, including one teacher, one administrator, and thirty evaluators, how many people in the class are male students?

    A. 78
    B. 80
    C. 91
    D. 114

Percentages

The definition of the word percent comes from the Latin word "cent," which means *one hundred*. This word is the root of other familiar words like "centipede" (*insect originally thought to have one hundred legs*), "century" (*period of one hundred consecutive years*), and "cent" (*unit of monetary value equal to 1/100 of a U.S. dollar*). A "per-cent" has to do with 100 too: it is a special ratio relating a number to one hundred, since "per" indicates division or proportionality. Just think of how you'd use the word "per" in other situations: "we need one ticket per person" means that the ratio of tickets to people must be 1 : 1.

Whenever you see a percent, you're seeing a value represented in shorthand of that value over one hundred or in a ratio to one hundred. Since percents are special proportions, we can readily interchange between different ways to represent them. A percent sign means "over one hundred" when converting to a fraction. To go from a decimal to a percent, just multiply it by 100 and add the percent sign.

$$24\% = \frac{24}{100} = 0.24 = 24 : 100$$

Thus, if this percent is the number of people who completed a survey, the 24 stands for that specific group, and the 100 for all the people who could have potentially completed it. After all, if the survey-senders got a 100% response rate, that means that everyone to whom they sent the survey would have returned it. But that's not the case: 24% of people returned the survey, so that means that for every 100 people they sent the survey to, 24 sent it back.

But what if they only sent the survey to 50 people? The percent still applies; it's an abstract value that doesn't rely on there actually being 100 surveys sent out. More or fewer could have been sent; the percent is a ratio that applies to however many we want to consider.

If we want to use a percent to figure out how many surveys were returned if 20 or 500 were sent out, we can do so by using the percent as a multiplier of that total value. Remember how 24% is just shorthand for $\frac{24}{100}$? Well, $\frac{24}{100} = 0.24$, and it's easier to multiply whole numbers with decimals than fractions.

$$24\% = \frac{24}{100} = 0.24$$

0.24 × 400 surveys = 96 surveys returned

0.24 × 50 surveys = 12 surveys returned

Here's a useful trick you can use when working with percent problems: read the problem to yourself, and whenever you have an unknown, write down an *x*. Whenever you say "is," write an equals sign, and whenever you say "of," write down a multiplication symbol. Thus, the problem,

"What is 36% of 540?" becomes the following:

$$x = 36\% \times 540$$

Of course, we'll need to change our percent into a decimal, but we get an easily solvable problem:

$$x = 0.36 \times 540 = 194.4$$

This also works with rearranged problems where you're asked to solve for the percent itself:

What percent of 300 is 42?

$$x \times 300 = 42$$

$$x = \frac{42}{300} = 0.14 = 14\%$$

Another method you can use to address percent problems is setting up a proportion and solving it for your unknown. Consider the question "26 is what percent of 84?" Since we are looking for the percent, we know that unknown value in our proportion will be in the numerator of our set of proportions, placed over a denominator of 100.

$$\frac{26}{84} = \frac{x}{100}$$

Cross multiply and solve for x to find the answer.

$$84x = 2600$$

$$x = \frac{2600}{84} = 30.95\%$$

Either method works—just use the one you feel more comfortable using, and remember: if you're using the "translating" method, you may need to convert your final answer into a percent by dividing it by 100 and adding the percent symbol. If you're using the ratio method, you've already done that by creating a fraction over 100, so you just need to add the percent symbol.

You're well on your way to being able to solve any percent problem the GRE can throw at you. Let's try some out now!

Many percent problems on the GRE can take the form of questions about sales and taxes. Sale problems will have you subtracting money from a given price, and tax problems will have you adding money to a price.

Sample Question #1

A pair of shoes costs $60. The store is having a sale in which there is 15% off on shoes. What is the sale price of the shoes? (Assume no sales tax.)

A. $9.00
B. $16.00
C. $44.00
D. $51.00

Sample Question #1 is a sale problem, so we'll be subtracting. We're given the initial, pre-sale price of a pair of shoes, and the percent of the price that the sale deducts. To solve this problem, we need to identify how much money that percent of the price represents so that we can subtract it and find the new price.

To find out how much money the sale is saving the shoe purchaser, we can ask ourselves: what is 15 percent of $60? That's language we can "translate":

What is 15% of $60?

$x = 0.15 \times 60 = 9$

Alternatively, you can we set up and solve a proportion:

$$\frac{15}{100} = \frac{x}{\$60}$$

At this point, we cross multiply to solve for x.

$100x = \$900$

$x = \$9$

$9.00 is not our final answer, though! This is the amount of money that is saved due to the sale. To find the final cost of the shoes, we subtract the saved amount of money from the original cost. The correct answer is D.

$60 – $9 = $51

Sample Question #2 is a two-part question about taxes on a car. First, we need to calculate the final price of the car after taxes. To do this, we multiply the original price by the tax proportion then add that number to the original price of the car.

$33504 \times 0.08275 = \$2772.46$

$2772.46 + \$33504 = \$36276.46$

Alternatively, we could have multiplied the original price of the car by the proportion of the sales tax percentage plus one: 1.08275. Why does this work? Well, our multiplier here consists of two parts: 1 + the tax proportion. If we multiply the value of the car by one, we get, well, the same number—the value of the car. If we multiply it by the tax percentage in decimal form, we get the amount of money that will be added to the cost of the car as tax. So really, by multiplying by 1.08275, we're just combining the steps of finding the amount of money taxed and adding it to the car's cost.

Sample Question #2

A brand new car is $33504. Sales tax is 8.275%. What proportion of the total price of the car after tax is the untaxed price of the car?

A.   0.8744
B.   0.8921
C.   0.9236
D.   0.9325

$$\$33504 \times 1.08275 = \$36276.46$$

Now we need to find the proportion (fraction) of the total after-tax price that was due to the price of the care alone (excluding tax). We set up a simple proportion/fraction:

$$\frac{\$33504}{\$36276.46} = 0.9236$$

Notice that our proportion does not have any units associated with it because it is a comparison between two relationships that are in the same units (dollars), so they cancel. The correct answer is C.

Sample Question #3

A $140 book is marked down by 25% and then by another 10%. What is its final sale cost?

   A. $92.50

   B. $94.50

   C. $96.25

   D. $98.25

Applying multiple sales to a single item can create a complex scenario: how do you go about applying two sales? Let's see how to approach such problems by answering a sample question.

You can do a problem like Sample Question #3 two ways: you can calculate the amount of money saved during each sale and subtract it from the original price, or you can consider the fraction of the original book's cost resulting after each sale and multiply the original cost by those sequential reductions as decimals.

For the first way, you can first multiply:

$$\$140.00 \times 0.25 = \$35.00$$

Subtracting this from $140, you get a price of $105. That's the price of the book after a 25% reduction. You do the same thing again for the 10% markdown:

$$\$105.00 \times 0.1 = \$10.50$$

Subtracting this from $105.00, you get $94.50.

A quicker way to do problems like this one is to notice that after the 25% discount, the price will be 75% of the original. Then, after the 10% discount, it will be 90% of that altered price. You can just multiply sequentially to get your amount:

$$\$140.00 \times 0.75 \times 0.90 = \$94.50$$

The correct answer is B. Either method of approaching the problem works consistently; just use whichever one makes the most sense to you.

Percent Increase and Percent Decrease

Another way in which the GRE might test your knowledge of percents is in the form of percent increases and percent decreases. In this problem paradigm, you'll be given two numbers and asked to quantify the change from one to the other as a percent of the original value. Thus, if an antique doubled in value over the course of one year, its value increased by 100%.

There's an important distinction to be made here. The new value of the antique would NOT be "100% of the old value." Remember how "of" implies multiplication? If we multiply the old value by 100%, we'd get the exact same value! No, we have to phrase it as "the value rose by 100%." That means that it would be 200% of its old value.

Let's consider another example and start applying our skills. Say one share of a stock was valued at $45.00 last year, and this year, it's valued at $37.00. What was its percent decrease from last year to this year? To find the percent decrease, we first need to quantify the change in value in dollars.

$45.00 – $37.00 = $8.00

Now we just need to figure out what percent of 45 8 is. The easiest way to mess up percent increase or percent decrease problems is to use the wrong value at this point. We want the original value of the stock *before* it decreased, not the value of the stock *after* it decreased. This makes sense: we want to know what percentage that original value decreased by, so we should focus on that number.

What percent of $45.00 is $8.00?

$x \times \$45.00 = \$8.00$

$x = \dfrac{8}{45} = 0.1778 \times 100 = 17.78\% \approx 18\%$

The stock's price decreased by 18% since last year. What if we're asked what percent of the original value the new value represents? Well, this is not a percent increase or decrease question. Instead, we'll need to make a fraction using the new value over the old value.

$\dfrac{37}{45} = 0.822\% = 82.22\% \approx 82\%$

This year, the stock is worth 82% of what it was worth last year. See how important subtle phrasing can be? Just pay attention to whether you're being asked to quantify the change in the original value or relate the new value to the old value. Figuring out whether you're focusing on the change or the new value can help distinguish these problems.

Percent increase or decrease: $\left(\dfrac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100 \right)\%$

What percent of old value is new value? $\left(\dfrac{\text{New Value}}{\text{Old Value}} \times 100 \right)\%$

For a bit more practice, let's try out another sample question. Remember how you need two values to solve a percent increase or a percent decrease problem? Well, the GRE isn't always going to just hand you those and say "go." They might be presented as part of a graph, table, or chart. This type of question works very well for data interpretation scenarios in which something is measured over time.

Week of Exercise Plan	0	1	2	3	4	5	6	7	8	9	10
Weight in lbs	130	127.5	129.5	126.5	121	123.5	124	120	122	117.5	115.5

Sample Question #4

A person exercises for ten weeks and tracks his weight each week. The data is shown at right.

By what percentage did his or her body weight decrease after five weeks of exercise? Round to the nearest hundredth of a percent.

A. 3.170%
B. 3.462%
C. 3.645%
D. 3.824%

Sample Question #4 is asking us for a percent decrease, whereas Sample Question #5 is asking us to find what percent of the exerciser's original weight his weight at the end of the exercise plan represents. For the first question, we need to subtract the exerciser's weight during week five from his initial weight to find the amount of weight he had lost at that point and divide it by his original weight. We can condense this into one step, and we find that the correct answer is B.

$$\frac{(\text{Weight}_{Old} - \text{Weight}_{New})}{\text{Weight}_{Old}}$$

$$\frac{(130 - 123.5)}{130} = \frac{4.5}{130} = 0.03462 \times 100 = 3.462\%$$

For Sample Question #5, we need to take the amount of weight lost and make it the numerator in a fraction where the denominator is the exerciser's original weight. Dividing the numerator by the denominator, we get a decimal, and multiplying that decimal by 100 and adding the percent symbol, we find that C is the correct answer.

$$\frac{\text{Weight}_{New}}{\text{Weight}_{Old}} = \frac{115.5}{130} = 0.08885 \times 100 = 88.85\%$$

Sample Question #5

What percent of his original weight did the exerciser weigh at the end of his exercise plan?

A. 84.50%
B. 86.75%
C. 88.85%€
D. 90.15%

Solving Exponents and Roots

When a given term is raised to the second power, it is said to be squared. In other words, when a number is squared, it has an exponent of 2 and is multiplied by itself. Three squared is equal to 3^2, which is equal to $3 \cdot 3$. The term being raised to a power is called the base. If the base is a variable and there is a number in front of it, e.g. the 2 in $2x^2$, that number is called the coefficient.

Numbers can be raised to powers other than that of two, though. Say you encounter x^3. This is pronounced "x to the third power" or "x cubed." Mathematically, you could write it out as $x \times x \times x$; Furthermore, any number that doesn't have a base specified can be seen as having a base of 1. (This accords with the *Identity Property of Exponents*, which we will review in a later lesson.)

If you want to add or subtract two bases with exponents, you can do so only if both the bases and the exponents are the same in each term.

$$4 + 1 = 5$$
$$3^2 + 3^2 = 2(3^2)$$
$$x + x = 2x$$
$$x^2 + x^2 = 2x^2$$

If the bases don't match, then you have to resolve them individually before adding their sums. If the bases are known numbers, then you can do this easily.

$$2^2 + 3^2 = (2^2) + (3^2) = 4 + 9 = 13$$

If part(s) of the expression are unknown variables, then the bases simply cannot be added together, and the addition expression is the most simplified form possible without additional information.

$$x^2 + y^2 = x^2 + y^2$$

Sample Question #1

Which of the following is equivalent to the expression $x^2 + 2y^3 + y^3 + x$?

A. $3x^2 + 3y^3$

B. $x + x^2 + 3y^3$

C. $x^2 x + 2y^3 y^3$

D. $x^3 + 2y^6$

Now, let's practice these rules by solving Sample Question #1. We need to figure out how the expression $x^2 + 2y^3 + y^3 + x$ can be simplified. Remember that two terms with powers can only be added together when their bases and exponents are the same. You don't simply add the bases together.

In this equation, the only terms that share the same base and power are the two terms that include y^3. These are the only terms we can combine. When combining them, remember that even when there is no number written in front of y^3, we need to assume that it has a coefficient of 1. This means that our addition expression really looks like this:

$$2y^3 + 1y^3 = 3y^3$$

Since none of the other terms in the given expression can be combined or simplified, our answer is $x + x^2 + 3y^3$. That means that the correct answer is B.

Square Roots

Taking the square root of a term is the inverse function of squaring. In other words, the solution to a square root answers the question, "What number can be squared to get the starting value in this problem?"

A perfect square is a term that is equal to a single number squared. A short list of perfect squares is given below:

$1^2 = 1$ $6^2 = 36$
$2^2 = 4$ $7^2 = 49$
$3^2 = 9$ $8^2 = 64$
$4^2 = 16$ $9^2 = 81$
$5^2 = 25$ $10^2 = 100$

A perfect square will always have a whole number square root. Since the square root operation is the inverse function of squaring, we can create a second list of equations showing the roots of each perfect square:

$\sqrt{1} = 1$ $\sqrt{36} = 6$
$\sqrt{4} = 2$ $\sqrt{49} = 7$
$\sqrt{9} = 3$ $\sqrt{64} = 8$
$\sqrt{16} = 4$ $\sqrt{81} = 9$
$\sqrt{25} = 5$ $\sqrt{100} = 10$

Sample Question #2

Simplify the following expression:

$$5^2 - \sqrt{49}$$

A. 15

B. 18

C. 19

D. 20

Looking at Sample Question #2, a basic example, we can see how these operations function. To find the final answer, simply evaluate each term. We can expand 5^2 to become $5 \cdot 5$, which is equal to 25.

$$5^2 - \sqrt{49} = 25 - \sqrt{49}$$

The square root of 49 is 7 because $7 \cdot 7 = 49$. 49 is a perfect square.

$$5^2 - \sqrt{49} = 25 - \sqrt{49} = 25 - 7 = 18$$

The final answer will be 18.

Simplifying Square Roots

The square root function is simplest when applied to perfect squares, but what about other numbers? On the GRE Quantitative section, you may be asked to simplify an expression that contains the square root of a number that is not a perfect square; however, you will not be asked to solve for the value of this term. Most often, solving for these values results in an irregular decimal or other such inconvenient term. To avoid these scenarios, the exam will focus on simplification of terms as opposed to direct solutions.

Simplifying a square root will ultimately lead to factoring out terms from under the square root and extracting factors until the term cannot be further simplified. Remember that taking the square root is the inverse function of squaring a number. If we can identify instances where there is a square underneath a square root, we can factor out that term. For example:

$$\sqrt{4^2} = 4$$

The easiest method for simplifying a square root is to identify all of the prime factors of the term under the square root operation, and then identify duplicate factors. These duplicates indicate squared terms. Prime factors can be found by building a factor tree or using any other method to identify factors. Let's look at $\sqrt{75}$ as an example. The prime factors of 75 are 3, 5, and 5 because $3 \cdot 5 \cdot 5 = 75$. There are two ways to represent this in a factor tree.
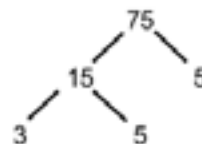
Once we have identified the factors, look for any duplicates. In the case of 75, we can see that 5 appears twice. In other words, $5 \cdot 5 \cdot 3 = 5^2 \cdot 3 = 75$. Since taking the square root is the inverse of squaring a number, we can factor out the 5 from under the square root.

$$\sqrt{75} = \sqrt{5^2 \cdot 3} = 5\sqrt{3}$$

Note that only the 5 can be factored out of the square root. Since the 3 is not a duplicate in the factor tree, it cannot be simplified. $5\sqrt{3}$ is the most simplified answer possible.

Let's solve Sample Question #3. The factors of 32 are 2, 2, 2, 2, and 2, so we can

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Sample Question #3

Which of the following is equivalent to $\sqrt{32}$?

A. $3\sqrt{2}$

B. $4\sqrt{2}$

C. $5\sqrt{2}$

D. $8\sqrt{2}$

rewrite $\sqrt{32}$ as follows:

$$\sqrt{32} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$

There are two pairs of 2's that can be factored out of this expression, but one unpaired 2 will remain under the square root. This means that our expression can be adjusted like this:

$$\sqrt{32} = 2 \cdot 2\sqrt{2} = 4\sqrt{2}.$$

The correct answer is B, $4\sqrt{2}$.

Equations with Square Roots

Now that we have looked at simplifying individual square root terms, we can start evaluating how these terms interact within equations and expressions. When a square root appears in an expression, the first step will always be to simplify the square root via the process we discussed previously. Once the square root cannot be simplified further, we can start applying other operations (e.g. addition, subtraction, multiplication, and division). Within expressions, the simplified square roots can be treated just like variables.

Let's start with a simple example of addition with square roots:

$$\sqrt{2} + \sqrt{2}$$

Treating the term "$\sqrt{2}$" like a variable, we can imagine that this expression is the same as $x + x$, which simplifies to $2x$. In the same way, $\sqrt{2} + \sqrt{2}$ simplifies to $2\sqrt{2}$.

$$\sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

The same process is used for simplification of addition and subtraction. As long as the same number is under the square root, the terms can be combined. Let's look at a full problem:

$$\sqrt{9} - \sqrt{12} + \sqrt{75}$$

Simplify the square roots.

$$3 - 2\sqrt{3} + 5\sqrt{3}$$

Combine terms that contain $\sqrt{3}$.

$$3 + 3\sqrt{3}$$

This problem cannot be simplified any further because the leading term is not associated with a square root. Remember, you can only combine terms with the same value under the square root; in this problem, we can only combine the terms that contain $\sqrt{3}$.

When there are different values under the square roots, the terms cannot be simplified. For example, $2\sqrt{3} + \sqrt{5}$ cannot be simplified any further.

Example:

$$\sqrt{40} + \sqrt{18}$$
$$2\sqrt{10} + 3\sqrt{2}$$

Because the base terms cannot be combined, the expression cannot be simplified.

When dealing with multiplication and division of square roots, there is a bit more flexibility. When multiplying or dividing square roots, the terms under the square root are combined.

$$\sqrt{2} \cdot \sqrt{3} = \sqrt{2 \cdot 3} = \sqrt{6}$$
$$\sqrt{14} \div \sqrt{7} = \sqrt{14 \div 7} = \sqrt{2}$$

Since the terms under the square root can be combined, multiplication and division can lead to additional simplification of the final answer. Many times, you will be able to factor out a term after multiplying two square roots.

Example:

$$\sqrt{40} \cdot \sqrt{15}$$

Simplify the square roots.

$$2\sqrt{10} \cdot \sqrt{15}$$

Multiply.

$$2\sqrt{10 \cdot 15}$$
$$2\sqrt{150}$$

Simplify the final square root.

$$2 \cdot 5\sqrt{6}$$
$$10\sqrt{6}$$

In this example, combining $\sqrt{10}$ and $\sqrt{15}$ lets us factor out a 5 before arriving at our final answer.

To solve Sample Question #4, we need to start by simplifying each term.

$$\sqrt{90} = 3\sqrt{10}$$
$$\sqrt{45} = 3\sqrt{5}$$
$$\sqrt{128} = 8\sqrt{2}$$

At this point, we can plug the simplified values back into the given equation.

$$\sqrt{90} \cdot \sqrt{45} - \sqrt{128} = 3\sqrt{10} \cdot 3\sqrt{5} - 8\sqrt{2}$$

Multiply to combine the first two terms, and then simplify the result.

$$3 \cdot 3\sqrt{10 \cdot 5} - 8\sqrt{2}$$
$$9\sqrt{50} - 8\sqrt{2}$$
$$9 \cdot 5\sqrt{2} - 8\sqrt{2}$$
$$45\sqrt{2} - 8\sqrt{2}$$

Since the two terms have the same value under the square root, they can be combined. After doing this, you can see that the correct answer is A.

$$45\sqrt{2} - 8\sqrt{2} = 37\sqrt{2}$$

When solving Sample Question #5, we start the same way: by simplifying each term.

$$\sqrt{216} = 6\sqrt{6}$$
$$\sqrt{24} = 2\sqrt{6}$$
$$\sqrt{18} = 3\sqrt{2}$$
$$\sqrt{32} = 4\sqrt{2}$$

Then, we can substitute the simplified values into the presented expression:

$$\sqrt{216} \div \sqrt{24} - \sqrt{18} \cdot \sqrt{32} = 6\sqrt{6} \div 2\sqrt{6} - 3\sqrt{2} \cdot 4\sqrt{2}$$

Divide the first two terms and multiply the last two terms, according to *order of operations*, and simplify the result.

Sample Question #4

Which of the following is equivalent to the following expression?

$$\sqrt{90} \cdot \sqrt{45} - \sqrt{128}$$

    A. $37\sqrt{2}$

    B. $40\sqrt{2}$

    C. $38\sqrt{3}$

    D. $42\sqrt{3}$

Sample Question #5

Which of the following is equivalent to the following expression?

$$\sqrt{216} \div \sqrt{24} - \sqrt{18} \cdot \sqrt{32}$$

    A. $14\sqrt{2}$

    B. $20\sqrt{3}$

    C. -21

    D. -40

$$6 \div 2 \sqrt{6 \div 6} - 3 \cdot 4 \sqrt{2 \cdot 2}$$
$$3\sqrt{1} - 12\sqrt{4}$$

Simplify the new square roots. Since both terms are perfect squares, we can factor out whole numbers.

$$3 \cdot 1 - 12 \cdot 2$$
$$3 - 24$$
$$-21$$

Algebra

One of the most significant mathematical topics the Quantitative Reasoning section tests is algebra. Put simply, algebra focuses on representing unknown quantitates as variables (such as x and y). When located in equations, specific mathematical steps can be taken to solve for the specific value of one or more variables. Our "Rules of Exponents and Variables" serves as a good introduction to the core rules of this type of math.

One can model different types of consistent relationships between two variables using either equations or the graphs associated with them. The Quantitative Reasoning section tests both presentations, so being able to move from one to the other seamlessly is an important skill to practice. With this in mind, we divide up our treatment of algebraic equations by type (e.g. linear or quadratic) and by format (e.g. equations and inequalities or graphs).

We conclude our Algebra section with a lesson looking at the definitions and conventions surrounding functions, and a lesson about the two specific formulae for calculating interest that may be relevant on test day.

If it's been a while since you worked with computational mathematics, algebra can easily seem imposing. Taking the time to read over the lessons in this section should help you break apart the challenge of refamiliarizing yourself with this branch of mathematics by treating it in distinct parts.

Section Outline

Rules of Exponents and Variables

Linear Equations and Inequalities

Systems of Linear Equations

Properties of Linear Graphs

Quadratic Equations and the Quadratic Formula

Properties of Quadratic Graphs

Function Notation

Word Problems: Simple Interest and Compound Interest

Word Problems: Working with Rates

Rules of Exponents and Variables

The GRE Quantitative section does not restrict itself to problems that make use of only the basic operations of addition, subtraction, multiplication, and division. You can expect problems featuring more complex mathematical concepts such as exponents and square roots to make an appearance on this section. In this lesson, we'll take a look at each of these topics and brush up on the specific rules used when mathematically working with them. By working through this lesson and its practice problems, you can refresh your knowledge of these concepts and be ready to demonstrate expert understanding of these subjects on test day.

Exponents

The rules of exponents are a significant part of the GRE Quantitative test. These rules may be tested as straightforward questions about the rules themselves (e.g. $x^2 \times x^3 = ?$), but they're more likely to be incorporated into broader questions associated with simplifying expressions. Given the GRE's penchant for word problems, it's likely that the rules of exponents will show up in questions asking about real-life scenarios.

Another important reason to know the rules of exponents for the GRE is because of the abilities of the digital calculator you are provided on the exam. It is easy for test makers to construct questions that, while not difficult, are easy to miss if you rely too heavily on a calculator. And the more complicated the exponential expression, the more likely you are to input the expression incorrectly.

For the purposes of the test, it is important to familiarize yourself with six rules of exponents. These can be divided into two groups of three: one group of basic rules to be aware of, and a group of more complex rules to be aware of when mathematically working with exponents. We'll refer to the first group as "Awareness" rules and the second as "Application" rules. Let's get started by going over each of them!

Awareness Rules	Application Rules
Identity Property of Exponents $$x^1 = x$$	Product of Exponents with the Same Base $$x^a \times x^b = x^{(a+b)}$$
Zero Exponent Rule $$x^0 = 1$$	Quotient of Exponents with the Same Base $$\frac{x^a}{x^b} = x^{(a-b)}$$
Negative Exponent Rule $$x^{-a} = \frac{1}{x^a}$$	Power-to-a-Power Rule $$(x^a)^b = x^{(a \times b)}$$

Identity Property	Any number raised to the power of 1 is equal to itself.

$$2^1 = 2$$

$$1,000,000^1 = 1,000,000$$

Zero Exponent Rule	Any number raised to the power of 0 is equal to 1.

$$2^0 = 1$$

$$5^0 = 1$$

$$1,000,000^0 = 1$$

Negative Exponent Rule	If you raise a number to a negative power, you need to divide the expression by 1 and change the sign on the exponent.

So, for instance, 5^{-3} becomes $\dfrac{1}{5^3} = \dfrac{1}{125}$.

Similarly, if you have a negative exponent in the denominator, that expression moves to the numerator and the exponent changes its sign:

$$\frac{1}{3^{-2}} = 3^2 = 9$$

Perhaps the most basic way to view a negative exponent is to look at your negative exponential expression and do two things:

1. Find its reciprocal (i.e. flip it)
2. Change the sign of its exponent

It is common for students to forget or misapply the rule for negative exponents. Test-takers generally want to change the sign of the base rather than the sign of the exponent. Don't let this happen to you, especially on test day! Negative exponents do not affect whether the final answer is positive or negative.

Examples:

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} \qquad\qquad \frac{1}{4^{-3}} = 4^3 = 64$$

$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9} \qquad\qquad \frac{1}{2^{-2}} = 2^2 = 4$$

| Product Rule | If you are multiplying exponential terms that have the same base, you can simply add the exponents, and raise the common base to that exponent. |

$$x^2 \times x^3 = x^{(2+3)} = x^5$$
$$2^5 \times 2^4 = 2^{(5+4)} = 2^9 = 512$$

| Quotient Rule | If you are dividing exponential terms that have the same base, you can subtract the exponent of the denominator from the exponent of the numerator. |

$$\frac{2^5}{2^4} = 2^{(5-4)} = 2^1 = 2$$
$$\frac{x^2}{x^3} = x^{(2-3)} = x^{-1} = \frac{1}{x^1} = \frac{1}{x}$$

| Power-to-a-Power Rule | When raising a power to a power, you multiply the exponents. |

$$(2^3)^2 = 2^{(3\times2)} = 2^6 = 64$$
$$((3^2)^3)^2 = 3^{(2\times3\times2)} = 3^{12} = 531,441$$

Try answering a few sample problems to really cement your knowledge of exponent rules! Sample Question #1 is a Quantitative Comparison question that requires you to use several of the rules we just reviewed. Let's start by simplifying the expression given as Quantity A:

$$\left(\frac{x^4}{x^3}\right)^{-2} = (x^{4-3})^{-2}$$

First, since both the numerator and the denominator of our fraction have the same base, x, we can turn our fraction into an exponent subtraction problem as per the Quotient Rule.

$$(x^{4-3})^{-2} = (x^1)^{-2}$$

Sample Question #1

Quantity A	Quantity B
$\left(\dfrac{x^4}{x^3}\right)^{-2}$	$(x^{-2})^2$

A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information given.

That subtraction results in a value of x^1. Since the entire expression in parentheses is raised to the power of negative two, we next need to multiply 1 and 2 as per the Power-to-a-Power Rule. This results in an expression of x^{-1}.

$$(x^1)^{-2} = x^{-2}$$

Finally, we can apply the Negative Exponent rule, making the positive value of our exponent a numerator over a denominator of the base, to find our simplified expression of Quantity A.

$$x^{-2} = \frac{1}{x^2}$$

Now to consider Quantity B! First, we need to apply the Power-to-a-Power Rule. This means multiplying 2 and –2, resulting in an exponent of –4. Then we apply the negative exponent rule.

$$(x^2)^{-2} = x^{-4} = \frac{1}{x^4}$$

At this point, we can compare the two simplified quantity expressions, $\frac{1}{x^2}$ and $\frac{1}{x^4}$. Is one necessarily of higher value than the other? No, and you can prove that to yourself by substituting in different values for x and comparing the results.

If $x = 1$, $A = B$.

$$\frac{1}{x^2} = \frac{1}{1^2} = \frac{1}{1} = 1$$
$$\frac{1}{x^4} = \frac{1}{1^4} = \frac{1}{1} = 1.$$

If $x = \frac{1}{4}$, $A < B$ $(16 < 256)$.

$$\frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\frac{1}{16}} = 1 \div \frac{1}{16} = 1 \times \frac{16}{1} = 16$$

$$\frac{1}{\left(\frac{1}{4}\right)^4} = \frac{1}{\frac{1}{256}} = 1 \div \frac{1}{256} = 1 \times \frac{256}{1} = 256$$

If $x = 2$, $A > B$, $\left(\frac{1}{4} > \frac{1}{16}\right)$.

$$\frac{1}{2^2} = \frac{1}{4}$$

$$\frac{1}{2^4} = \frac{1}{16}$$

That means that the correct answer to this question is D: the relationship cannot be determined from the information given.

Linear Equations and Inequalities

A "linear" equation is a special type of algebraic relationship in which a certain change in the x-value creates a consistent change in the y-value on a coordinate plane. Put a slightly different way, the ratio of the change in *y* to the change in *x* is always the same. This ratio is referred to as the **slope** of the equation. You might hear slope described as "rise over run"—this essentially means the same thing. Written algebraically, this generates the following expression:

$$m = \frac{\Delta \text{rise}}{\Delta \text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

There are two conventional forms in which you can write a linear equation.

> Standard (slope-intercept) form: $y = mx + b$
>
> Point-slope form: $y - y_1 = m(x - x_1)$

In both forms, *m* is the slope of the equation. In slope-intercept form, *b* is its y-intercept, or the y-coordinate of the point at which $x = 0$—the point at which the graphed line intersects the y-axis. Note that no variable is raised to a power other than 1 in either form; this is part of the detinition of a linear equation, and if you encounter squared terms or terms raised to larger powers, you are not dealing with a linear equation or, graphically, a simple straight line.

The GRE Quantitative section may test your abilities to write linear equations by presenting you with different types of data. You can find the equation of a line if you know any of the following:

1. The **slope** of the line and its **y-intercept**

2. The **slope** of the line and the (x, y) coordinates of **one point** on the line

3. The (x, y) coordinates of **two points** on the line

Let's write the equation of a line using each of the three methods.

1. Finding the equation from the slope and y-intercept

We can solve Sample Question #1 using the easiest method. Since you are given the slope and a y-intercept, you can just use the slope-intercept formula, substituting your slope for *m* and your y-intercept for *b*. Thus, the equation of a line with a slope of 2.5 and a y-intercept of –7 is $y = 2.5x - 7$. None of our answer choices match this, but that's because they're not in standard form and they need to be reduced. Let's consider each one and see which one matches. The correct answers are B and D.

A. $10x + 4y = 28$

$10x + 4y - 28 = 0$

$10x - 28 = -4y$

$-4y = 10x - 28$

$y = -\dfrac{10}{4}x - 7$

$y = -2.5x - 7$

B. $3y - 21 = 7.5x$

$3y = 7.5x + 21$

$y = 2.5 + 7$

C. $0 = 3y + \dfrac{15}{2}x - 21$

$-3y = \dfrac{15}{2}x - 21$

$3y = -\dfrac{15}{2} + 21$

$3y = -7.5x + 21$

$y = -2.5 + 7$

D. $-\dfrac{y}{2} + \dfrac{5}{4}x + 28 = 42$

$-\dfrac{y}{2} + 1.25x = 14$

$-\dfrac{y}{2} = -1.25x + 14$

$-2(-\dfrac{y}{2}) = [-2(-1.25)] - [-2(14)]$

$y = 2.5 + 7$

2. Finding equation from slope and a point on the line

Sample Question #2 is much more straightforward, but it asks us to find an equation of a line from different data. We're given the slope, 2, but not the y-intercept. In other words, we are given a point through which the line passes, though: (–3, 1). We are thus told a starting point and the relative direction in which the line progresses; therefore, we can use this information like the starting point on a map and directions in order to calculate where the line must necessarily cross the y-axis. We can thus solve this problem by substituting in the slope and the *x* and *y* coordinates of the point on the line into the equation and then solving for *b*, the only remaining unknown variable.

Sample Question #1

Multiple answers may be correct. Select all that apply.

Which of the following is the equation of a line with a slope of 2.5 and a y-intercept of –7?

A. $10x + 4y = 28$

B. $3y - 21 = 7.5x$

C. $0 = y + \dfrac{15}{2}x - 21$

D. $-2y - \dfrac{5}{4}x + 28 = 42$

Sample Question #2

What is the equation of a line that passes through the point (–3,1) and has a slope of 2?

A. $y = 3x - 2$

B. $y = 2x + 7$

C. $y = 2x + 4$

D. $y = 3x + 1.5$

$$y = mx + b$$
$$1 = 2(-3) + b$$
$$1 = -6 + b$$
$$b = 7$$

After finding b, we now know the two things that we need to construct the equation of the line: its slope (m) and its y-intercept (b). We can substitute these into standard form to find the equation.

$$y = mx + b$$
$$y = 2x + 7$$

The correct answer is B!

You could also solve this problem by using the point-slope formula. The approach would be similar: we would substitute in the information we have about the slope and the point's coordinates into the formula. It doesn't matter whether we substitute our point's coordinates into the equation as x and y or x_1 and y_1—we just have to be consistent and not mix and match these pairs of coordiantes, e.g. don't substitute in the point's values for x and y_1 or x_1 and y. Since there is no b in this form, we would just simplify the equation and rearrange it into slope-intercept form to match the answer choices.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 2(x - (-3))$$
$$y - 1 = 2(x + 3)$$
$$y - 1 = 2x + 6$$
$$y = 2x + 7$$

Note the major difference between these approaches is that the slope-intercept form (as the name suggests) involves solving for b and then plugging it into the equation along with the provided slope, whereas the point-slope formula doesn't involve b and yields an equation. Once you put that equation into standard form, b becomes apparent, but you don't directly solve for it. Use whichever method works best for you—just make sure to distinguish between them!

3. Equation from two points on the line

This is the most difficult equation to determine. Many students get stuck because they don't know the slope of the line, and don't realize that they need to calculate that before applying one of the equations.

In this case, you know two points. The slope-intercept form requires you to know the slope and the y-intercept, and you have neither; the point-slope form requires the slope and one point, and you only have half of the information you need. In both cases, you need the slope, so you need to use the formula for the slope of a line between two points before you can answer the question.

Let's use this information to solve Sample Question #3. First, use the slope formula and substitute $(-3, 6)$ in for (x_1, y_1) and $(7, 11)$ in for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11 - 6}{7 - (-3)}$$

$$m = \frac{5}{10} = 0.5$$

Now that you have the slope, you can use either the slope-intercept form or the point-slope form to find the equation of the line that passes through them. We'll demonstrate both here, starting with the point-slope form. To complete the equation in this form, we'll also need to solve for its y-intercept. Substitute in the coordinates of either of the given points on the line as well as the slope and solve for b:

$$y = mx + b \qquad\qquad y = mx + b$$
$$6 = 0.5(-3) + b \qquad 11 = 0.5(7) + b$$
$$6 = -1.5 + b \qquad\quad 11 = 3.5 + b$$
$$b = 7.5 \qquad\qquad\quad b = 7.5$$

Knowing the slope m (0.5) and the y-intercept b (7.5), you can now write the equation of the line.

$$y = mx + b$$
$$y = 0.5x + 7.5$$

Sample Question #3

Which of the following equations passes through the points $(-2, 4)$ and $(7, 11)$?

    A. $y = 0.5x + 7.5$

    B. $y = 0.75x + 4.5$

    C. $y = \frac{1}{3}x + 5$

    D. $y = 1.25x + 3.5$

Instead of using the slope-intercept form and solving for the y-intercept, you can substitute either point on the line and the slope into the point-slope equation, simplify it, and rearrange it into slope-intercept form to find the answer.

$$y - y_1 = m(x - x_1)$$
$$y - (6) = 0.5(x - (-3))$$
$$y - 6 = 0.5x + 1.5$$
$$y = 0.5x + 7.5$$

$$y - y_1 = m(x - x_1)$$
$$y - (11) = 0.5(x - 7)$$
$$y - 11 = 0.5x - 3.5$$
$$y = 0.5x + 7.5$$

Either way, you end up with the same answer: A.

We've seen how to solve for the y-intercept in the examples above using the slope-intercept form, but what if you're asked to solve for the x-intercept? This is the x-value of the equation when $y = 0$. Graphically, it corresponds to the point at which the equation's line crosses the x-axis. In order to solve for the x-intercept of a line in Sample Question #4, you need to know its equation in slope-intercept form or its equation in point slope form as well as one point on the line.

Sample Question #4

Quantity A	Quantity B
The x-intercept of the line $y = 8x + \dfrac{4}{14}$	$-\dfrac{1}{25}$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

$$y = 8x + \frac{4}{14}$$
$$0 = 8x + \frac{4}{14}$$
$$-\frac{3}{11} = 8x$$
$$\frac{1}{8} \times -\frac{4}{14} = \frac{1}{8} \times 8x$$
$$x = -\frac{4}{112} = -\frac{1}{28}$$

When $y = 0$, $x = -\dfrac{1}{28}$. $-\dfrac{1}{28} > -\dfrac{1}{25}$ —if you didn't get that answer, consider that $\dfrac{1}{28} < \dfrac{1}{25}$, and that the number with the smaller absolute value will be the larger number when comparing two negatives. An analogous situation would be comparing –2 and –3; 3 may be greater than 2, but –2 is greater than –3 because it is closer to zero and thus more positive. A is correct!

Absolute Value Equations

Absolute value is denoted using the notation $|x|$ and signifies the distance between x and 0 on a number line. Even if you're dealing with the distance from a negtive number to zero, distance to zero is only ever considered a positive value, so absolute value bars make positive any value inside of them.

When simplifying and solving absolute value equations, you'll need to get rid of the absolute value bars. (Make sure you simplify to eliminate any elements outside of the absolute value bars first.) To omit the bars from the equation, rewrite your equation without absolute value bars, and write a second equation that lacks absolute value bars and in which you also make the right side of the equation negative instead of positive. These two equations are associated by the conjunction "or" since one value can't be true for both of them. (Note that this approach assumes that the right side of the equation is positive, and given that absolute value must be positive, this just excludes zero as a possibility.)

Written algebraically, this approach is

$$|x| = y$$

$$x = y \quad \text{or} \quad x = -y$$

Let's solve the following sample absolute value equation as an example:

$$2|4x - 15| = 98$$

Absolute value equations are simplified and solved by applying steps to each side of the equation, just like regular equations are. Let's begin by isolating the absolute value bars on the left side of the equation. We can do this by dividing both sides by two:

$$|4x - 15| = 49$$

Now we need to rewrite our equation. Omit the bars, and make one of the two equations include –49 instead of 49.

$$4x - 15 = 49 \quad \text{or} \quad 4x - 15 = -49$$

Now we just solve each equation to find our compound answer.

$$4x - 15 = 49 \quad \text{or} \quad 4x - 15 = -49$$

$$
\begin{aligned}
4x - 15 &= 49 \quad \text{or} \quad 4x - 15 = -49 \\
4x &= 64 \qquad\qquad\quad 4x = -34 \\
x &= 16 \qquad\qquad\quad x = -\frac{34}{4} = -8.5
\end{aligned}
$$

x can equal either 16 or –8.5 and it will work in our original absolute value equation.

Linear Inequalities

Solving inequalities is very similar to solving equations, but there are a few key differences. As the name suggests, the quantities on either side of an inequality may not be equal to each other. Rather, they are compared using relative terms such as greater than or less than. Inequalities can be related using the terms "greater than or equal to" and "less than or equal to," each of which allows for the possibility that the two quantities are equal but does not limit the quantities' relationship to this one possibility. The sample inequalities below demonstrate the four symbols used when representing inequalities mathematically.

Greater than: $7 > 4$

Less than: $3 < 10$

Greater than or equal to: $5 ″ 9$

Less than or equal to: $8 ≥ 8$

Inequalities are "read" from left to right. The symbol used in the first inequality means "greater than" and that used in the second means "less than." Thus, the first two statements are true: seven is indeed greater than four and three is less than ten. The symbol used in the third inequality translates as "less than or equal to" and that used in the last equation means "greater than or equal to." The last two statements are thus true as well: five is less than or equal to nine (it's less than nine) and eight is less than or equal to eight (it's equal to eight).

The first two inequalities listed are called "non-inclusive" or "exclusive" inequalities because they "exclude" the potential of an equal value. For instance, $7 > 7$ would not be a correct statement, and neither would $3 < 3$. The latter two symbols are called "inclusive" because they do allow for this "equals" possibility, e.g. $5 ″ 5$ would be correct and $8 ≥ 8$ is.

If you have a hard time remembering what the symbols mean, there are a few tricks you can use to recall the difference between them. One way to remember the definitions of these inequalities is to think of the symbols as the open mouth of a hungry alligator. The alligator always opens his mouth toward and "eats" the larger number. Furthermore, inclusive inequalities use a lower bar ($≤$ and $≥$) and end up with two lines in the symbol, just like an equals sign.

Solving Inequalities

Like equations, whatever operation is done to one side of the inequality must be done to the other side. Consider the following sample inequality:

$x + 2 < 10$

To solve this inequality, we must subtract 2 from both sides of the inequality.

$$x + 2 - 2 < 10 - 2$$

Simplify.

$$x < 8$$

The same is true for multiplication and division.

$$2x \geq 20$$

To solve this inequality, we must divide both sides by 2.

$$\frac{2x}{2} \geq \frac{20}{2}$$

Simplify.

$$x \geq 10$$

The one important rule when solving inequalities that is different from solving equations is that **if we multiply or divide by a negative number, then the direction of the inequality switches**. Consider the following inequality:

$$-\frac{1}{2}x > 14$$

To solve this inequality for x, we must multiply both sides by (-2):

$$(-2)-\frac{1}{2}x > 14(-2)$$

Remember that when we multiply or divide by a negative number, our inequality changes direction! Thus,

$$x < -28$$

Note that this is true (although easy to miss) for multiplying both sides by (-1). Consider the following inequality:

$$-x \geq 7$$

This means that:

$$x \;'' -7$$

To illustrate this property, let's pick a number that satisfies the previous inequality, say, -10. -10 is indeed less than or equal to -7 (it is less). Now, if we plug in -10 to the original inequality, we are left with the inequality $-(-10) \geq 7$ or just $10 \geq 7$, which are both true.

Compound Inequalities

Because inequalities describe the value of a variable relative to another number, it's possible for a single variable to be stated to be larger than one number but smaller than another one. Inequalities that take this form look like this:

$$-2 \leq x \leq 5$$

This inequality tells us that the value of x is in between –2 to 5 inclusive. Remember, "inclusive" in this context means that x could potentially equal –2 or 5.

When solving these types of inequalities, you can use the same tactics that you use to simplify equations; however, you need to perform these operations on all three parts of the inequality statement. Let's simplify the inequality shown below as an example.

$$-72 \leq -9x \leq 36$$

We'll need to divide each part of the inequality by –4 to end up with the variable by itself in the middle. Keep in mind, when simplifying inequalities, not all inequalities will end up with just the variable in the middle; some may involve coefficients in their simplest forms. Also, since we are dividing by a negative number, remember to flip the inequality signs! If you don't, your answer won't make sense and will be incorrect.

$$\frac{-72}{-9} \leq \frac{-9x}{-9} \leq \frac{36}{-9}$$

$$8 \geq x \geq -4$$

We can test out our answer by substituting in the bounds of x in the simplified inequality into the original form of the inequality to test that they still hold. If $x = 8$, the original inequality holds because $8 \times -9 = -72$. The original inequality holds if $x = -4$ too, because $-4 \times -9 = 36$.

Absolute Value Inequalities

When you encounter an inequality that features absolute value, you'll need to consider two possibilities as different mathematical statements in order to correctly simplify and solve it. Consider the following sample inequality:

$$|3x + 1| \leq 10$$

Say we approach it the usual way and simplify it, subtracting one from each side and then dividing by three. This yields

$$|x| \leq 3$$

But wait! What if we put in a negative number like –3.5 into the original equation?

$$|3x+1| \le 10$$

$$|3(-3.5)+1| \le 10$$

$$|-10.5+1| \le 10$$

$$|-9.5| \le 10$$

$$9.5 \le 10$$

Hmm, –3.5 produces a valid result, yet doesn't fall in the range predicted by $|x| \le 3$. This demonstrates that we can't solve absolute value inequalities the same way that we can solve regular inequalities—we have to use a special approach.

When working with "less than" (<) or "less than or equal to" (≤) absolute value inequalities where the right side of the inequality is positive, you can write them as two statements: one that remains unchanged, and another that flips the sign and makes the right side of the inequality negative. These statements are associated with the conjunction "or" since the ranges are exclusive—one result can't fall into both of them. The absolute value signs are removed when you break the inequality into two separate statements.

$$|3x+1| \le 10$$

$$3x+1 \le 10 \quad \text{or} \quad 3x+1 \ge -10$$

You then solve both statements to come up with the answer. In this case, after simplifying each statement, we get

$$x \le 3 \text{ or } x \ge -\frac{11}{3}$$

This gives us the lower bound $\left(-\frac{11}{3}\right)$ and the upper bound (3) of the possible values of x that work in the original inequality. If you test these values in the original inequality, they work, and they explain why –3.5 works; it's greater than $-\frac{11}{3}$.

When working with "greater than" (>) or "greater than or equal to" (≥) inequalities where the number on the right side of the equation is positive, you create a compound inequality using the negative version of the number on the right-hand side, as shown below.

$$|0.75x-1| < 1$$

$$-1 < 0.75x-1 < 1$$

To simplify this compound inequality, remember to perform each step on all three parts of the inequality. (You can rewrite 0.75 as $\frac{3}{4}$ to make working with fractional values easier.)

$$-1 < \frac{3}{4}x - 1 < 1$$

$$0 < \frac{3}{4}x < 2$$

$$0 < x < 2 \times \frac{4}{3}$$

$$0 < x < \frac{8}{3}$$

In summary, if $y > 0$:

$$|x| > y \rightarrow x > y \text{ or } x < -y$$

$$|x| < y \rightarrow -y < x < y$$

These statements also apply to the corresponding inclusive inequalities ($\geq$ and $\leq$).

Let's test out our knowledge of inequalities by answering Sample Question #5. We can rewrite Quantity A as

$$-29 \leq 3x + 4 \leq 29$$

To solve this, perform operations on each part of the inequality. First, subtract 4:

$$-33 \leq 3x \leq 25$$

Then, divide by 3:

$$-11 \leq x \leq \frac{25}{3}$$

Now let's rewrite and simplify Quantity B:

$$-18 \leq 2y + 6 \leq 18$$

Subtract 6:

$$-24 \leq 2y \leq 12$$

Then, divide by 2:

$$-12 \leq y \leq 6$$

The smallest value for x is -11, while the smallest value for y is -12, so Quantity A is greater.

Sample Question #5

$$|3x + 4| \leq 29$$

$$|2y + 6| \geq 18$$

Quantity A | Quantity B

The smallest possible value for x | The smallest possible value for y

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

Systems of Linear Equations

Systems of linear equations are like simple linear equations in the sense that we need to use algebra to find the value of one (or more) unknown(s). The main difference lies in the number of unknowns and equations. Simple linear equations such as $4x + 3 = 11$ include a single variable and a single equation that can be solved relatively easily using algebra. In this case, $x = 2$. When dealing with systems of equations, there are multiple variables that are related via multiple equations. Note that the number of different variables is equal to the number of different equations. Let's take a look at linear equations that contain two variables.

In Sample Question #1, we are asked to find the value of two variables (x and y) given two equations that each include x and y. We have two options when solving systems of linear equations like this one: substitution and linear combination (elimination). To solve a system of linear equations using substitution, we need to first manipulate one of the equations to express one of the variables in terms of the other. At first glance, we can quickly express y in terms of x using the first equation by subtracting $4x$ from each side of the equation. This yields $y = 16 - 4x$. The next step is to substitute this value for y into the second equation wherever we see the variable y and solve for x using algebra.

> ### Sample Question #1
>
> Find x and y.
>
> $$4x + y = 16$$
> $$2x + 3y = 18$$
>
> **A.** $x = 1, y = 4$
>
> **B.** $x = 4, y = 1$
>
> **C.** $x = 3, y = 4$
>
> **D.** $x = 4, y = 3$

$2x + 3(16 - 4x) = 18$

$2x + 48 - 12x = 18$

$-10x = -30$

$x = 3$

Now, we can substitute this numerical value for x into either of the two equations to solve for y. Let's plug it into the first equation.

$4(3) + y = 16$

$12 + y = 16$

$y = 4$

Thus, the solution to this system of equations is $x = 3$ and $y = 4$ (answer choice C), which can be expressed as the point $(3, 4)$. If you have extra time on your exam, feel free to plug these values into both equations to make sure they produce true statements; however, it is important to note that some incorrect answers might be true for one equation but not the other.

Now that we have practiced solving a system of linear equations by substitution, let's use linear combination (elimination) to solve Sample Question #2.

When solving systems of equations using the elimination method, the goal is to be able to eliminate one of the variables by making the coefficients equal in magnitude and either adding or subtracting the equations from one another. This usually requires multiplying an entire equation by a coefficient. For example, if we multiply the bottom equation by two, we will have 4x in both equations, which can be eliminated if we subtract one equation from the other.

$$4x - 6y = 12$$
$$2(2x + 2y = 6)$$

After distributing the 2, we are left with

$$4x - 6y = 12$$
$$4x + 4y = 12$$

To eliminate the x's, we will subtract the bottom equation from the top equation.

$$4x - 6y = 12$$
$$- \ 4x + 4y = 12$$
$$\overline{ - 10y = 0}$$
$$y = 0$$

At this point, elimination and substitution are the same. Plug in the newly discovered value ($y=0$) into either of the equations to solve for the other variable (x). Let's substitute $y=0$ into the second equation.

$$2x + 2(0) = 6$$
$$2x = 6$$
$$x = 3$$

Thus, the final answer to this system of linear equations is answer choice A (3,0). Note that this is the point on the coordinate plane at which these linear equations' lines intersect. If you take a minute to put these equations into slope-intercept form and graph them, you will find that the point (3,0) is a solution for both lines.

Sample Question #2

Find x and y.
$$4x - 6y = 12$$
$$2x + 2y = 6$$

A. $x = 3$, $y = 0$

B. $x = -3$, $y = 0$

C. $x = -2$, $y = 4$

D. $x = 2$, $y = -4$

As is the case for many Quantitative Reasoning questions, we're presented with some information in Sample Question #3 that we will need to convert into one or more solvable equations. Defining variables is almost always a great place to start. Let n equal Nick's current age, and let s equal Sarah's current age.

Looking at the first part of the given statement, we know that Sarah is three times as old as Nick. Mathematically, this is equivalent to

$s = 3n$

The second part of the statement tells us that in two years, Sarah will be twice as old as Nick. We can express this mathematically as follows:

$s + 2 = 2(n + 2)$

Now we can solve the system of equations using either substitution or elimination. Since the first equation already states s in terms of n, substitution will probably be quicker and easier. Substitute $(3n)$ for s in the second equation and solve for n.

$3n + 2 = 2(n + 2)$

$3n + 2 = 2n + 4$

$n = 2$

Now we can plug this value into either of the equations to find s.

$s = 3(2)$

$s = 6$

Thus, Sarah is 6 years old and Nick is 2 years old, which corresponds to answer choice B.

The GRE Quantitative section is not limited to systems of equations involving two equations and two variables. Let's take a look at an example in which we're asked to take our knowledge of solving systems of linear equations one step further.

Sample Question #4 might seem daunting at first, but you can solve it by relying on what you know about solving systems of linear equations. We can proceed using either substitution or linear combination (elimination). Let's solve via substitution.

Sample Question #3

Nick's sister Sarah is three times as old as him, and in two years will be twice as old as he is then. How old are they now?

A. Nick: 1, Sarah: 3

B. Nick: 2, Sarah: 6

C. Nick: 2, Sarah: 4

D. Nick 1, Sarah: 2

Sample Question #4

Solve the system of equations.

$7x + 4y + 9z = 0$

$8x - 3y - 6z = 12$

$2x - y - 3z = 5$

A. $x = -1, y = 2, z = -\dfrac{5}{3}$

B. $x = 1, y = 2, z = \dfrac{5}{3}$

C. $x = \dfrac{1}{2}, y = \dfrac{3}{2}, z = -3$

D. $x = 5, y = -9, z = 3$

Looking at all three variables in all three equations, it seems that solving for *y* in the third equation is the quickest and easiest first step.

$$2x - y - 3z = 5$$
$$2x - 3z - 5 = y$$

Now, we need to substitute $(2x - 3z - 5)$ into the other two equations for *y*.

$$7x + 4y = 9z = 0 \;\rightarrow\; 7x + 4(2x - 3z - 5) + 9z = 0$$
$$8x - 3y - 6z = 12 \;\rightarrow\; 8x - 3(2x - 3z - 5) - 6z = 12$$

Simplify the first equation.

$$7x + 8x - 12z - 20 + 9z = 0$$
$$15x - 3z - 20 = 0$$

Simplify the second equation.

$$8x - 6x + 9z + 15 - 6z = 12$$
$$2x + 3z + 15 = 12$$

Now we have two equations with two unknowns. We can find the value of *x* and *z* via substitution or elimination. In this case, elimination looks a bit easier since, if we add the two equations to each other, the *z* terms cancel out, which allows us to solve for *x*.

$$15x - 3z - 20 = 0$$
$$+ \; 2x + 3z + 15 = 12$$
$$\overline{17x - 5 = 12}$$
$$17x = 17$$
$$x = 1$$

Next, we can plug in 1 for all *x*'s in the original equations:

$$7x + 4y + 9z = 0 \;\rightarrow\; 7(1) + 4y + 9z = 0 \;\rightarrow\; 4y + 9z + 7 = 0$$
$$8x - 3y - 6z = 12 \;\rightarrow\; 8(1) - 3y - 6z = 12 \;\rightarrow\; -3y - 6z + 8 = 12$$
$$2x - y - 3z = 5 \;\rightarrow\; 2(1) - y - 3z = 5 \;\rightarrow\; -y - 3z + 2 = 5$$

From here, we can use any two of the equations to solve for either *y* or *z* by substitution or elimination. Let's again choose substitution by solving for *y* in the third equation.

$$-y - 3z + 2 = 5$$
$$y = -3z - 3$$

We can plug in this value for *y* into either of the other two equations. Let's choose the first one.

$$4(-3z - 3) + 9z + 7 = 0$$
$$-12z - 12 + 9z + 7 = 0$$
$$-3z - 5 = 0$$
$$-3z = 5$$
$$z = -\frac{5}{3}$$

Now, we use our calculated values for *x* and *z* to find *y*. Again, it does not matter which equation we use, but remember that time is of the essence! We already rearranged an equation to isolate *y*. Let's reuse that equation to make finding y as quick as possible.

$$y = -3z - 3$$
$$y = 3\left(-\frac{5}{3}\right) - 3$$
$$y = 5 - 3$$
$$y = 2$$

Our final answers are: $x = 1$, $y = 2$, and $z = \frac{5}{3}$; therefore, answer choice B is the correct answer. In the event that you have time to check your answer, plug in all three values into all three equations to make sure they are all true statements. For practice, you can try solving this Sample Question #4 via elimination first; you will get the same values for *x*, *y*, and *z*.

Properties of Linear Graphs

Learning to interpret equations graphically and graphs in terms of the equations that they depict is an extremely important skill necessary for success on the GRE Quantitative section.

Linear equations and inequalities are always straight lines when graphed. Slope is defined as change in y over change in x. Since the same change in y always causes the same change in x at all points on the line in a linear equation, the line's slope is consistent at all points. Let's imagine that for a given linear equation, for every 1.0 units y increases, x decreases by 2.0 units. This would produce a consistent slope that we can express using the following equation:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{1.0}{-2.0} = -\frac{1}{2}$$

Even if a slope isn't presented as a fraction, you can read it in terms of rise over run to figure out how it would appear graphically. Say a linear equation has slope of –3; put differently, this would be a slope of $\frac{-3}{1}$. This means that for every 3.0 units y is decreased, x increases by 1.0 units.

The most common form in which equations of lines are written is slope-intercept form:

$$y = mx + b$$

In this equation, y is the y-coordinate of the point of interest on the line, m is the slope of the line, x is the x-coordinate of the point of interest on the line, and b is the y-intercept (i.e. the y-coordinate of the point on the line that intersects the y-axis).

Another form in which equations of lines are commonly written is point-slope form:

$$y - y_1 = m(x - x_1)$$

In this equation, (x_1, y_1) is a point on the line, (x, y) is any point in the coordinate plane, and m is the slope of the line. Point-slope form is useful when, as the name suggests, we know a point on a line and the slope of that line. If the point (x, y) is on the line, we know that the slope between the two points (x, y) and (x_1, y_1) is m. Point-slope form is useful if we are given the slope of a particular line and a point through which it passes, as we can use the
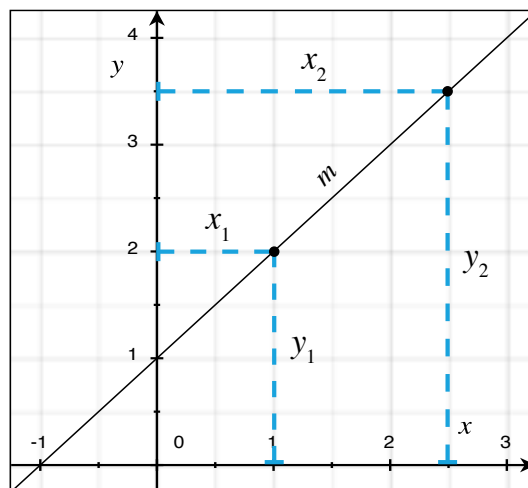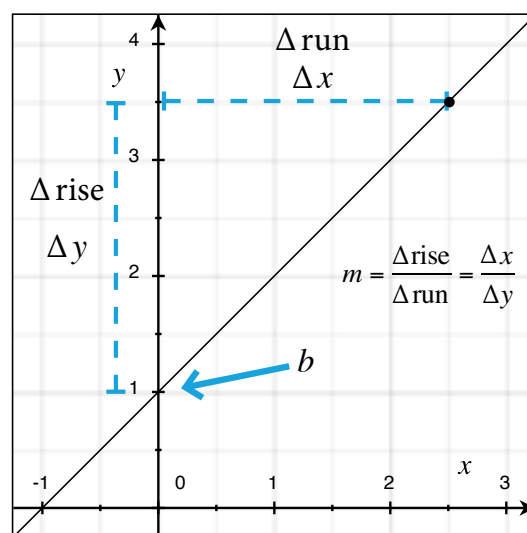
y-intercept as a point with the x-coordinate of 0.

To find the equation of the line in Sample Question #1, we'll need to figure out what the line's y-intercept is. We can find the y-intercept by substituting in the given x-value, y-value, and slope into slope-intercept form and solving for *b*:

$$y = mx + b$$
$$1 = 2(-3) + b$$
$$1 = -6 + b$$
$$b = 7$$

Once we know *b*, all we have to do is substitute in the values of *m* and *b* into the slope-intercept form; the resulting equation will be the equation of the line.

$$y = 2x + 7$$

Alternatively, we can simply plug this point into point-slope form to find the equation of the line:

$$y - 1 = 2(x - (-3))$$
$$y - 1 = 2x + 6$$
$$y = 2x + 7$$

No matter which form of a linear equation you use, you get the same answer: $y = 2x + 7$. That means that the correct answer is B.

The GRE Quantitative section might also give you the equation of a line and ask you to solve for its x- or y-intercept. Sample Question #2 asks for both!

The x-intercept is the x-coordinate of the point on the line that intersects the x-axis. By definition, this point will have the form $(x, 0)$ since it is on the x-axis. The y-intercept is the y-coordinate of the point on a line that intersects the y-axis. By definition, this point will have the form $(0, y)$ since it is on the y-axis. Since this equation is already in slope-intercept form, we know that the y-intercept, *b*, is –9. Thus, the line intersects the y-axis at the point $(0, -9)$.

Sample Question #1

What is the equation of a line that passes through the point $(-3, 1)$ and has a slope of 2?

- **A.** $y = 3x - 2$
- **B.** $y = 2x + 7$
- **C.** $y = 2x + 4$
- **D.** $y = 3x + 1.5$

Sample Question #2

Find the x- and y-intercepts of the line that has the equation $y = 3x - 9$.

- **A.** x-intercept = $(3, 0)$
  y-intercept = $(0, -9)$
- **B.** x-intercept = $(5, 0)$
  y-intercept = $(0, -3)$
- **C.** x-intercept = $(9, 0)$
  y-intercept = $(0, -3)$
- **D.** x-intercept = $(5, 0)$
  y-intercept = $(0, -9)$

If you didn't notice this shortcut, don't worry: you can also solve for the y-intercept algebraically. Just plug zero in for the x-value in the equation of the line and solve for *y*.

$$y = 3x - 9$$
$$y = 3(0) - 9$$
$$y = -9$$

To find the x-intercept, we must use our knowledge that the x-intercept has a y-coordinate of zero. Thus, we plug in zero for *y* into our equation and solve for *x*.

$$0 = 3x - 9$$
$$-3x = -9$$
$$x = 3$$

Thus, the *x*-intercept is $(3, 0)$ and the *y*-intercept is $(0, -9)$, so the correct answer is A.

Vertical Lines

Suppose there is a vertical line that has an x-intercept of 4. A vertical line has slope that is undefined by definition. Slope is written algebraically as $\frac{\Delta \text{rise}}{\Delta \text{run}}$. (The triangular symbol, delta, means "change in"). A vertical line does not have any change in *x*; thus, the denominator of our expression for the slope of a vertical line is zero. Now, it is a rule that we cannot divide any number (change in rise) by zero. Thus, the slope of a vertical line is said to be undefined. The equation for the vertical line shown at right is $x = 4$. Notice how it crosses the x-axis at 4.
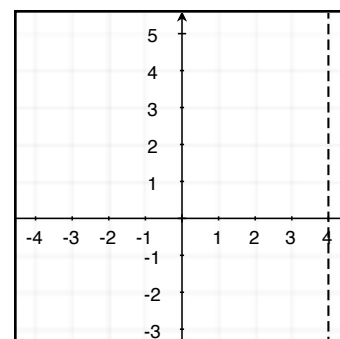
Horizontal Lines

In the graph of a horizontal line, all of the points on that line will have the same y-value, and an infinite number of x-values. The slope of a horizontal line graph is thus $\frac{0}{\text{infinity}}$. By definition, any fraction that has zero in the numerator is equal to zero, so the slope of a horizontal line is zero. Another way to remember this is by thinking of a graph with a very small positive slope, say $\frac{1}{100}$. The more this slope flattens out the closer it gets to zero (i.e. $\frac{1}{1000}, \frac{1}{10000}$, etc.). Note that this is the opposite case for vertical lines, where the highest positive slope approaches verticality. The equation of the horizontal line shown at right is $y = 3$.
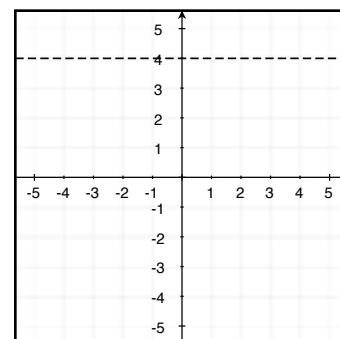
Parallel Lines

Parallel lines in the same plane never intersect. The only way for this to be true is for the slopes of the lines to be equal. Depicted in the graph to the right are some examples of parallel lines in the same plane: The equations of the lines are as follows:

$y = x - 2$

$y = x + 2$

Any line that also has a slope of 1 would be parallel to these two lines. All of the following lines would be parallel to the depicted lines:

$y = x - 37$

$2y = 2x + 8$

$0 = 3.5x - 3.5y - 40$
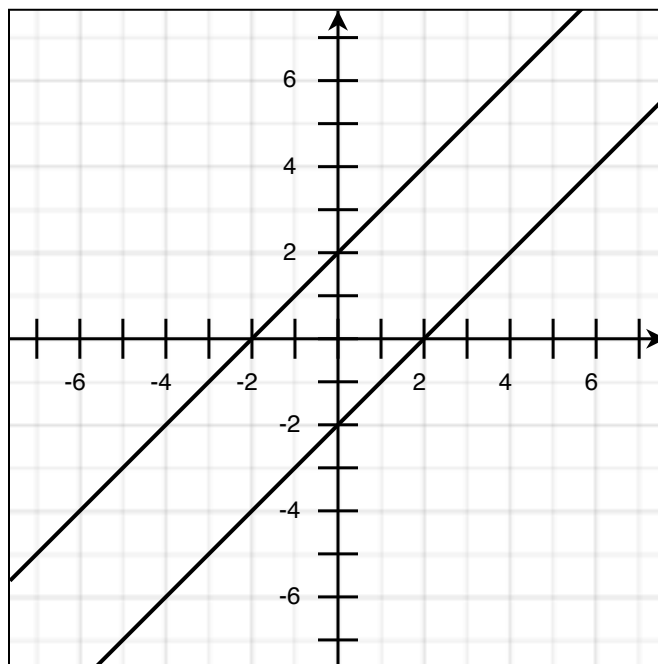
Perpendicular Lines

Perpendicular lines intersect at 90° angles and create a pair of right angles. Perpendicular lines have slopes that are opposite reciprocals of one another. For example, a line with a slope of -3 will be perpendicular to any line with a slope of $\frac{1}{3}$.

The lines pictured at right have the following equations:

$y = -4x + 7.5$

$y = \frac{1}{4}x + 1$
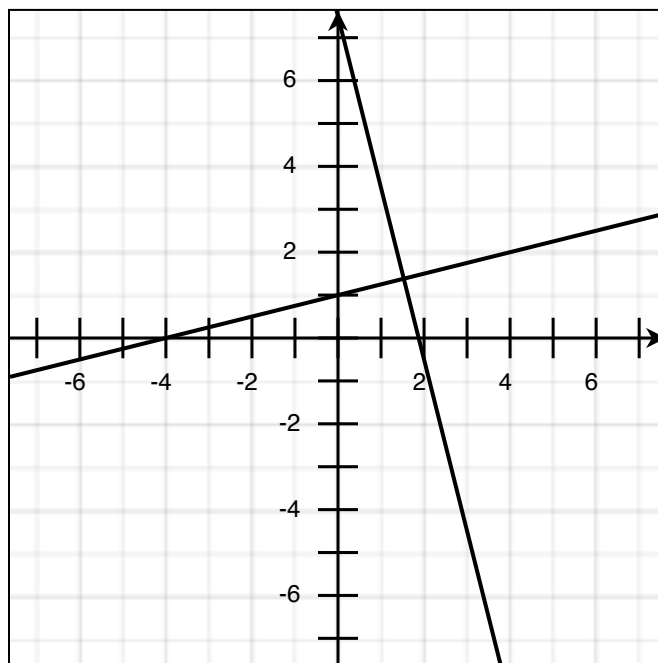
Sample Question #3 takest the equation-assembly paradigm a bit further, asking you to find a line perpendicular to a different line that is not directly provided.

First, we need to recognize what this question is asking us to calculate. We are looking for the equation of the line that passes through the point $(1, 2)$ and is perpendicular to the line that has a slope of –6 and passes through $(5, 3)$. We know that our line of interest is perpendicular to the one with a slope of –6, so our slope is the opposite reciprocal of –6, which is $\frac{1}{6}$. The perpendicular line passes through $(1, 2)$, so we set up an equation in point-slope form:

$$y - 2 = \frac{1}{6}(x - 1)$$

Simplify this into slope-intercept form so that you can tell which of the answer choices it matches:

$$y - 2 = \frac{1}{6}x - \frac{1}{6}$$
$$y = \frac{1}{6}x - \frac{1}{6} + 2$$
$$y = \frac{1}{6}x - \frac{1}{6} + \frac{12}{6}$$
$$y = \frac{1}{6}x + \frac{11}{6}$$

Sample Question #3

What is the equation of the line passing through the point $(1,2)$ that is perpendicular to a line that passes through the point $(5,3)$ and has a slope of –6?

A. $y = \frac{5}{6}x + \frac{1}{6}$

B. $y = \frac{5}{6}x + \frac{4}{5}$

C. $y = \frac{1}{6}x + \frac{7}{6}$

D. $y = \frac{1}{6}x + \frac{11}{6}$

The correct answer is D. Note that to answer this question we did not have to use the extraneous information about the point $(5, 3)$. All we needed to know about the line passing through this point was its slope.

Use the graph shown to the right to answer the bolded questions.

1. What is the slope of the line?

Pick any two points on the line and use the slope formula:

$$\frac{\Delta \text{rise}}{\Delta \text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's pick the points $(0, 4)$ and $(1, 1)$.

$$m = \frac{1 - 4}{1 - 0} = -3$$

2. What is the equation of this line?

Since we already calculated the slope, and we can easily find a point through which the line passes, we can use point-slope form to find the equation for this line. Let's use the point $(0, 4)$.

$$y - 4 = -3(x - 0)$$
$$y - 4 = -3x$$
$$y = -3x + 4$$

Double check to make sure the answer looks reasonable. Does the line have a negative slope? Yes—the line points down and to the right. Where does the line cross the y-axis? From the graph, we see that it intersects the y-axis at the point $(0, 4)$. This means that in our equation for this line, the y-intercept in slope-intercept form (b) is also 4.

3. What is the x-intercept of this line?

At first glance, this line does not look like it crosses the x-axis at a whole number. This would have been the quickest way to find the x-intercept; however, we can move on to our next option: substituting values into the equation of the line. The x-intercept is the point at which the line intersects the x-axis. We know that this point has a y-coordinate of zero; therefore, in our equation, we will plug in 0 for y.

$$0 = -3x + 4$$
$$3x = 4$$
$$x = \frac{4}{3}$$

Thus, the x-intercept is $\left(\frac{4}{3}, 0\right)$.
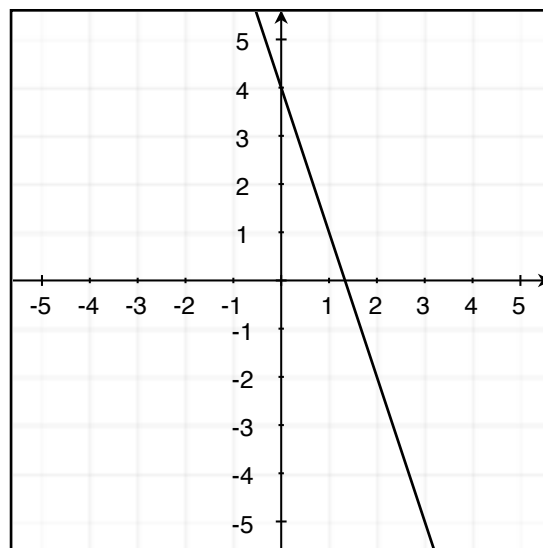
Transformations and Reflections of Linear Equations

Certain algebraic adjustments to a linear equation result in specific changes to that line's graph. When preparing for the GRE, it's important to learn to identify these shifts, whether they're presented as in equation form or on the coordinate plane.

A linar equation's graph can be shifted up or down through adjustments made to its y-intercept. The solid line shown at right is the equation $y=-2x$. This equation has no apparent b value even though the equation is otherwise in standard form ($y=mx+b$). This means that for this line, $b=0$. This is confirmed graphically: the line passes through the origin. Positive shifts in the b-value correspond to upward shifts, and negative shifts in the b-value correspond to downward ones. The dotted line shifted upwards is the line $y=-2x+4$. Because its b-value has been increased by 4, the line has been shifted upwards by 4 units. The same holds true for the dotted line that has been lowered four units: its equation is $y=-2x-4$.
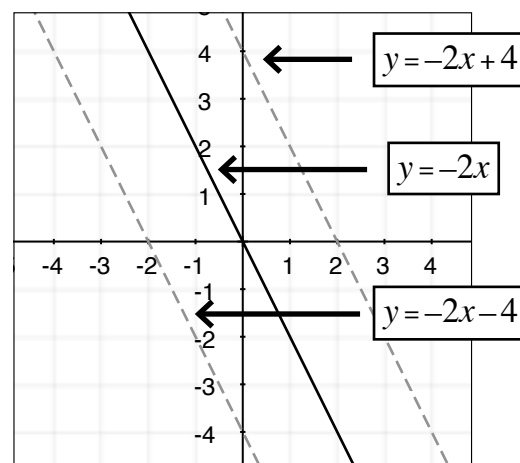
Lines can also be shifted horizontally through a different algebraic adjustment. This time, the adjustment has to be made to the x-variable within parentheses, before any coefficients are multiplied. Increasing the variable by adding a positive number in this way shifts the graphed line left that many units, and subtracting a positive number shifts it right that many units. The dotted line shifted right by four units is that of the equation $y=-2(x-4)$, and the dotted line shifted left by four units is that of the equation $y=-2(x+4)$. Note that by changing the equations in this way, they could be further simplified to reveal new y-intercept values—8 for the equation shifted right and –8 for the equation shifted left.
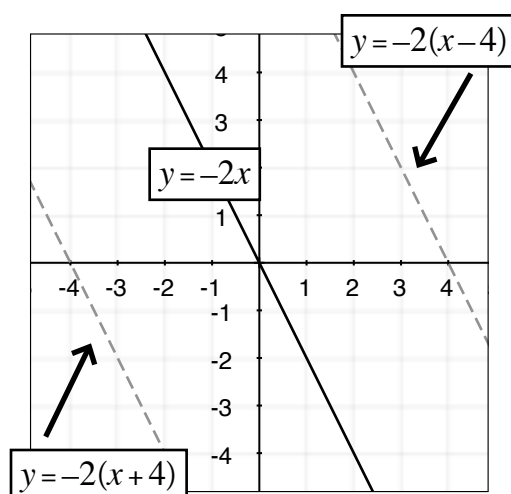
Graphs of linear equations can also be reflected over the x- or y-axis through the insertion of negative signs into their equations. A line is reflected around the x-axis when a negative sign that applies to the entire equation is added; reflections about the y-axis are generated by applying a negative symbol only to the x-variable using parentheses. In the graph shown to the right, the solid line is the graph of $y=0.25x-3$. The dashed-line equation with the same y-intercept is the graph of $y=0.25(-x)-3$. It has been reflected around the y-axis. The dashed line with positive values has been reflected over the x-axis; it is the graph of $y=-(0.25x-3)$.
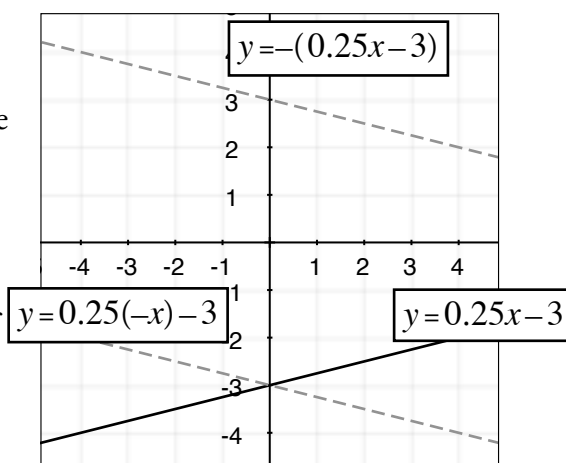
Graphing Absolute Value Functions

Absolute value functions appear as "V"-shapes and are subject to the same horizontal and vertical shift rules as other linear equations. Because we're dealing with absolute value, $y=|x|$ and $y=|-x|$ end up having the same graph. This makes sense not only algebraically, but also graphically: because absolute value functions are symmetrical around the x-axis, so reflecting them about that axis doesn't result in any changes. Adding a negative coefficient to the absolute value results in the equation being flipped over the y-axis.

In the graph shown at right, the basic absolute value function $y=|x|$ is shown with a solid line. The function $y=|x-7|$ shifts this base function 7 units to the right, and the function $y=|x|+2$ shifts it up by two units. Adding a negative sign outside of the absolute value bars inverts the function without changing its vertex point. Note that adding a negative sign inside the absolute value bars does nothing because absoluve value graphs are symmetrical around the y-axis, so rotating it around the y-axis doesn't change anything; however, if we other numbers are present in the absoluve value bars, the negative sign would apply to them and potentially shift the graph in a different direction horzontally.

The coefficient of x influences the width of the V shape of an absolute value equation. At right, you can see that as the coefficient increases above 1, the "V" gets less wide, and as the coefficient decreases between 1 and 0, the "V" gets wider.
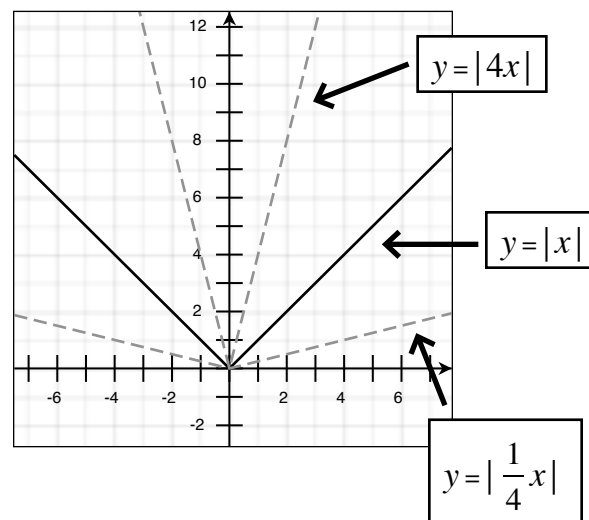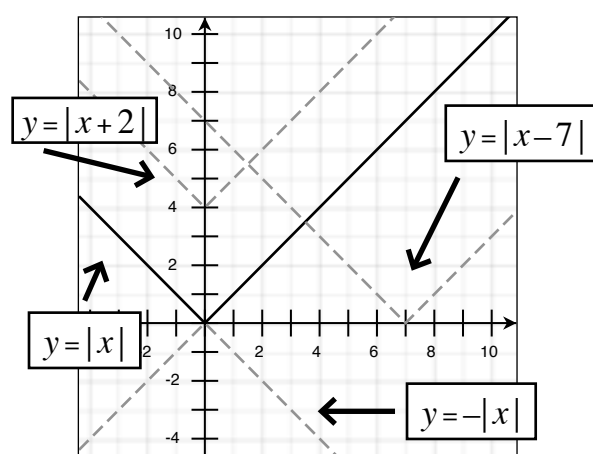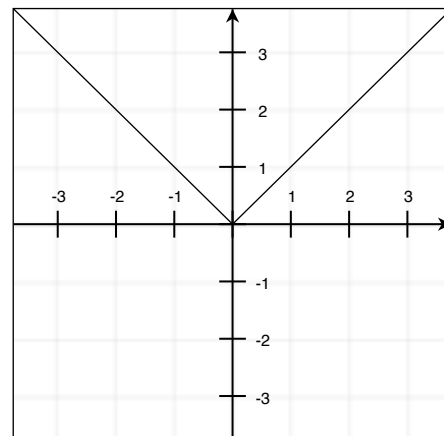
Consider the most basic absolute value function, $f(x) = |x|$. The graph of this function is a "V"-shaped graph with a vertex at $(0,0)$. What kind of shifts would we have to perform to turn this basic function into the function in Sample Question #4? Answering this question will shed light on the movement of the function's vertex.

Let's rewrite the question stem's function so it looks a little bit more like an equation in vertex form.

$$f(x) = 10 - |x + 10|$$

$$f(x) = -|x + 10| + 10$$

Relative to the unadjusted absolute value function, the one in the question stem is shifted ten units left (based on the +10 inside the absolute value bars) and ten units up (based on the 10 added outside the absolute value bars). It is also inverted, based on the negative sign that precedes the absolute value bars. Where does this put its vertex? Well, the vertex was originally at $(0,0)$, and it's shifted in just the same way as the rest of the function. Ten units left means that its x-coordinate is -10, and ten units up means that its y-coordinate is 10. The inversion of the function doesn't affect its vertex. Thus, the vertex of the given function $f(x)$ is at $(-10, 10)$ and B is the correct answer.

Graphing Linear Inequalities

Graphing linear inequalities is similar to graphing linear equations, but the process involves two major differences. The first difference is in regards to the line itself, depending on whether it is an inclusive or non-inclusive inequality. For inequalities that are inclusive (″ or ≥), a solid line is used. For inequalities that are non-inclusive (< or >), a dashed line is used.

The next aspect of graphing an inequality involves shading a region. If an inequality reads "y is less than" or "y is less than or equal to," the area below the line will be shaded. If an inequality reads "y is greater than" or "y is greater than or equal to", the area above the line will be shaded. If you're ever unsure about which side of an inequality to shade, you can always use a test point.

Consider the inequality $y > x - 4$. Let's say that we're unsure about which side of the inequality to shade. We can choose the value of $(0,0)$ and plug it into the equation:

$$0 > 0 - 4$$
$$0 > -4$$

With these values inserted, the inequality holds true, so we want to shade the side of the line on which the origin $(0,0)$ falls. Any point can function as your test point; the origin is just an example.

The following are graphs of linear inequalities.
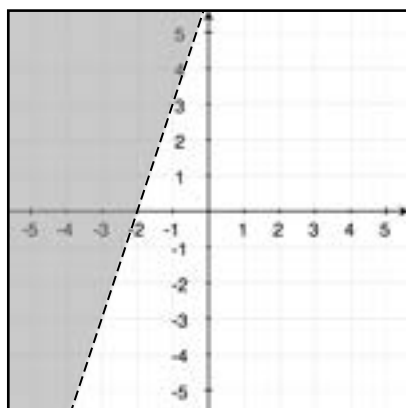
$$y > 3x + 6$$
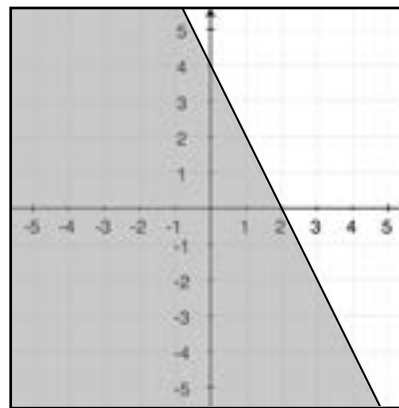
$$y \leq -2x + 4$$

Quadratic Equations and the Quadratic Formula

A quadratic equation is a special type of polynomial in which the highest power to which a number or variable is raised is two. When graphed, these equations create smooth parabolas. Quadratic equations can contain up to three terms: one squared term, one single-variable term, and one constant term. The general form of a quadratic equation is shown below.

$ax^2 + bx + c = 0$

Although quadratics can have three terms, it doesn't mean that they always do. The important characteristic to remember, and the determining factor that identifies a quadratic equation, is that the highest power any of its variables are raised to is two. Here are some examples of quadratic equations:

$y = x^2$

$y = 3x^2 - 1$

$y = x^2 + 2x + 4$

Solving Quadratic Equations

Most often, when you are asked to algebraically solve quadratic equations, you're specifically asked to identify the value(s) of x when $y = 0$. These particular values of x are the x-intercepts of the equation—the point(s) at which the parabola crosses the x-axis. Because parabolas are curved, it's possible for a quadratic equation to cross the x-axis in one location, two locations, or not at all. These x-intercept x-values are sometimes called the "roots" of the equation.

When taking the GRE, there are two efficient routes to solving quadratic equations: factoring the equation and using the quadratic formula. Depending on the specific quadratic equation you're faced with, factoring may or may not be the easiest method; however, all quadratic equations can be solved quite easily using the quadratic formula. Let's consider how to use each of these methods now.

The Quadratic Formula

The quadratic formula can be used to solve a quadratic equation that is in the form of $ax^2 + bx + c = 0$, where a is the coefficient on the quadratic term, b is the coefficient on the single variable term, and c is the constant term. The quadratic formula is written as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$, the part of the quadratic formula underneath the radicand (the square root symbol), is called the discriminant. If the discriminant is positive, the quadratic equation you're solving will have real solutions. If it is negative, its solutions will be imaginary numbers, becuase you'll end up needing to take the square root of a negative number, which produces imaginary-number results.

To solve an equation using the quadratic formula, plug in the coefficients of the equation into the formula. For example, let's use the quadratic formula to solve the following equation:

$$3x^2 - 4x + 1$$

Let's first identify the coefficients to be used in the quadratic formula: $a = 3$, $b = -4$, and $c = 1$. When you plug these values into the quadratic formula, you get the following equation:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(1)}}{2(3)}$$

Recall that when a negative sign is applied to a negative number, the number becomes positive. This is also true when a negative number is squared; squaring a negative number means that a negative number is multiplied by itself. It becomes a positive number; therefore, this particular solution becomes:

$$x = \frac{4 \pm \sqrt{16 - 12}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{4}}{6}$$

$$x = \frac{4 \pm 2}{6}$$

$$x = \frac{2 \pm 1}{3}$$

With most quadratic equations, there will be two solutions.

$$x = \frac{3}{3} \text{ and } \frac{1}{3}$$

$$x = 1 \text{ and } \frac{1}{3}$$

What does this mean in graphical terms? It means that this quadratic equation crosses the x-axis at the points (0, 1) and (0, 0.33).
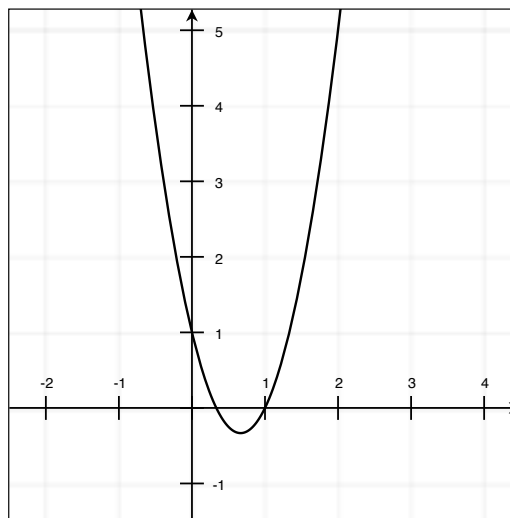
Sample Question #1 asks us to tackle another quadratic equation, and we can use the quadratic formula to do so. First, we need to identify the equation's coefficients:

$$a = 6, b = -2, c = -7$$

We can then plug the coefficients into the quadratic formula. Our discriminant is positive, so we will get real-number answers. At this point, we don't need to solve for a single value—the answer choices are given as equations that result from the quadratic formula. Let's simplify our equation and see which answer choice it matches.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-7)}}{2(6)}$$

$$x = \frac{2 \pm \sqrt{4 - (-168)}}{12}$$

$$x = \frac{2 \pm \sqrt{172}}{12}$$

At this point, we're going to need to simplify our square root into its component factors in order to pick out the correct answer.

$$x = \frac{2 \pm \sqrt{2 \times 2 \times 43}}{12}$$

$$x = \frac{2 \pm 2\sqrt{43}}{12}$$

At this point, we can further simplify our fraction and arrive at our final answer, A:

$$x = \frac{1 \pm 1\sqrt{43}}{6}$$

Let's try solving another quadratic, this time focusing on one that might give you a bit of a hard time if you don't recognize it as such. The equation featured in Sample Question #2 doesn't have a single-variable term, but it's still a quadratic equation because its highest power is 2; furthermore, we can still use the quadratic formula to solve it, using 0 as the coefficient for the x-term and therefore the value of b.

Sample Question #1

Solve $6x^2 - 2x - 7$.

A. $x = \dfrac{1 \pm 1\sqrt{43}}{6}$

B. $x = \dfrac{2 \pm 1\sqrt{43}}{3}$

C. $x = \dfrac{2 \pm 3\sqrt{37}}{5}$

D. $x = \dfrac{1 \pm 3\sqrt{37}}{2}$

Sample Question #2

Solve $x^2 - 18$.

A. $x = \pm\sqrt{9}$

B. $x = \pm\sqrt{14}$

C. $x = \pm\sqrt{18}$

D. $x = \pm 6$

Our first step is to identify our coefficients.

$a = 1, b = 0, c = -18$

Now we can plug our coefficients into the quadratic formula.

$$\frac{-0 \pm \sqrt{(0)^2 - 4(1)(-18)}}{2(1)}$$

At this point, all we have to do is simplify the right side of the equation. The correct answer is C.

$$x = \frac{\pm\sqrt{72}}{2}$$

$$x = \frac{\pm 2\sqrt{18}}{2}$$

$$x = \pm\sqrt{18}$$

Factoring Quadratics

Factoring is a key concept used to solve quadratics. In many cases, factoring a quadratic is more time-efficient than using the quadratic formula. Recall the form of a quadratic: $ax^2 + bx + c$. Factoring a quadratic results in two binomials that are multiplied together in the form $(ex \pm d)(fx \pm h)$. The sign within each binomial depends on the signs of the b and c terms. Let's assume the sign of a is positive; however, if a is negative, then factor out a negative one, and then the relationships below will hold true.

When a is positive . . .

Sign of b term	Sign of c term	Result
+	+	$(x + \_)(x + \_)$
-	+	$(x - \_)(x - \_)$
+	-	$(x - \_)(x - \_)$
-	-	$(x + \_)(x - \_)$

Once the signs in the binomials are found, values for the positions within the binomials can be found. Use the variable in the binomial terms as $(ex + d)(fx + h)$ for the factorization of $ax^2 + bx + c$. The terms e and f are going to be factors of a, and d and h are factors of c. More specifically, the factors of c, when multiplied with the factors of a, will sum to equal the term b.

Let's next walk through an example of factoring the following quadratic equation:

$$x^2 - 7x + 12$$

Step 1: Identify the *a*, *b*, and *c* terms.

$a = 1$, $b = -7$, $c = 12$

Step 2: Identify the signs that will be in the binomials. Since the *b* term is negative and the *c* term is positive, both binomials will have negative signs.

$$(x - \_)(x - \_)$$

Step 3: Identify the factors of the c term.

$$12 : (-1, -12) \ (-2, -6) \ (-3, -4)$$

Step 4: Identify which factors of *c*, when added together, result in the *b* term.

$$-3 + -4 = -7$$

Step 5: Substitute the factors into the binomial factors.

$$(x - 3)(x - 4)$$

At this point, if we needed to solve for the "roots" (x-intercepts) of the equation, we would take each parenthetical term, set it equal to zero, and solve for the value of *x*.

$(x - 3) = 0$ $\qquad$ $(x - 4) = 0$

$(x - 3) + 3 = 0 + 3$ $\qquad$ $(x - 4) + 4 = 0 + 4$

$x = 3$ $\qquad$ $x = 4$

This quadratic equation crosses the x-axis at the points (0,3) and (0,4), as shown at right.
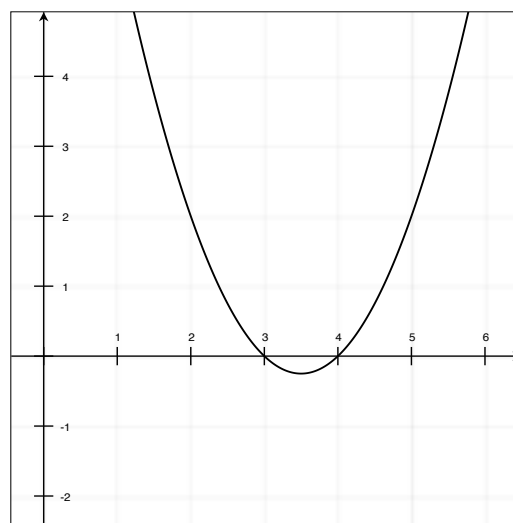
Try this next question on your own!

First, identify the *a*, *b*, and *c* terms.

$a = 3$, $b = 4$, $c = 1$

Next, identify the signs that will be in the binomials. Since *b* and *c* are both positive, both of the signs in the binomials are positive.

$(x + \_)(x + \_)$

Identify the factors of *a* to fill in the first blank in each binomial.

$3x^2 : (3x, x)$
$(3x + \_)(x + \_)$

Identify the factors of *c* which, when multiplied by the factors of *a* and added together, result in *b*.

$1 : (1,1)$

Substitute the factors into the binomials. The correct answer is D.

$(3x + 1)(x + 1)$

To solve Sample Question #4, you'll need to construct an equation to model the situation described. We're told that the consecutive numbers are positive multiples of three, so let *x* equal the first positive number and $x+3$ equal the second positive number. The product of these numbers is 504, so the equation to solve is:

$x(x + 3) = 504$

We could factor the equation at this point, but it might save us time to proceed directly through the quadratic equation.

$x^2 + 3x - 504 = 0$

$a = 1$, $b = 3$, $c = -504$

Sample Question #3

Factor $3x^2 + 4x + 1$.

A. $(2x + 1)(x + 2)$
B. $(3x - 1)(x - 1)$
C. $(2x - 1)(x - 2)$
D. $(3x + 1)(x + 1)$

Sample Question #4

Two consecutive positive multiples of three have a product of 504. What is the sum of the two numbers?

A. 33
B. 39
C. 45
D. 51

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-3 \pm \sqrt{(3)^2 - 4(1)(-504)}}{2(1)}$$

$$\frac{-3 \pm \sqrt{9 + 2016}}{2}$$

$$\frac{-3 \pm \sqrt{2025}}{2}$$

$$\frac{-3 \pm \sqrt{2000 + 25}}{2}$$

$$\frac{-3 \pm 40 + 5}{2}$$

$$\frac{-3 \pm 45}{2}$$

$$\frac{-3 + 45}{2} = \frac{42}{2} = 21$$

$$\frac{-3 - 45}{2} = \frac{-48}{2} = -24$$

Ignore the negative sign; it's not relevant to our question. The numbers are 21 and 24, which sum to 45, making C the correct answer.

Alternatively, if you prefer factoring, you can solve this problem using that method. The equation (and the problem itself) tells us that our numbers need to have a product of 504, and we can also see that they need to have a difference of 3 to generate the 3 coefficient on the $3x$. What are the factors of 504? Listing them out, we find that only one pair fits the definition of positive consecutive multiples of three: 21 and 24.

1×504

2×252

3×168

4×126

6×84

7×72

8×63

9×56

12×42

14×36

18×28

21×24

Properties of Quadratic Graphs

Many standardized tests, including the GRE, evaluate students' knowledge of the graphs associated with various functions. It is not necessary to graph a function to know how the graph will appear on the coordinate plane. Each function's equation has markers that can identify different characteristics of its graph. To quickly identify the graph characteristics of a quadratic function, it is helpful to first manipulate the standard-form equation into vertex form.

> Standard form of a quadratic: $f(x) = ax^2 + bx + c$
>
> Vertex form of a quadratic: $f(x) = a(x - h)^2 + k$

First things first: by definition, quadratics form parabolas, u-shaped lines. In the vertex form, the vertex of the parabola is located at the point (h, k). In standard form, the x-value of the vertex is $-\dfrac{b}{2a}$, and you can solve for the y-value by substituting the x-value into the function. A parabola's axis of symmetry occurs at $x = h$. The example graph at right shows the equation $y = x^2 - 8x + 12$. This parabola's vertex occurs at (4, –4). To put this equation into vertex form, we can substitute 4 for h and –4 for k in the vertex form equation; because no coefficient precedes x^2, $a = 1$.

$$f(x) = a(x - h)^2 + k$$
$$f(x) = 1(x - 4)^2 + (-4)$$
$$f(x) = (x - 4)^2 - 4$$

The parabola's axis of symmetry, indicated by a dotted line, occurs at $x = 4$. If we were asked to solve the equation for its roots, we'd come up with two answers: 2 and 6. We can see this in the graph shown at top right, as the parabola intersects the x-axis at 2 and 6. Note that these two points are both the same distance, two units, away from the parabola's line of symmetry.
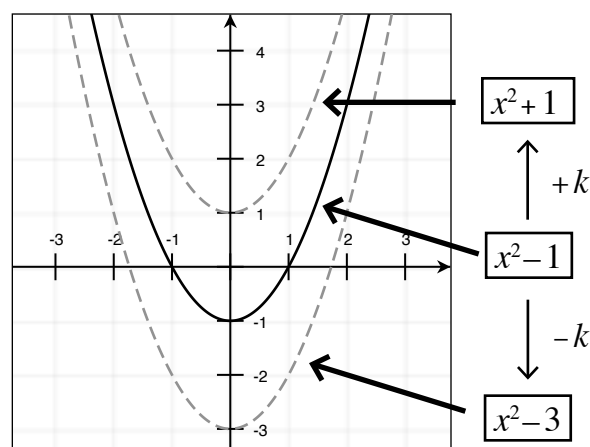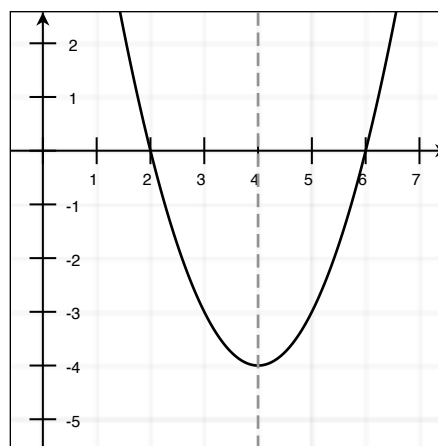
By increasing k, a parabola can be shifted upward. Decreasing k results in the parabola being shifted downward. At bottom right, the quadratic function $x^2 - 1$ is shown with a solid line. Above it, the function $x^2 + 1$ is shown. Because k is increased by two units, this raises the parabola on the y-axis by two units. The dotted-line parabold under $x^2 + 1$ is the parabola $x^2 - 3$. Because k decreases by two units between these equations, the parabola is lowered two units.

Parabolas can be shifted left or right by making changes within the parentheses to the x variable's modifier—changes to h. Shifts to the left are caused by decreases in h (negative changes to x within the parentheses); this makes sense because as you move left on the x-axis, positive numbers get smaller and negative numbers get increasingly negative. By the same logic, increases in h (positive changes to x within the parentheses) shift the parabola right.

$(x+2)^2-1$ ← x^2-1 → $(x-2)^2-1$

$+h$ $-h$
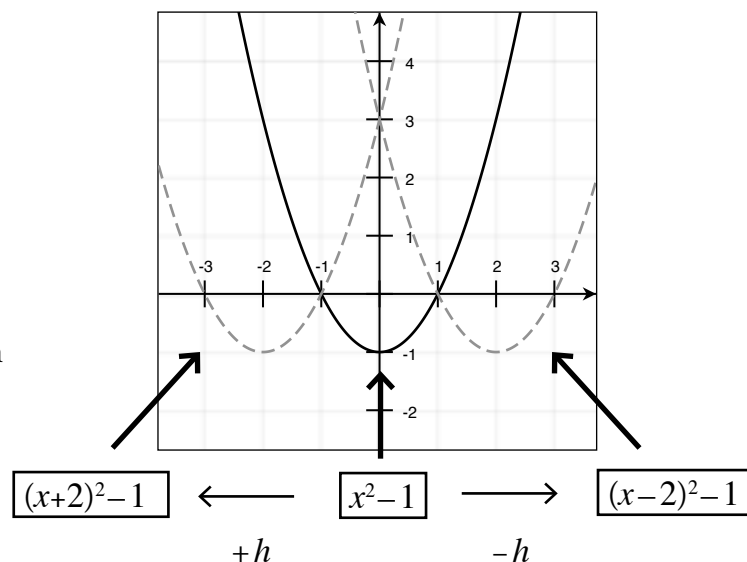
Many graphical changes specific to parabolas depend on the variable a. The sign of a determines whether the parabola opens upward or downward. A positive a means the parabola opens upward; a negative a means the parabola opens downward. Our example equation x^2-1 has a positive a: 1. (Keep in mind that we could write our equation as $1x^2-1$ if we so chose.) If we make the coefficient negative (-1), we invert the parabola without changing its vertex; the parabola of the equation $-x^2-1$ is graphed with a dotted line at right.

x^2-1 (+)

$-x^2-1$ (−)
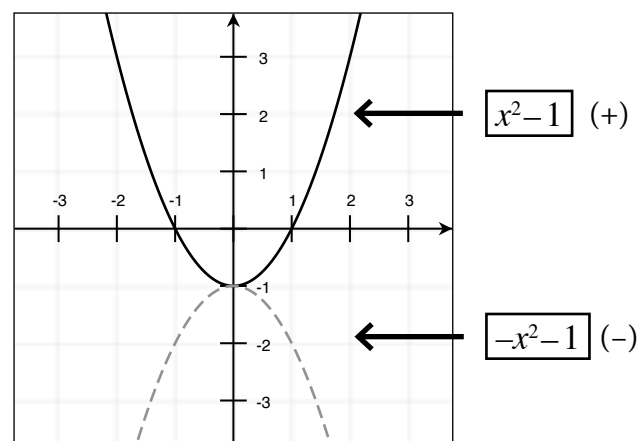
The width of a parabola also depends on the value of a in its equation. As the absolute value of a ($|a|$) decreases below 1, the parabola becomes wider. On the other hand, as the absolute value of a increases above 1, the parabola becomes narrower. Graphed at right is our sample equation x^2-1 (the solid line). The dotted-line equation with a wider width is $\frac{1}{4}x^2-1$, Because $|\frac{1}{4}|<1$, the parabola's width is greater than that of our initial equation where $|a|=1$. The dotted-line equation with the narrower width is $4x^2-1$. $|4|>1$, so the parabola's width is less than that of our main example equation.

$4x^2-1$

$\frac{1}{4}x^2-1$

x^2-1

$-|a|>1$ $+|a|>1$
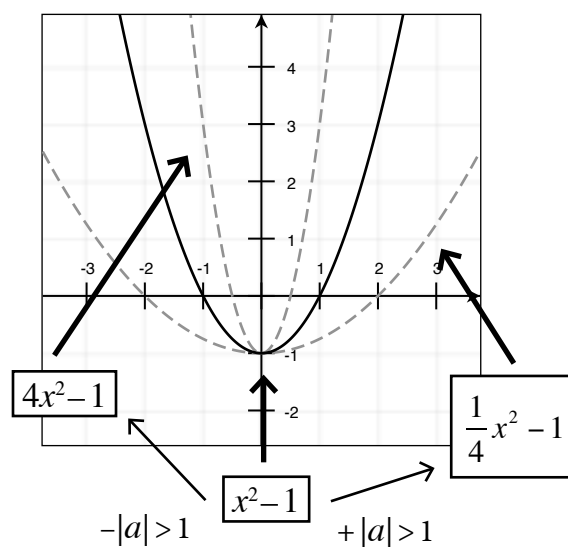
To find the vertex of the function given in Sample Question #1, you'll need to consider the function as it relates to the vertex form in which quadratic equations can be presented, shown below.

$$f(x) = a(x - h)^2 + k$$

In our equation, $a = 2$ and $k = 9$. Correctly identifying h takes a bit more work, because vertex form involves subtracting h from x, but in our equation, 1 is being added to x. That means that h must equal -1, not 1. Algebraically, this makes sense; look what happens when we substitute in -1 for h in vertex form:

$$f(x) = a(x - h)^2 + k$$
$$f(x) = a(x - (-1))^2 + k$$
$$f(x) = a(x + 1)^2 + k$$

Thus, in this particular function, $h = -1$. The vertex of a parabola is located at (h, k), so that of this parabola is located at $(-1, 9)$, making D correct. The solution holds true even when observed graphically, as seen at right.

Sample Question #1

At what point is the vertex of the function $f(x) = 2(x + 1)^2 + 9$ located?

   A. $(1, 9)$
   B. $(-1, 2)$
   C. $(2, 1)$
   D. $(-1, 9)$

Sample Question #2 presents four statements about a pair of quadratic equations in vertex form to analyze. Let's work through these one at a time.

A. The width of a parabola is determined by $|a|$. For A, $a = \frac{1}{3}$, and for B, $a = -2$. Remember that we need to consider the absolute value, though; $|a|$ is greater in B than in A, so A will have the wider parabola and answer choice A isn't true.

B. The direction in which a parabola faces is determined by the sign of a. A positive a translates to an upward-opening parabola and a negative a translates to a downward-opening parabola. For B, $a = -2$, so B's parabola opens downward and B is incorrect.

Sample Question #2

Consider the following equations:

A: $f(x) = \frac{1}{3}(x - 4)^2 + 2$

B: $f(x) = -2(x + 3)^2 + 1$

Which of the following is true?

   A. A's parabola is narrower than B's parabola.
   B. B's parabola opens upward.
   C. B's line of symmetry is found at $x = -3$.
   D. The vertexes of both A and B are found above $x = 0$.

C. A parabola's line of symmetry is found at $x=h$, and for B, $h=-3$. It's negative because vertex form subtracts h from x, and here we're adding it, so for that sign to change, h must be negative; therefore, C is incorrect. We can see this algebraically as shown at right.

$$f(x) = a(x-h)^2 + k$$
$$f(x) = a(x-(-3))^2 + k$$
$$f(x) = a(x+3)^2 + k$$

D. The vertex of a parabola is found at (h,k), so A's vertex is located at $(4,2)$ and B's is located at $(-3,1)$. Both of these points have y-values above zero, so D is correct in its claim that the vertexes of both equations are located above $x=0$.

Graphing Square Root Functions

Square root functions take the form of half of a parabola on its side. If you can work with quadratic functions, you can work with square root functions: they involve the same rules applied to a different base shape. The function with the solid line at right is $y = \sqrt{x}$. Square root functions follow the same rules of being shifted up, down, left, and right as linear and quadratic functions. The dashed-line function that has been shifed right two units from our main example is $y = \sqrt{(x-2}$, and the dashed-line function that has been shifted down two units is $y = \sqrt{(x}-2$.

Like quadratic functions, square root functions' widths are determined by a coefficient. Instead of considering the coefficient of a parenthetical, though, we need to consider the coefficient of the square root. As the absolute value of this coefficient increases above one, the square root function gets taller; as its absolute value decreases below 1, the square root function gets shorter. Remember, it's not just the coefficient, but the coefficient's *absolute value* that determines height square root function height.

The trickiest thing to deal with when working with graphed square root functions is the difference between two ways of reflecting them. The dashed-line function reflected over the y-axis—the one that "opens" to the left instead of to the right, in Quadrant II—is $y = \sqrt{-x}$. The dashed-line function reflected over the x-axis—the one in Quadrant IV—is $y = -\sqrt{x}$.
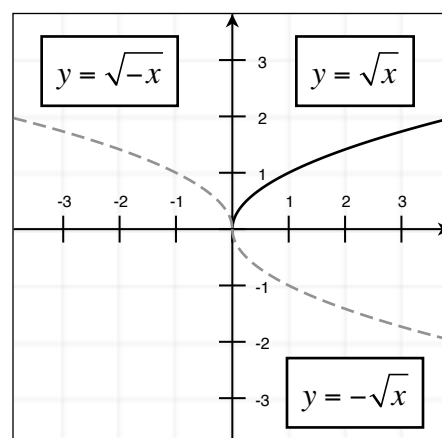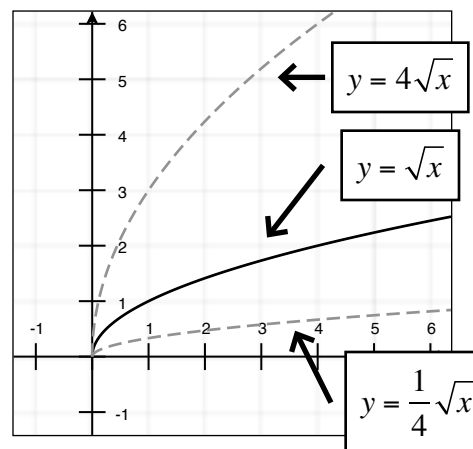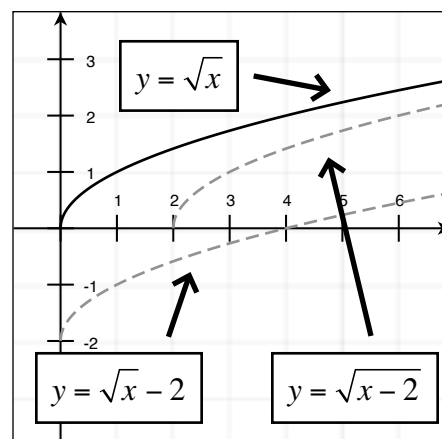
Graphing Circles

The standard form of the equation of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

The highest power to which any of these variables are raised is 2; therefore, circle equations qualify as a kind of quadratic equation.

r refers to the radius of the circle. Consider the solid-line circle shown at right. It's centered on the origin and in its equation, $r = 2$, so the edges of the circle cross the x-axis at $(2, 0)$ and $(-2, 0)$. When working with circles graphically, keep in mind that you're working with radius, not with diameter. Incorrect answer choices may be designed to trip you up on this specific detail.

h determines the circle's horizontal shift, and *k* determines its vertical shift. It's easy to distinguish between these in this case because *h* influences *x* and *k* influences *y*. Note that *h* and *k* are subtracted from *x* and *y*, respectively—you'll need to be on the lookout for questions asking about negative variables! For example, in the dotted-line circle that's been raised by two units in the figure shown to the right, *k* is actually positive two because the negative is provided by the equation. If the equation read $(y + 2)$, the circle would be lowered by two units and *k* would be –2, because its negative and the standard form's negative would cancel. Similarly, the dotted-line circle that

has been shifted left has been shifted as a result of an h-value of –2, even though the equation reads $(x + 2)$. This is again because of the negative sign provided by the equation. If the equation read (x–2), *h* would equal 2 and the circle's graph would be shifted up two units, not down two units. Be very careful when working with circles equations' *h* and *k* values and the respecitve shifts they cause!
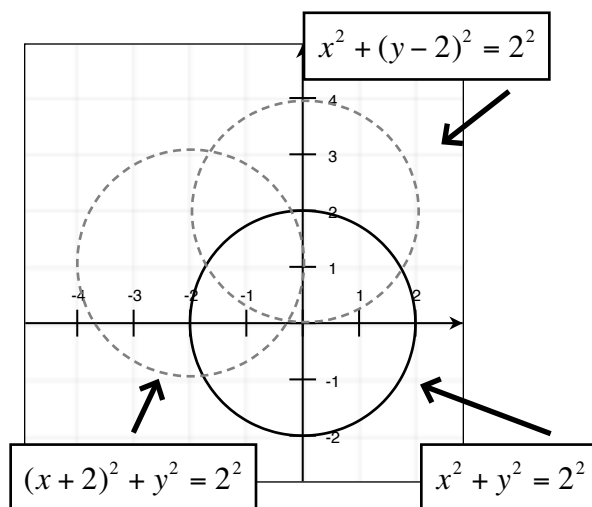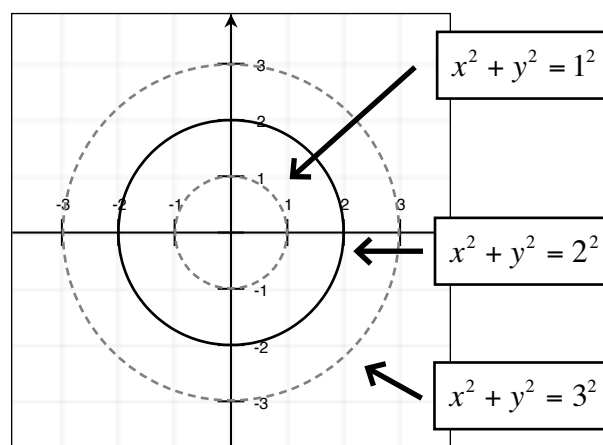
Sample Question #1 gives us an equation and asks us to make several graphical shifts to it. We need to make three changes: one horizontal, one vertical, and one inversion. We can consider our equation as being in either standard or vertex form, since it does not involve a first-power variable. Let's rewrite it explicitly in vertex form.

$$f(x) = a(x - h)^2 + k$$
$$f(x) = 3(x - (0))^2 + (-9)$$

h is the horizontal shift, and the parabola shifts right as $(x-h)^2$ becomes increasingly negative. k is the vertical shift; down correlates to increasingly negative values. So, we need to make –9 into –11. Thus, the correct answer includes $(x-4)^2$ and –11. That narrows our choices down to B and D.

Now we need to invert our parabola. B does this by applying it outside the parenthetical, and D does this by applying the negative within the parenthetical. If you're not sure which one to pick, assume x is positive and consider this: if you square a negative number, what happens? You end up with a positive number. Thus, D can't be correct, as it doesn't change the resulting values (and thus, the parabola) at all. B is the correct answer.

Sample Question #3

Which of the following gives the equation of the function $f(x) = 3x^2 - 9$ after it has been shifted down two units, to the right four units, and inverted?

    A. $f(x) = 3(-x - 2)^2 - 13$

    B. $f(x) = -3(x - 4)^2 - 11$

    C. $f(x) = -3(x - 2)^2 - 13$

    D. $f(x) = 3(-x - 4)^2 - 11$

Sample Question #4

Quantity A	Quantity B
The y-coordinate of the vertex of the parabola $f(x) = 2x^2 + 6x + 5$	$\dfrac{1}{3}$

    A. Quantity A is greater.

    B. Quantity B is greater.

    C. The two quantities are equal.

    D. The relationship cannot be determined from the given data.

Next, let's investigate Sample Question #4. While the vertex of a parabola in vertex form is (h, k), finding the vertex of a parabola in standard form requires a bit more algebra. The x-coordinate of the vertex is $-\dfrac{b}{2a}$, so we'll need to find this first and then substitute it into the function and solve for y ($f(x)$). In our equation, $b = 6$ and $a = 2$, so

$$x = -\frac{b}{2a} = -\frac{(6)}{2(2)} = -\frac{6}{4} = -1.5$$

Now we can substitute –1.5 into the function for x and solve for the y-coordinate:

$$y = 2(-1.5)^2 + 6(-1.5) + 5 = 2(2.25) - 9 + 5 = 4.5 - 9 + 5 = 0.5$$

$y = 0.5$ and $0.5 > \dfrac{1}{3}$, so A is correct.

Sample Question #5 asks us to find the equation of a circle presented as a graph. Let's begin by considering the equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Let's start with r, the radius of the circle. Remember, this is radius, not diameter! The circle extends horizontally from –1 to 11, making its diameter 12 and its radius 6. That means that B and D are incorrect, as they square the circle's diameter, not its radius, to get 144, when r^2 should be 6^2, or 36.

Now for the horizontal and vertical shifts. To identify these, we'll need to first identify the center of the circle. This is the point halfway between its horizontal and vertical diameters. So, it's the midpoint between –1 and 11 $\left(\dfrac{(-1)+11}{2} = \dfrac{10}{2} = 5 \right)$ and between 4 and –8 $\left(\dfrac{4+(-8)}{2} = \dfrac{-4}{2} = -2 \right)$. The center

of this circle is thus (5, –2), and we need to consider it "shifted" from the origin five units to the right and two down. h is the horizontal shift and k is the vertical shift, so we need to substitute in 5 for h and –2 for k, as negative values here correlate with shifts right. Make sure to actually plug these values into the circle equation, though, as negatives will cancel! The correct answer is A.

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (5))^2 + (y - (-2))^2 = 5^2$$
$$(x - 5)^2 + (y + 2)^2 = 36$$

Sample Question #5

Which of the following gives the equation of the circle graphed below?
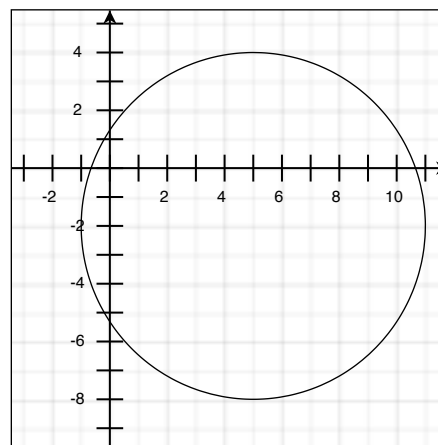
A. $(x - 5)^2 + (y + 2)^2 = 36$

B. $(x + 5)^2 + (y - 2)^2 = 144$

C. $(x + 2)^2 + (y - 5)^2 = 36$

D. $(x - 2)^2 + (y + 5)^2 = 144$

Let's solve one more sample question—specifically, one about functions that involve square roots. Function A already includes some features that shift it away from the origin. The –2 under the square-root symbol shifts the function two units to the right, and the +3 outside of the square-root symbol shifts it three units up. This is not what we're asked about, though: we need to figure out what shift occurs when moving from Function A to Function B. The –2 under the square-root sign becomes a +1; this is a change of positive three. Within parentheses or underneath a square-root symbol, positive equates to movement left. So, moving from Function A to Function B involves shifting left three units. This allows us to narrow our potential answer choices down to B and D. The only other difference between Functions A and B is the presence of a negative sign outside of the square-root symbol in Function B. When the negative is outside of the square-root symbol, it translates to a vertical flip. If the negative sign were within the square-root symbol, it would create a horizontal flip, but this is not what's happening in this problem. The correct answer is B; we can see that this is indeed the case by considering the two functions graphed below. Function A is shown as a solid line, and Function B as a dashed line.

Which of the following graphical shifts are expected moving from Function A to Function B?

Function A: $f(x) = \sqrt{x-2} + 3$

Function B: $f(x) = -\sqrt{x+1} + 3$

A. A vertical flip and a shift right three units

B. A vertical flip and a shift left three units

C. A horizontal flip and a shift right three units

D. A horizontal flip and a shift left three units
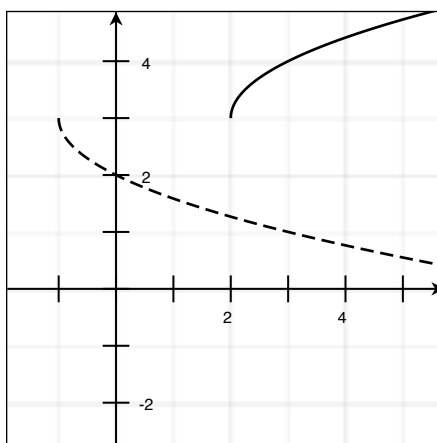
Function Notation

Function notation is an alternative form of formatting certain equations. A function is defined as an algebraic equation for which each input term has exactly one output term. In general, this means that for every value of x, there is only one possible solution for y. Multiple inputs can result in equal outputs (multiple values of x can give the same value of y), but a single input cannot give two solutions.

By this definition, all functions will adhere to the vertical line test. In other words, when a function is graphed, it will be impossible for a vertical line on the graph to intersect multiple points.
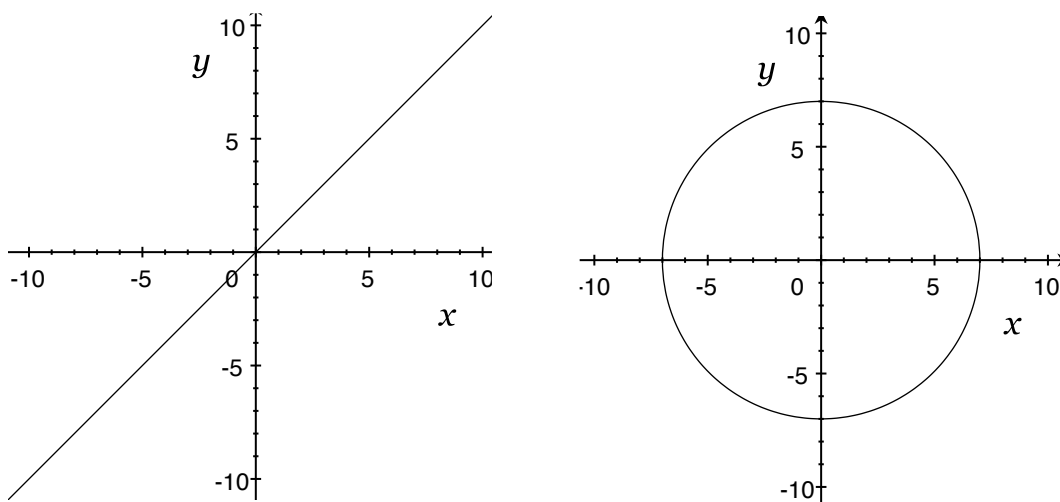
Note that for the equation $x = y$, a vertical line cannot possibly intersect the graph more than once. In contrast, there are values on the graph of a circle where a vertical line will intersect the graph twice. All linear equations are functions, with the exception of those that generate a vertical line such as $x = 5$. By definition, these graphs would fail the vertical line test. All vertically-oriented parabolas (i.e. those that open upwards or downwards) are also functions.

Function notation provides a different method of writing these special equations. Instead of the traditional format using an input variable (typically x) and an output variable (typically y), function notation implies a function being applied to a set of input variables, generating a set of outputs.

Let's examine a simple linear function:

$y = 4x + 3$

We can rewrite this equation using function notation:

$f(x) = 4x + 3$

The implication is that for any value of x, the function can be applied to give a single output. $F(x)$ denotes "the function of x," or the set of all output values corresponding to the available set of input values. In many cases, it is still practical to think of $f(x)$ as equal to y, but function notation provides a framework to look at trends in the graph of the function as opposed to singular values.

Consider evaluating the function above when x is equal to 5. In function notation, this would be written as $f(5)$. On the GRE Quantitative Section, this question could be asked in the format shown in Sample Question #1.

To solve this question, you need to plug in 5 as the value of x and evaluate the result. The correct answer is B.

$$f(5) = 4(5) + 3 = 20 + 3 = 23$$

In function notation, the term in parentheses is the input. Though this may seem like an obvious statement, consider Sample Question #2.

Instead of a single variable input, function notation allows us to evaluate more complex applications of the given equation. To solve this problem, we can start by isolating the term $f(x^2 + 3)$ on the left side of the equation by adding x^2 to both sides of the equation.

$$f(x^2 + 3) - x^2 + x^2 = x + 7 + x^2$$
$$f(x^2 + 3) = x^2 + x + 7$$

Now, we need to replace every instance of x in the right side of the equation with the term given as the input: $x^2 + 3$.

$$f(x^2 + 3) = (x^2 + 3)^2 + (x^2 + 3) + 7$$

Simplify to find the final answer.

$$f(x^2 + 3) = (x^2 + 3)(x^2 + 3) - (x^2 + 3) + 7$$
$$f(x^2 + 3) = x^4 + 6x^2 + 9 - x^2 - 3 + 7$$
$$f(x^2 + 3) = x^4 = 5x^2 + 4$$

The correct answer is A.

Sample Question #1

$f(x) = 4x + 3$. Find $f(5)$.

   A. 20
   B. 23
   C. 25
   D. 28

Sample Question #2

Find $f(x^2 + 3) - x^2 = x + 7$

   A. 20
   B. 23
   C. 25
   D. 28

In some instances, the GRE Quantitative Section may ask you to apply one function to another. These questions will provide two separate functions, then ask you to evaluate an input for one function in terms of another. The format of these questions will appear as $f(g(x))$.

What does $g(f(6))$ mean? We need to apply the function $g(x)$ to the outcome of $f(6)$. To start, we need to find $f(6)$.

$$f(6) = (6)^2 - 14 = 36 - 14 = 22$$

Now, we can rewrite $g(f(6)$ as $g(22)$, because $f(6) = 22$. To find the final answer to this question, simply solve for $g(22)$.

$$g(22) = -(22) + 4 = -18$$

So, the final answer will be C, because $g(f(6)) = -18$.

Sample Question #4 tests your knowledge of functions in a more abtract way: by making sure you understand what defines a function. Keep in mind that the question asks you to pick out any answer choices that are NOT functions, so we're looking for any that do not pass the straight-line test or don't qualify in other, more general, ways. A is instanty recognizable as the equation of a circle, and since a circle doesn't pass the vertical line test, A isn't a function. B and C are functions, but D doesn't include a y-variable at all, so it can't be a function. Similarly, E includes both *x* and *y*, but doesn't have an equals sign to equate them. D and E are simply expressions, not equations, so they cannot be functions. The correct answers are A, D, and E.

Sample Question #3

$$f(x) = x^2 - 14$$
$$g(x) = -x + 4$$

Find $g(f(6))$.

A. 11

B. 12

C. −18

D. −21

Sample Question #4

Which of the following are NOT functions?

> Multiple answers may be correct. Select all that apply.

A. $(y + 2)^2 + (x + 2)^2 = 16$

B. $f(x) = \sqrt{2x} + 4y$

C. $y = -x^{12}$

D. $54x + 29^4$

E. $\dfrac{xy}{24}$

Word Problems: Simple Interest and Compound Interest

In economics, "interest" refers to the amount of money someone collects on a lump sum kept in a bank or the amount of additional money one must pay to a creditor along with an amount loaned. Mathematically, interest is a defined percentage or proportion of the principal amount that is added or subtracted to the principal amount as time passes.

The GRE Quantitative section tests two related but distinct types of interest: simple interest and compound interest. Both types of interest deal with a principal amount of money and the interest accrued on that specific amount of money. Let's go over each type of interest individually while focusing on the details that tell you which type is relevant to a given problem.

Simple interest reflects the interest taken on an amount after one given period of time based on a rate, percentage, or ratio. To solve for simple interest, you need to know three things: the principle (initial amount of money borrowed or invested), the interest rate, and the amount of time. If you are given three of the four values, you can solve for any of these four, including the simple interest itself.

Compound interest differs from simple interest in that simple interest models one discrete "collection" of interest based on an unchanging principle, while compound interest models interest in situations in which the interest is added to the principle over time. Thus, the principle changes over time, and a different equation must be used to reflect this. Compound interest still involves the same four variables as simple interest, but it adds in one more: n, the number of times the amount is compounded per the specified time. You can think of this as how many distinct times interest is "collected" and added to the principle per amount of time, t.

Simple Interest	**Compound Interest**
$I = P \times r \times t$	$A = P(1 + \dfrac{r}{n})^{nt}$
P = Principal Amount (Starting amount of money) r = Rate/Percentage/Ratio of Increase t = Time	A = Principal plus interest earned P = Principal Amount (Starting amount of money) r = Rate/Percentage/Ratio n = Number of times amount is compounded per specific time t = Time

So, how do you distinguish between problems that call for simple interest calculations and those that call for compound interest calculations? It's all in the wording of the problem, specifically in the way that the problem refers to time. If you're told that a loan or investment is "compounded" at a specified regular interval, you need the compound interest formula. If nothing seems to suggest that the interest was re-invested after regular periods—that is, the problem only describes one single period of investment—you need the simple interest formula.

Pay careful attention to how each problem is worded. Are you solving for the interest, final sum, or some other variable? Amount of interest will be defined as the variable *I*, but total final amount will be *I+P*, so you'll need to perform one last simple calculation to find the total final amount. In addition, there's a good chance some of the answer choices will present the alternate option, making it all too easy to miss your mistake should you make one at this point in the problem. Read and interpret carefully!

Let's put these concepts into practice by working through Sample Question #1. Our first step is to identify the type of interest the question concerns. We're asked to solve for the total amount of money in the account after one period of fifteen years. No mention is made of interest being reinvested, and we're not given any information about the periods of time at which such reinvestment might occur (*n*). So, this is a simple interest question.

Sample Question #1

Billy invests $1500 in a fifteen-year CD that has a 2.3% interest rate. How much money will be in the CD's account at the end of the fifteen-year period?

A. $517.50
B. $604.25
C. $2017.50
D. $2104.25

Now that we know which type of interest we're discussing, we can start our calculation by listing the given information.

$P = 1500$
$r = 2.3\% = 0.023$
$t = 15$
$I = ?$

Now, we can write the formula and substitute in the given information to solve for the amount of interest that accrues for the account over fifteen years.

$I = P \times R \times T$
$I = \$1500(0.023)(15)$
$I = \$517.5$

This is the amount of interest Billy's money has made. But you're not done solving this problem yet! The question asks you to find the amount of money that will be in the CD's account after the fifteen-year investment period, *T*. That means we need to add the interest accrued to the initial principal amount to answer the question. C is the correct answer.

$T = I + P$
$T = \$517.5 + \$1500$
$T = \$2017.50$

In working through Sample Question #2, we first need to recognize which type of interest is in play so that we can make use of the correct equation.

We're told that the interest on Dan's loan is "compounded quarterly," so we need to use the compound interest formula. Now we can proceed to identifying the information the question stem gives us. The trickiest part of this process is translating "compounded quarterly" into mathematical terms. Since there are four quarters in a year, we need to assign the value of $n = 4$.

Next, we can write the interest equation and substitute in the given information.

Sample Question #2

Dan took out a $2000 from his bank. His loan is compounded quarterly at a rate of 1.95%. How much interest will Dan owe on his loan after 3 years?

A. $94.72

B. $98.34

C. $108.24

D. $120.19

$P = 2000$

$r = 1.95\% = 0.0195$

$t = 3$

$n = 4$

$A = ?$

$$A = P(1 + \frac{r}{n})^{nt}$$

$$A = 2000(1 + \frac{0.0195}{4})^{4 \cdot 3}$$

$$A = 2000(1 + 0.004875)^{12}$$

$$A = 2000(1.004875)^{12}$$

$$A = 2000(1.06)$$

$$A = 2120.188$$

$$A = \$2120.19$$

The compound interest formula that we use gives us the total amount of the loan after the specific time period because we are adding 1 to the rate. So, we are calculating the initial amount added to the interest made. To figure out how much of Dan's amount owed is interest accrued on his initial loan, we need to subtract the amount of the initial loan from the result of the compound interest equation. Doing this, we can see that the correct answer is D.

$$A = I + P$$

$$A - P = I$$

$$\$2120.19 - \$2000.00 = I$$

$$I = \$120.19$$

Sample Question #3 presents a new twist on the types of interest problems we've seen so far in this lesson. Instead of solving for a final sum of money, the final sum is directly provided, and we're asked to calculate one of the other variables in the interest equation. In this problem, we're asked to solve for the rate of interest. We can tell that we need to concern ourselves with the compound interest equation because the question stem specifies that the friend's account is "compounded every six months." Now, we need to list out the data we're given and identify n. In this case, n equals 2, because every six months means the same thing as twice yearly.

Another wrinkle in this problem is the fact that we're not given the amount of interest accrued directly; we're instead told the balance of the account on the day it opened and a year later. Now we can list out our information:

Sample Question #3	

Your friend deposits one thousand dollars into a savings account that is compounded every six months. One year after your friend opens the account, she checks her balance and finds that the account contains $1103.45.

Quantity A	Quantity B
The rate of interest accrued on your friend's bank account	9.0%

   A. Quantity A is greater.
   B. Quantity B is greater.
   C. The two quantities are equal.
   D. The relationship cannot be determined from the information given.

$A = \$1103.45$

$P = \$1000.00$

$r = ?$

$n = 2$

$t = 1$

Substituting these values into the compound interest equation, we get an equation with only one variable in it: r. This means that we can solve for r algebraically.

$$A = P(1 + \frac{r}{n})^{nt}$$

First, we need to substitute our known values for variables into the equation.

$$1103.45 = 1000\left(1 + \frac{r}{2}\right)^{2(1)}$$

At this point, we can multiply the exponents to simplify that part of the equation.

$$1103.45 = 1000\left(1+\frac{r}{2}\right)^2$$

We can continue by dividing both sides by 1000.

$$\frac{1103.45}{1000} = \frac{1000\left(1+\frac{r}{2}\right)^2}{1000}$$

$$\frac{1103.45}{1000} = \left(1+\frac{r}{2}\right)^2$$

At this point, we can get rid of the power by taking the square root of both sides.

$$\sqrt{\frac{1103.45}{1000}} = \sqrt{\left(1+\frac{r}{2}\right)^2}$$

$$\sqrt{\frac{1103.45}{1000}} = 1+\frac{r}{2}$$

We can now subtract the one from both sides of the equation.

$$\left(\sqrt{\frac{1103.45}{1000}}\right)-1 = \left(1+\frac{r}{2}\right)-1$$

$$\sqrt{\frac{1103.45}{1000}} - 1 = \frac{r}{2}$$

Finally, we can isolate r on one side of the equation by multiplying both sides by 2.

$$2\left(\sqrt{\frac{1103.45}{1000}}-1\right) = 2\left(\frac{r}{2}\right)$$

Now that we've isolated r, all we have to do is simplify the right side of the equation.

$$r = 2\left(\sqrt{\frac{1103.45}{1000}}-1\right) = 2(\sqrt{1.10345}-1) = 2(1.05045-1) = 2(0.05045) = 0.1009$$

At this point, we have a decimal value that we need to turn into a percent value. We can do that by multiplying it by 100.

$$0.1009 \cdot 100 = 10.09\%$$

This is a larger value than Quantity B's 9.0%, so A is the correct answer.

Word Problems: Working with Rates

Certain problems on the GRE Quantitative section involve rates—the quantitative representation of some sort of measure of progression over some sort of measure of time. You might measure the rate at which a faucet fills a bucket; in this case, the units of the rate might be gallons per minute. Alternatively, you might measure the number of times some sort of action is performed in a given time frame. We could measure the number of words a person reads per minute, or the number of cell phones a factory can manufacture per hour.

Rates like these can be represented mathematically in a few different forms. Fractions provide an easy way to represent them, with the "progress made" number acting as the numerator and the "time taken" number acting as the denominator. If a faucet dumps three gallons of water into a bucket every minute, the fraction representing this rate would be:

$$\frac{3 \text{ gallons}}{1 \text{ minute}}$$

Fractions are an especially convenient way to display rates because they are easy to mathematically manipulate in order to get them to apply to other circumstances. For instance, say we want to know how many gallons of water our faucet could emit in an hour, we could find this by converting our time units from minutes to hours. How do we do that? The specific operation needed will depend upon the particular units with which you're working. There are sixty minutes in one hour, so we'll need to multiply our fraction by another fraction so that we end up with a time unit of hours in the denominator instead of minutes. Using dimensional analysis—writing out the fraction and making sure the proper units cancel—can make this process much easier.

$$\frac{3 \text{ gallons}}{1 \; \cancel{\text{minute}}} \times \frac{60 \; \cancel{\text{minutes}}}{1 \text{ hour}} = \frac{180 \text{ gallons}}{1 \text{ hour}}$$

Of course, we might be given rates in non-fractional forms. We could convey our faucet's rate in any of the following forms—it represents the same rate:

$$\frac{3 \text{ gal}}{1 \text{ min}} = 0.05 \text{ gal/sec} = 180 \text{ gal/hour} = \frac{1 \text{ gal}}{20 \text{ sec}}$$

As you can see, rate problems center around working with a given ratio—either algebraically adjusting it so that you're representing the same rate using different units, or—as we'll see next—working with multiple rates. Either way, pay particular attention to units when working on a problem involving rate—slipping up here can result in accidental errors that the test-writers fully anticipate some people to make, so incorrect answer choices might be the results you get if you make those particular mistakes.

Finding and Applying a Rate

Finding a rate is simple if you're given the information you need—you just need to construct a fraction to represent it. Sample Question #1 asks you not only to calculate a rate, but to adjust it.

What are we told? There's a lot of information going on here—we're working with three concepts: number of machines, number of devices, and time in minutes. We can calculate the rate as work done over time and then incorporate the number of workers as a multiplier.

Unit of Work Done = Number of Workers × Rate × Time

If 24 machines can make 5 devices in 30 minutes, how many hours will it take 4 machines to make 15 devices?

    A. 6 hours
    B. 7 hours
    C. 8 hours
    D. 9 hours

In our case, we are given 24 "workers," able to make 5 "units of work," in 0.5 "time." The rate is not given overtly, but can be solved for with our given information.

Units of work = 5

Number of workers = 24

Time = 0.5 hours (30 minutes)

$$\text{Rate} = \frac{\text{Units of work done}}{\text{Number of workers} \times \text{Time}} = \frac{5}{24 \times 0.5} = \frac{5 \text{ devices}}{12 \text{ workers} \times \text{hours}}$$

Now, since we know the rate, which does not chage, we can solve for the new time when the number of machines is decreased and the number of devices is increased. D is the correct answer!

Unit of Work Done = Number of Workers × Rate × Time

$$15 = 4 \times \frac{5}{12} \times t$$

$$15 = \frac{20}{12}t = \frac{4}{3}t$$

$$15 \times \frac{3}{4} = t$$

$$9 = t$$

Solving for One of Two Rates Based on Result

Some problems might involve multiple sources of input toward a common goal. Multiple machines might be building units at a factory, or multiple people might be frosting cupcakes for a large order. No matter the context, if you're told the goal and one rate— or given enough information to find these— you can approach the problem the same way mathematically.

Sample Problem #2 gives the rate of Hose A instead of the time it takes to fill the shark tank. Let's convert the rest of the problem information into rates as well. The tank is 10,000 gallons, and it takes the two hoses together 4 hours to fill the tank. Therefore, the combined rate is

$$\frac{10000 \text{ gallons}}{4 \text{ hours}} = 2500 \, \frac{\text{gallons}}{\text{hour}}$$

Sample Question #2

A 10,000 gallon shark tank is filled by two hoses. Hose A fills at a rate of 1,000 gallons per hour. When Hose A and Hose B are both on, they fill the shark tank in 4 hours. At what rate does Hose B fill the tank?

A. 1250 gallons per hour
B. 1500 gallons per hour
C. 1750 gallons per hour
D. 2000 gallons per hour

Now we know that Hose A can deliver 1,000 gallons in an hour, and together Hose A and Hose B can deliver 2,500 gallons in an hour. So, we can write out the following:

$$1,000 \frac{\text{gallons}}{\text{hour}} + x \frac{\text{gallons}}{\text{hour}} = 2500 \frac{\text{gallons}}{\text{hour}}$$

Then Hose B fills the tank at 2500 – 1000 = 1500 gallons/hour, so the correct answer is B.

Combining Two Rates into One Result

Perhaps the most difficult type of rate-mathematics problem you're likely to encounter on the GRE is the kind that ask you to find two individual rates and find how long it will take them to accomplish a quantified goal or how much quantified work they can get done in a specified amount of time. Sample Question #3 is just such a problem, made even more daunting by the fact that we're only given two numbers, not four.

First, find what fraction of the job is completed per hour. John can paint the room in 4 hours, so he paints

$$\frac{1\text{ room}}{4\text{ hours}} = \frac{1\text{ room}}{4\text{ hour}} = \frac{1}{4}\text{ room per hour}$$

Similarly, Susan can paint the same room in 6 hours, so she paints

$$\frac{1\text{ room}}{6\text{ hours}} = \frac{1\text{ room}}{6\text{ hour}} = \frac{1}{6}\text{ room per hour}$$

At this point, we have individual rates calculate for both John and Susan, so to find the rate at which they could paint one room together, we can just add together the individual rates. We need to create a common denominator for them, so let's use the LCM, 12:

$$\frac{1}{4}+\frac{1}{6}=\left(\frac{1}{4}\right)\frac{3}{3}+\left(\frac{1}{6}\right)\frac{2}{2}=\frac{3}{12}+\frac{2}{12}=\frac{5\text{ rooms}}{12\text{ hours}}=\frac{5}{12}\text{ room per hour}$$

Five-twenfths of the job is completed per hour. To find the number of hours for the whole job (1 room), we just need to do one more calculation, using the combined rate of room-painting with an input of 1 room and an output of time taken.

$$1\text{ room} \times \frac{12\text{ hours}}{5\text{ rooms}} = \frac{12}{5}\text{ hours}$$

Our answer choices are in minutes, so we need to convert hours to minutes. C is correct!

$$\frac{12}{5}\text{ hours} \times \frac{60\text{ minutes}}{1\text{ hour}} = \frac{720}{5}\text{ minutes} = 144\text{ minutes}$$

Sample Question #3

John can paint a room in 4 hours, and Susan can paint the same room in 6 hours. How many minutes would it take them to paint the room together?

A. 136 min
B. 140 min
C. 144 min
D. 160 min

Geometry

Geometry is the study of two- and three-dimensional shapes and their specific measurements. This branch of math involves the construction of diagrams, the memorization of equations, and the use of a great deal of vocabulary; therefore, it's reasonable to feel overwhelmed at the start of your geometry review. We've partitioned our review of geometry content into a number of to-the-point lessons about the particular topics you'll need to master to be fully prepared for test day. Many of these can overlap—for example, a question about a pair of similar right triangles might involve topics covered in the "Right Triangles" and "Congruent Figures and Similar Figures" lessons—but we distinguish them so that you can isolate one concept to review at a time. After reading through the lessons in this section, you'll be aware of the skills you need to master to be prepared for geometry questions when you take your exam; furthermore, you'll possess the knowledge to be able to devote extra time to the aspects of geometry that give you the most trouble. So, even if you're feeling a bit overwhelmed, check out one of our geometry lessons and start mastering this branch of math bit by bit!

Section Outline

Length and Midpoint

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Length and Midpoint

SAT Math questions that concern lines and line segments may ask you to calculate their lengths and midpoints. Even if such lines are graphed on a Cartesian plane, finding their length isn't as simple as counting the number of spaces between the endpoints. The lines that appear in these questions will most likely be diagonal lines that are impossible to accurately measure by eye. Diagonal lines present similar problems for identifying the midpoint of a line, which may not fall on a point in which the coordinates are whole numbers. Luckily, there are two formulae that you can use just in these instances. They're not provided on the GRE, so you'll need to memorize them. Let's review them now.

The Distance Formula

Consider a line with endpoints A and B. $A = (x_1, y_1)$ and $B = (x_2, y_2)$. The following formula yields the distance between the two points.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Basically, what this formula is doing is treating the distance between the two points as the hypotenuse of a triangle formed by the rise (Δx) and run (Δy) between them.
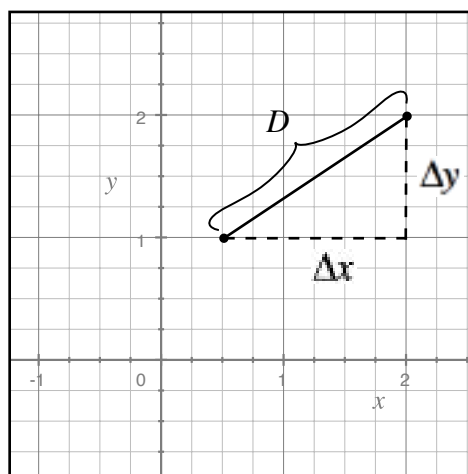
To demonstrate this, we can derive the distance formula from the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

Let's rename our variables to keep things consistent.

$$(\Delta x)^2 + (\Delta y)^2 = D^2$$

Let's define Δx and Δy with variables related to our two points.

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = D^2$$

Starting to look familiar? All we have to do now is take the square root of each side to solve for D:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1^{\,2})}$$

Voila! There's our formula. If you forget it on test day, just remember, it's just the Pythagorean Theorem written a little differently so as to be easy to apply to points in the coordinate plane. Let's next try out a sample problem making use of this formula. Practicing how to use formulae to solve problems is a great way to help you remember them!

Let's start by solving Sample Question #1. First, we need to assign the values in our points to the correct variables in the distance formula. It can be very helpful to write out which value is which in order to prevent confusion, especially in a test situation. It also makes checking your work much easier, and therefore faster, should you want to glance over how you arrived at your answer. It doesn't matter which coordinates you assign to be which points as long as you keep them consistent. After all, it's the same distance from $(-3,3)$ to $(4,4)$ as it is from $(4,4)$ to $(-3,3)$! For this problem, let's call $(-3,3)$ point 1 and $(4,4)$ point 2. That means that $x_1 = -3$, $y_1 = 3$, $x_2 = 4$, and $y_2 = 4$. Now we can plug our data into the distance formula and solve.

Sample Question #1

How far is the point $(-3,3)$ from the point $(4,4)$ on a standard coordinate plane?

A. $3\sqrt{5}$

B. $4\sqrt{3}$

C. $7\sqrt{3}$

D. $5\sqrt{2}$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$D = \sqrt{(4 - (-3))^2 + (4 - 3)^2}$$
$$D = \sqrt{7^2 + 1^2}$$
$$D = \sqrt{49 + 1}$$
$$D = \sqrt{50}$$

At this point, we can simplify the square root to come up with our final answer.

$$D = \sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

It is $5\sqrt{2}$ units from $(-3,3)$ to $(4,4)$, so D is the correct answer!

Let's now try a more complex sample question.

Sample Question #2

A scientist is taking ice core samples in Antarctica near her base camp. The first site at which she plans to drill for ice cores, Site A, next is located three miles north of the base camp, and the second site at which she plans to collect samples, Site B, is located four miles east of her base camp.

The scientist plans to use a 15 mpg snowmobile to travel to Site A, then to Site B, and return back to base camp. How much fuel will she use in traveling from Site A to Site B?

A. $\frac{1}{4}$ gallon

C. $\frac{1}{2}$ gallon

B. $\frac{1}{3}$ gallon

D. $\frac{2}{5}$ gallon

Don't let the context of Sample Question #2 fool you—it looks complex, but really, it's just a distance problem combined with a rate of fuel use. So, to solve it, we need to first calculate how far it is from Site A to Site B. Then, we need to take that distance and multiply it by the snowmobile's fuel consumption rate, 15 miles per gallon.

Even though this problem isn't presented as being on a coordinate plane, we can imagine it on a plane in which the units are miles and the base camp is the origin point $(0,0)$. Site A is located at the point $(0,3)$ and Site B at the point $(4,0)$.

Now that we've identified our points, we can substitute them into the distance formula and solve. Let's call Site A point 1 and Site B point 2. That means that $x_1 = 0$, $y_1 = 3$, $x_2 = 4$, and $y_2 = 0$.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$D = \sqrt{(4 - 0)^2 + (0 - 3)^2)}$$
$$D = \sqrt{(4)^2 + (-3)^2}$$
$$D = \sqrt{16 + 9}$$
$$D = \sqrt{25}$$
$$D = 5$$

It is exactly five miles from Site A to Site B. We can now use this information to calculate how much gas the scientist's snowmobile will use when making this trip. Using dimensional analysis can help keep units in an easy-to-view order. You can see that the miles cancel and leave us with gallons as our unit.

$$5 \text{ ~~miles~~} \times \frac{1 \text{ gallon}}{15 \text{ ~~miles~~}} = \frac{5}{15} \text{ gallon} = \frac{1}{3} \text{ gallon}$$

The scientist's snowmobile will use $\frac{1}{3}$ gallon of gas when traveling between Site A and Site B, so B is the correct answer.

Midpoint Formula

If you know the endpoints of a line segment and need to figure out its midpoint, the following formula can help you find it. Consider a line with endpoints A and B. $A = (x_1, y_1)$ and $B = (x_2, y_2)$.

$$(x_n, y_n) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

This formula is not using the Pythagorean Theorem like the distance formula is. Instead, it is simply taking the average of the x-coordinates and the y-coordinates, which yields the "average" of the two points—the midpoint. We don't have to worry about using the Pythagorean Theorem in this scenario because we can treat the x- and y-variables independently instead of having to find the length of a diagonal, which requires consideration of both.

Let's try out some practice problems to cement your knowledge of this formula!

Sample Question #3

Point A is located at $(2,2)$ and Point B is located at $(0,3)$. Find the midpoint of $\overline{AB}$.

A. $(1, 2.5)$ **C.** $(1,1)$

B. $(1, 1.5)$ **D.** $(1,2)$

In order to solve Sample Question #3, we first need to assign our data to the correct variables. Let's call Point A point 1 and Point B point 2. That means that $x_1 = 2$, $y_1 = 2$, $x_2 = 0$, and $y_2 = 3$. Substitute these values into the midpoint equation, and you'll then be able to solve for the correct answer.

$$(x_n, y_n) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$(x_n, y_n) = \left(\frac{2+0}{2}, \frac{2+3}{2} \right)$$
$$(x_n, y_n) = \left(\frac{2}{2}, \frac{5}{2} \right)$$
$$(x_n, y_n) = (1, 2.5)$$

The midpoint of $\overline{AB}$ is the point $(1, 2.5)$, so A is the correct answer.

Midpoint questions can also be presented in a slightly different manner. Certain problems might give you the midpoint of a line segment and one of the endpoints and ask you to calculate the other endpoint. Let's practice this type of problem next.

To solve Sample Question #4, create two separate equations to figure out where B is located: one that relates to the x-values of A, B, and the midpoint, and another that relates to their y-values.

We know that $x_1 = 4$ and $y_1 = 5$. We also know that the x-value of the midpoint is 1. Knowing that that midpoint's x-value must be the average of the x-values of points A and B, we can write the following equation:

$$\frac{4 + x_2}{2} = 1$$

Solving for x_2, we get:

$$4 + x_2 = 2$$
$$x_2 = -2$$

The x-coordinate of B is –2. Now for the y-coordinate! We can set up the same equation using information that relates to the y-coordinates of the relevant points. $y_1 = 5$, and the y-coordinate of the midpoint is 2.

Sample Question #4

Point A is located at $(4,5)$. The midpoint of $\overline{AB}$ is $(1,2)$. What are the coordinates of Point B?

A. $(-3,-1)$

B. $(-2,-1)$

C. $(2,-1)$

D. $(-2,0)$

$$\frac{5 + y_2}{2} = 2$$

Solving for y_2, we get:

$$5 + y_2 = 4$$
$$y_2 = -1$$

The y-coordinate of point B is −1. Putting our results together, we find that the coordinates of point B are $(-2, -1)$. That means that B is the correct answer!

Let's tackle one more midpoint problem, this time one presented as a word problem.

Sample Question #5

Emily is setting up flags for a relay race. The race's starting line is one hundred meters north and one hundred meters east from the concession stand, and it will be run in a straight line to the finish line, located five hundred meters north of the concession stand and three hundred meters east of it. Emily has been instructed to set up flags halfway to the finish line. Where should she set up the flags relative to the concession stand?

A. 200 m north and 300 m east   **C.** 300 m north and 200 m east

B. 250 m north and 250 m east   **D.** 350 m north and 200 m east

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Sample Question #5 might look overwhelming at first. There's a lot of data being thrown around, but once you take a moment to process it, you'll realize that you're being asked to find the midpoint of a line. This line isn't presented on a traditional coordinate plane, though, which may have you wondering how to approach it. How is the line defined? The points are presented as distances north and east from the concession stand. Aha! If we label the location of the concession stand the origin, increasing x-values "north," and increasing y-values "east," we can then translate the distances from it into coordinates. Dividing each of the distances by 100 makes them much more manageable:

Starting point: 100 meters north, 100 meters east → (1,1)

Finish line: 500 meters north, 300 meters east → (5,3)

We have our points; the next step is to assign the values to the variables in the midpoint formula. Calling the starting point point 1 and the finish line point 2, $x_1 = 1$, $y_1 = 1$, $x_2 = 5$, and $y_2 = 3$. We can substitute these values into the equation and find our answer in coordinates.

$$(x_n, y_n) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(x_n, y_n) = \left(\frac{1+5}{2}, \frac{1+3}{2} \right)$$

$$(x_n, y_n) = \left(\frac{6}{2}, \frac{4}{2} \right)$$

$$(x_n, y_n) = (3, 2)$$

The midpoint of the race route is found at (3,2). But wait, we need to identify that midpoint in terms of meters from the concession stand! All we need to do is multiply each of the terms by 100 m. Remember, x-values relate to distance north from the concession stand, and y-values relate to distance east from it.

$(3,2) \rightarrow$ 300 m north, 200 m east

Emily needs to set up the flags 300 m north from the concession stand and 200 m east of it. C is the correct answer.

Angles, Parallel Lines, and Perpendicular Lines

Part of succeeding on the GRE Quantitative section involves knowing your way around line diagrams and being able to calculate the measures of angles indicated on them. While they may appear similar to graphed linear equations, these lines are distinguished by one important feature: they're not placed in the context of a coordinate plane. Instead, they convey information about the relationships between different lines and the angles formed by their intersections. In this lesson, we'll go over how to read a line diagram, relationships between collinear angles, and various crucial types of relationships between lines that you need to know to solve certain angle problems. Having mastered these elementary but important points, you'll have the groundwork knowledge required to handle any line diagram that the GRE Quantitative section can throw at you. Let's get started!

Reading Line Diagrams: The Basics

Line diagrams can seem intimidating at first glance due to the numerous specific symbols that can appear in them. Let's review them and make sure that you have the basic knowledge needed to understand what's being asked of you in a line diagram problem.

Points, whether they are on lines or not, are given single-letter names and indicated with dots. Points A, B, and C are shown below. Points that are said to be "colinear" are located on the same line. In the diagram below, points B and C are colinear, but points A and B are not.
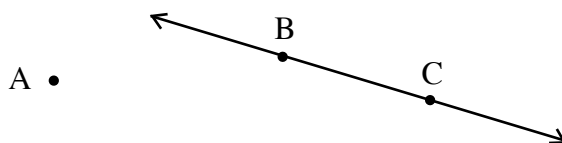
A line is named after points that are on it. Points B and C are on a line. This line is indicated with the following notation: $\overleftrightarrow{BC}$. A line segment refers to only the part of the line that falls between the two indicated points. Line segments are indicated with the following notation: $\overline{BC}$.

Note that lines can have multiple names. If there were a point D between points B and C on the same line, we could refer to the line as $\overleftrightarrow{BD}$, $\overleftrightarrow{DC}$, or $\overleftrightarrow{BC}$ and be talking about the same line. Note, also, that the order in which we refer to the points does not matter; $\overleftrightarrow{BC}$ and $\overleftrightarrow{CB}$ refer to the same line.
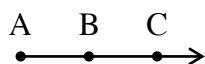
A ray is a line that begins at a specified point. The first point in the name of a ray is its starting point, and the second point is a point through which it passes. We could refer to the following ray as either $\overrightarrow{AB}$ or $\overrightarrow{AC}$, but calling it $\overrightarrow{BC}$ would be incorrect.
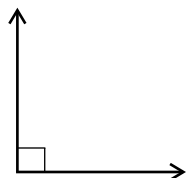
A symbol that looks like two sides forming a square in the corner of two intersecting lines indicates that the two lines form a 90° angle. This means that the lines are perpendicular to one another.
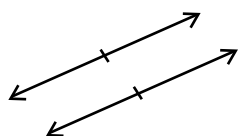
A pair of lines marked by a pair of tick marks are parallel to one another.

Pairs of lines that are either parallel or perpendicular lines may also be indicated in the question stem through the use of the following notation:

AB ∥ CD   This means that line AB and line CD are parallel.

AB ⊥ CD   This means that line AB and line CD are perpendicular.

Angles are named from the points that compose them using the following notation: "m∠ABC" means "the measure of angle ABC." Always use three points to refer to an angle, or it could become unclear to which angle you are referring. In the diagram shown on the right, m∠ABC is 180°; it's called a "straight angle" because it forms a straight line. m∠CBE is 90°. Note that the middle letter in an angle name stands for the vertex. The order of the other letters doesn't matter; we could refer to the ninety-degree angle in the diagram as either ∠EBC or ∠CBE.
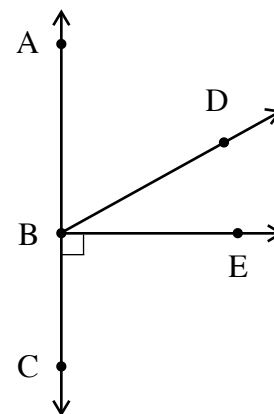
While these definitions may seem simple, they can form the basis of some challenging questions, like this one:

Sample Question #1

A, B, and C are distinct points. Which of the following establishes that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are the same ray?

A. Points A, B, and C are colinear.

B. $\overline{AB} + \overline{BC} = \overline{AC}$

C. It is farther from Point A to Point B than from Point B to point C.

D. Points A and B are conlinear, but Points A and C are not.

In Sample Question #1, Answer choice A does not prove the rays to be the same or different, as seen in the following diagram:
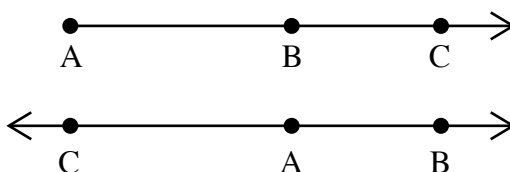
In both figures, A, B, and C are collinear, satisfying the condition of answer choice A. But in the top figure, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are the same ray, since C is on $\overrightarrow{AB}$; in the bottom figure, since C is not on $\overrightarrow{AB}$, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are distinct rays.

Answer choice C does not provide any information relevant to determining whether $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are the same ray or not. As you can see in the diagrams above, the distance between the three points could be varied without affecting the name of the ray(s) they form.

Answer choice D is not correct. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ can't be the same ray if A and B are colinear, but A and C are not. In this case, the three points would form a triangle.

The correct answer is B. If $\overline{AB} + \overline{BC} = \overline{AC}$, this means that point B must fall between points A and C.

We can prove this using segment addition and contradiction. Consider the bottom arrow diagram, in which A falls between C and B and $\overrightarrow{AB}$ is different from $\overrightarrow{AC}$.

$$\overline{AB} + \overline{AC} = \overline{BC}$$
$$\overline{AB} + \overline{AC} + \overline{AB} = \overline{BC} + \overline{AB}$$
$$\overline{AB} + \overline{BC} = \overline{AC} + 2(\overline{AB}) > \overline{AC}$$

This result contradicts what we're told in answer choice B, meaning that answer choice B tells us that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are the same ray.

Complementary and Supplementary Angles

Adjacent angles that sum to 90Yare called complementary angles.
Adjacent angles that sum to 180Yare called supplementary angles.

Memorizing these definitions is crucial to being able to solve line diagram questions on the GRE Quantitative section, because sometimes, not all of the information you need will be contained in the diagram itself; the question stem might reveal that two of the angles are either complementary or supplementary, and you'll need to be able to recognize the mathematical significance of such a clue.

Consider Sample Question #2, which demonstrates just such a situation:

Sample Question #2

What is the measure of angle B?

A. 60Υ

B. 70Υ

C. 110Υ

D. 150Υ

110Υ

B
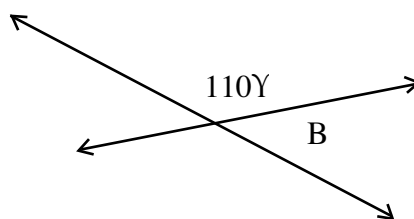
One thing you can assume on the GRE Quantitative section is that in a line diagram, a line that appears to be a straight line is a straight line. This may seem like a minor point, but it's all you need to answer the above question. You can think of a straight line as forming a 180Υangle on each of its sides, and when such a line is intersected, the 180Υangle is split into multiple angles that still sum to 180Υ. The angle labeled as being 110Υand angle B are supplementary angles—they are adjacent and must necessarily sum to 180Υ. Thus, you can find angle B by subtracting 110Υfrom 180Υ:

$$180° - 110° = 70°$$

The measure of angle B is 70Υ, so the correct answer is B.

Let's try another sample question that tests the other relationship we touched upon: complementary angles.

Sample Question #3

m∠ABD = 60˚. What is m∠DBE?

A. 30Υ

B. 45Υ

C. 60Υ

D. 90Υ

D   E

A   B   C
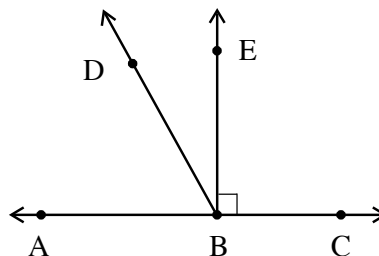
There's a lot more going on in Sample Question #3's diagram, but the question still boils down to recognizing the significance of a simple mathematical relationship. Let's take stock of what we know. If we only knew that m∠ABC = 60°, we wouldn't be able to solve this problem; however, we're also told that m∠CBE = 90°. Don't scan the question stem for that information—it's not there. It's in the diagram—note the right angle symbol. Now we can solve the problem! m∠ABD, m∠DBE, and m∠EBC must necessarily sum to 180° because they are all contained on one side of AC, and you can think of straight lines as being 180° angles. We can write the following equation:

m∠ABD + m∠DBE + m∠EBC = 180°

We know that m∠ABD = 60° and that m∠EBC = 90°, so we can substitute in those values to create an equation with a single variable that we can then solve for.

60°+m∠DBE + 90°= 180°

150°+m∠DBE = 180°

m∠DBE = 30°

The correct answer is A.

Parallel and Perpendicular Lines

If you are told that a pair of lines is parallel or perpendicular, you can be confident that this information will be relevant to any questions you are being asked about these lines or the angles they form. A number of rules become apparent when you combine angle measurement problems with parallel and perpendicular lines.

Let's begin with perpendicular lines, the simpler case. Perpendicular lines by definition form four 90° angles. If you are asked to find the measurement of an angle in one "quadrant" of a pair of perpendicular lines, you can do so by using the fact that all of the angles in each of the spaces between the lines must sum to 90°. In the diagram below, AB ⊥ CD. This establishes the following relationships between the angles:
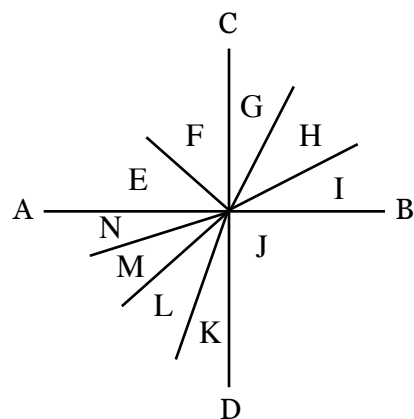
∠E + ∠F = ∠G + ∠H + ∠I = ∠J = ∠K + ∠L + ∠M + ∠N = 90°

A Shortcut: Vertical Angles

Intersecting lines create vertical angles: a distinct, easily identifiable relationship between pairs of angles that are formed by the intersection of two lines and have exactly one angle between them. (Put more casually, they are "vertical" from one another.) Two intersecting lines form two pairs of vertical angles. In the following diagram, ∠A and ∠C are vertical angles, and ∠B and ∠D are also vertical angles. Each angle in a pair of vertical angles has the same angle measurement as the other in the pair. We can prove this relationship by considering the following scenario.
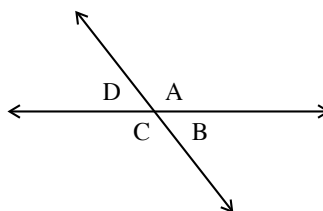
A straight line has an angular measurement of 180˚. Because of this, we know that m∠A and m∠B must sum to 180˚, and that m∠C and m∠D must also sum to 180˚. Furthermore, m∠D and m∠A must sum to 180˚, and m∠B and m∠C must also sum to 180˚—we're dealing with two straight lines, after all, so this makes sense.

$$m\angle A + m\angle B = 180°$$
$$m\angle B + m\angle C = 180°$$
$$m\angle C + m\angle D = 180°$$
$$m\angle D + m\angle A = 180°$$

Because of the tight-knit relationship between these angles, if you define the value of one of them, you define the value of the other three. Let's say that a question stem tells us that m∠A = 120˚. If we fill in that information to the equations listed above, we get:

$$120° + m\angle B = 180°$$
$$m\angle B + m\angle C = 180°$$
$$m\angle C + m\angle D = 180°$$
$$m\angle D + 120° = 180°$$

We can now solve for m∠B and m∠D. From the equations, you can see that they both result in a measurement of 60˚. Note that they're vertical angles and have the same measurement, as we just found. We know m∠B and m∠D, so we can replace the middle two equations with the following:

$$60° + m\angle C = 180°$$
$$m\angle C + 60° = 180°$$

Solving either of these equations reveals that m∠C = 120˚ —just like m∠A! These relationships holds true for any intersecting lines. Consider vertical angles when you work with line diagrams.

Sample Question #4

∠FCB and which angle are examples of a pair of vertical angles?
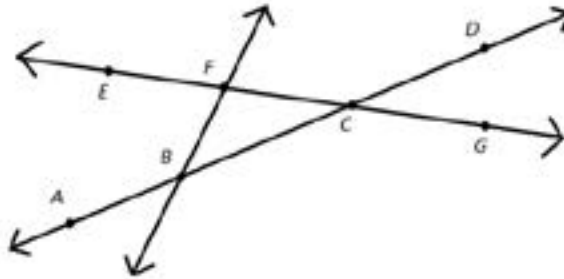
A. ∠DCG

B. ∠DCF

C. ∠ECB

D. ∠FCB

Two angles are vertical if they have the same vertex and if their sides form two pairs of opposite rays. In Sample Question #4, the correct choice will have its vertex at point C, which is the vertex of ∠FCB. Its rays will be the rays opposite CF and CB, which are CG and CD, respectively. The angle that fits this description is ∠DCG, so answer choice A is correct.

Another Shortcut: Transverse Lines + Vertical Angles

Adding consideration of transverse lines to your knowledge of vertical angles can help you easily discern even more information contained in a line diagram at first glance. A transverse line is a line that intersects both of a pair of parallel lines. EF is a transverse line in the following diagram, as it intersects the parallel lines AB and CD.
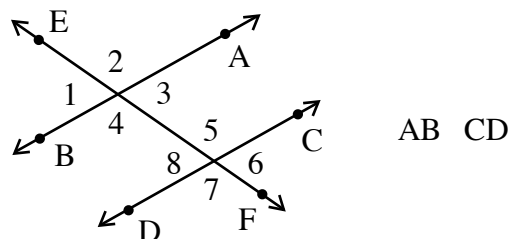
What does the fact that EF is transverse tell us? A lot. Because EF is a straight line and AB and CD are parallel, EF crosses AB at the same angle at which it crosses CD. This means that the measures of certain pairs of vertical angles at the intersection are going to be identical to certain other pairs of vertical angles.

$$m\angle 1 = m\angle 3 = m\angle 6 = m\angle 8$$
$$m\angle 2 = m\angle 4 = m\angle 5 = m\angle 7$$

"Corresponding angles" are pairs of angles in the same relative ("corresponding") position formed by each of the parallel lines and the transversal. ∠1 and ∠8 are corresponding angles, as are ∠2 and ∠5, ∠3 and ∠6, and ∠4 and ∠7. Note that each of these pairs of angles is identical in measurement.

Keep these particular relationships in mind when dealing with any diagram that presents you with a transverse line!

Triangles and Quadrilaterals

Keep in mind, the lines that make up a line diagram can form familiar shapes with rules governing their angles—rules you can make use of to solve problems. If three lines form a triangle, the interior angles of that triangle must sum to 180Y. Equilateral triangles contain three 60Y angles, and isosceles triangles are 45-45-90 triangles.

If four lines form a quadrilateral, the interior angles must sum to 360Y. Don't get confused and assume this rule only holds true for regular rectangles and squares, where each of the shape's interior angles is 90Y.

It's easy to forget about these rules as you may have trained yourself to consider them only in other specific scenarios—when answering questions about the length of a triangle's hypotenuse, for example. Combining your knowledge of mathematics from different contexts may be necessary in this case to solve some of the line problems the GRE Quantitative section presents.

Solving Line Diagrams for Missing Angles

Let's try some sample problems that require you to work with line diagrams. The following question asks you to use your knowledge of geometric relationships to find the measure of an angle.

Sample Question #5

Refer to the diagram below. m∠DCG = 43°. What is the measure of ∠FCB?

A. 43ϒ

B. 47ϒ

C. 133ϒ

D. 137ϒ
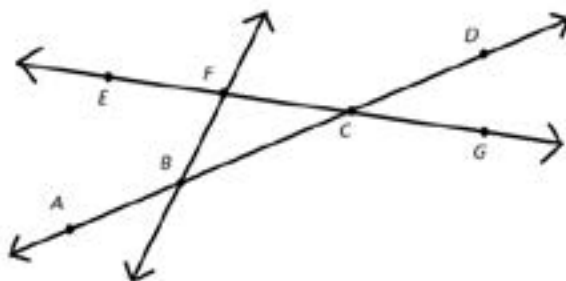
In Sample Question #5, ∠FCB and ∠DCG are vertical angles, which must be congruent, so if we know m∠DCG = 43°, then m∠FCB = 43° also. The correct answer is A!

Sample Question #6

Statement A: m∠3 + m∠4 + m∠5 = 131

Statement B: m∠6 + m∠7 + m∠8 = 131

Referring to the figure and statements, evaluate m∠1 + m∠2.

A. 90°

B. 98°

C. 104°

D. 110°
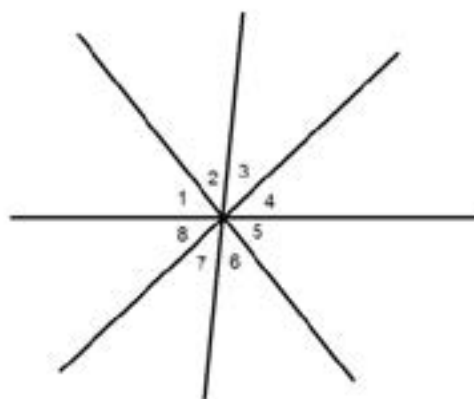
Let's consider the first statement in Sample Question #6 by itself. $\angle 2$, $\angle 3$, $\angle 4$, and $\angle 5$ together form a straight angle, so their degree measures total $180°$.

$$m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 180°$$
$$m\angle 2 + 131° = 180°$$
$$m\angle 2 = 49°$$

Without further information, no other angle measures, including that of $m\angle 1$, can be found. So, let's consider the second statement. $m\angle 1$, $m\angle 6$, $m\angle 7$, and $m\angle 8$ together form a straight angle, so their degree measures total $180°$.

$$m\angle 1 + m\angle 6 + m\angle 7 + m\angle 8 = 180°$$
$$m\angle 1 + 131° = 180°$$
$$m\angle 1 = 49°$$

We've found both $\angle 1$ and $\angle 2$, so all we have to do is add them together to solve the problem:

$$m\angle 1 + m\angle 2 = 49° + 49° = 98°$$

The correct answer is B.

Sample Question #7

Refer to the diagram shown at right.
   OPQ is an equilateral triangle. Evaluate $m\angle 3 + m\angle 4$.

A. $60°$

B. $75°$

C. $120°$

D. $150°$
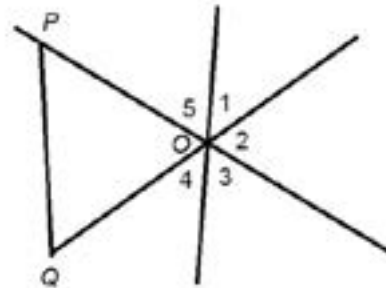
In Sample Question #7, the angles of an equilateral triangle all measure $60Y$, so $\angle POQ = 60°$. $\angle 3$, $\angle 4$, and $\angle POQ$ together form a straight angle, so we can write the following equation:

$$m\angle 3 + m\angle 4 + m\angle POQ = 180°$$
$$m\angle 3 + m\angle 4 + 60° = 180°$$
$$m\angle 3 + m\angle 4 = 120°$$

Note that we don't have to find the individual measurements of $\angle 3$ and $\angle 4$, just their sum. The correct answer is C, $120Y$.

Solving Line Diagrams for Linear Relationships

Some line diagrams may not provide you with information about parallel and perpendicular lines and ask you to calculate the measure of a missing angle. Instead, they may ask you to work "in reverse" and provide you with information about the angles in a diagram in order to ask about which lines must necessarily be parallel or perpendicular based on those angles' relationships. Let's take a look at an example of this type of question.

Sample Question #8

Statement A: $m\angle 2 = 89°$

Statement B: $m\angle 3 = m\angle 6$
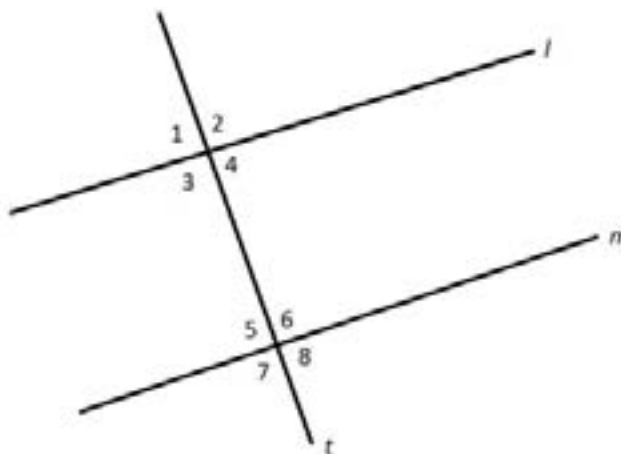
Given the above figure and two statements, which of the following is true?

A. Line m is perpendicular to line t

B. Line m is not perpendicular to line t

C. $m\angle 4 = m\angle 7$

D. $m\angle 1 = m\angle 6$

In Sample Question #8, Statement A alone establishes by definition that line l is not perpendicular to line t, but it does not establish any relationship between line m and line t.

By Statement B alone, since alternating interior angles are congruent, line l is parallel to line m. We can't yet draw any conclusion about the relationship of line t, since we haven't found the actual measures of the angles.

By Statement B, line l is parallel to line m. $\angle 2$ and $\angle 6$ are corresponding angles formed by a transversal across parallel lines, so $m\angle 6 = m\angle 2 = 89°$. $\angle 6$ is not a right angle, so line m is not perpendicular to line t and B is the correct answer.

Sample Question #9

Consider the line diagram shown at right. Which of the following pieces of information is enough to prove that AB is parallel to CD?
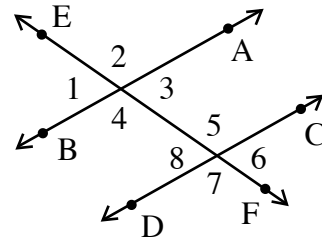
A. $m\angle 1 = m\angle 3$

B. $m\angle 4 = 120°$ and $m\angle 8 = 60°$

C. $m\angle 5 = 120°$ and $m\angle 6 = 60°$

D. $m\angle 2 = m\angle 6 = 100°$

In Sample Question #9, we know that $m\angle 1 = m\angle 3$ because they are vertical angles, so this information doesn't tell us anything about the relationship between AB and CD. The same thing can be said of answer choice C; $\angle 5$ and $\angle 6$ form a straight-angle pair that sums to 180°, but knowing the specific angle values associated with each doesn't tell us anything about whether AB and CD are parallel.

Answer choice D provides information that, if true, means that AB is not parallel to CD. For the lines to be parallel, corresponding angles would need to have the same measurements. That is, $\angle 2$ would need to have the same measurement as $\angle 5$, and $\angle 3$ would need to equal $\angle 6$. If $m\angle 2 = m\angle 6 = 100°$, $m\angle 3$ and $m\angle 5$ would each equal 80°. This means that the corresponding angles would have different measurements, meaning that AB would not be parallel to CD.

The correct answer choice is B. By learning that $m\angle 4 = 120°$ and $m\angle 8 = 60°$, we have enough information to determine that $m\angle 1 = 60°$ (because it forms a straight angle with $\angle 4$) and that $m\angle 7 = 120°$ (since it forms a straight angle with $\angle 8$). At this point, we can see that the pairs of corresponding angles, $\angle 1$ and $\angle 8$ and $\angle 4$ and $\angle 7$, match in value. This means that AB is parallel to CD.

Defining Polygons

A polygon is defined as a figure in two-dimensional space that possesses three or more sides. Several examples of polygons are depicted and named below. The GRE will require you to understand and manipulate the relationships between the sides and angles of polygons in order to calculate concepts such as area and perimeter. Problems might require you to pick up on subtle clues conveyed by the definition of a particular kind of shape. In order to be equipped to solve complex and multi-step problems on test day, you need to master the rules and jargon of polygons.
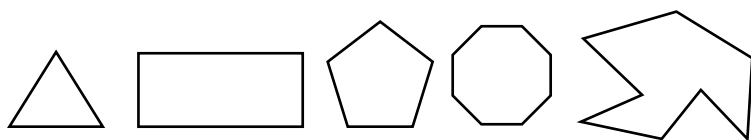
Number of Sides	Name of Shape
Three	Triangle
Four	Quadrilateral
Five	Pentagon
Six	Hexagon
Seven	Heptagon
Eight	Octagon
Nine	Nonagon
Ten	Decagon
Eleven	Hendecagon
Twelve	Dodecagon
n	n-gon

Types of Triangles

Triangles may also be defined by their largest angles.
If a triangle contains an angle that is greated than 90°, it is called an **obtuse** triangle. If it contains a 90° angle, or a right angle, it is called a **right** triangle, and many other specific rules apply to it; we cover these in our *Right Triangle* lesson. Finally, if each of a triangle's angles are smaller than 90°, the triangle is called an **acute** triangle.

Triangles can be also categorized by their side lengths, their angles, or both. If we define triangles by how their side lengths relate to one another, there are a few options. A triangle can have sides that are all equal in length to one another; such a triangle is called **equilateral**. A triangle might have two sides that are the same length and one side that is different; this kind of triangle is called **isosceles**. Finally, a triangle's sides all might be different lengths. Such a triangle is referred to as **scalene**.
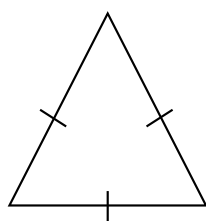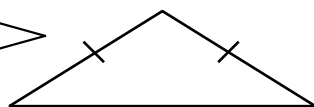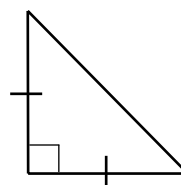
Equilateral Triangle
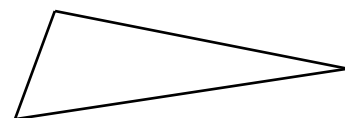(Acute by definition)
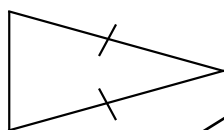
Acute Isosceles Triangle

Obtuse Isosceles Triangle

Right Triangle
(Isosceles by definition)

Obtuse Scalene Triangle

The information that a given triangle is equilateral or isosceles conveys more information than you might otherwise notice. If we know something about how a triangle's sides relate, we also know something about its angles. This is because the length of a triangle's side and the size of the angle across from that side are in a definite ratio.

Since the sides of an equilateral triangle are all equal, this tells us that the angles must all be equal as well. Since every triangle has an interior angle sum of 180˚, we can divide this by three to find the measure of any interior angle in an equilateral triangle: 180˚ divided by 3 is 60˚. This is valuable information to have when solving a problem. It works in reverse as well: if you are told or determine that a triangle has three 60˚ interior angles, that triangle must be equilateral.
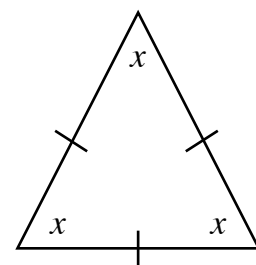
$$x + x + x = 180°$$
$$3x = 180°$$
$$\frac{3x}{3} = \frac{180°}{3}$$
$$x = 60°$$

We can glean similar information from the definition of an isosceles triangle. Because the sides and angles are in proportion to one another, if two of the triangle's sides are the same length, we know that two of the triangle's angles must be the same; however, unlike with equilateral triangles, we can't assign a specific degree measure to those angles. We just know that they have to be equal. If we know the third angle, though—the one that matches the unique side—we can solve for the measure of each of the other angles.
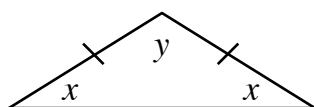
$$x + x + y = 180°$$
$$y = 115°$$
$$x + x + 115° = 180°$$
$$2x + 115° = 180°$$
$$(2x + 115°) - 115° = 180° - 115°$$
$$2x = 65°$$
$$x = \frac{65°}{2} = 32.5°$$

Consider the triangle shown to the left. We know that the angles across from each of the equal sides must be equal, so let's call each of them x. Let's call the unique angle y. Say we're told that $y = 115°$. We can define the rest of the angles in the triangle by using the fact that they all must sum to 180˚, and that the two angles labeled x must be the same. As you can see, each of the angles labeled x must be 32.5˚.

We can even learn something about a scalene triangle's angles through its definition. Since all of a scalene triangle's sides are unique, its angles all have to be unique as well! To figure out one of the angles, we would need to know two of the others, and subtract their sum from 180˚.
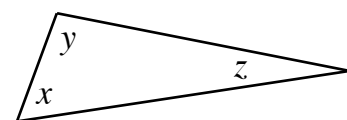
$$x + y + z = 180°$$

Triangle Inequality Theorem

One more rules follows from the fact that a triangle's side and the angle opposite it are in proportion with one another. This rule is called the Triangle Inequality theorem, and it states that the sum of any two sides of a triangle must be greater than the third side. The following relationships are true for **all** triangles; A, B, and C refer to their side lengths.

$$A + B > C$$
$$B + C > A$$
$$C + A > B$$

Sample Question #1 requires application of the Triangle Inequality theorem. If two sides of a triangle are known and the angles of the triangle are unknown, the length of the missing side is limited by the difference and sum of the other two sides.

$$|a - b| < c < |a + b|$$
$$|5 - 16| < c < |5 + 16|$$
$$11 < c < 21$$

For a triangle with sides and , there is no way a side could be 22 cm. Thus, Quantity B is greater, and B is the correct answer.

Sample Question #2 deals with angles instead of sides, and requires you to use your knowledge of different types of triangles. The triangle on the left is an isosceles triangle with one labeled angle, and the one on the right is a scalene triangle with two labeled angles. This gives you enough information to calculate x and y.

For the isosceles triangle:

$$x + x + 20° = 180°$$
$$2x + 20° = 180°$$
$$(2x + 20°) - 20° = 180° - 20°$$
$$2x = 160°$$
$$\frac{2x}{2} = \frac{160°}{2}$$
$$x = 80°$$

For the scalene triangle:

$$y + 30° + 65° = 180°$$
$$y + 95° = 180°$$
$$(y + 95°) - 95° = 180° - 95°$$
$$y = 85°$$

$x < y$, so $A < B$, so B is the correct answer.

Sample Question #1

One of a triangle's sides is 5 cm long. Another is 16 cm long.
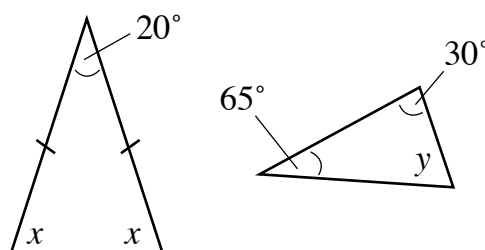
Quantity A	Quantity B
The length of the triangle's missing side	22 cm

    A. Quantity A is greater.
    B. Quantity B is greater.
    C. The two quantities are equal.
    D. The relationship cannot be determined from the information given.

Sample Question #2
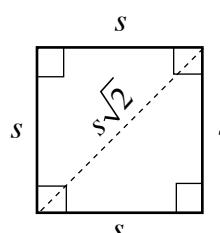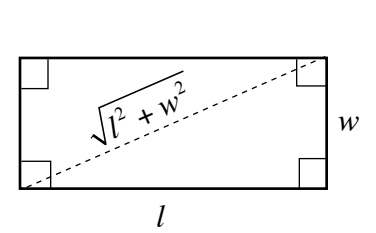

(Drawing not to scale.)
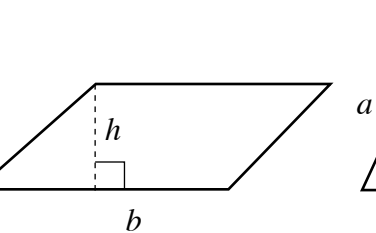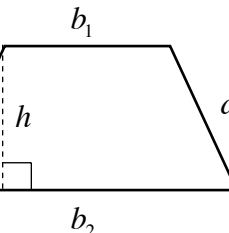
Quantity A	Quantity B
x	y

    A. Quantity A is greater.
    B. Quantity B is greater.
    C. The two quantities are equal.
    D. The relationship cannot be determined from the information given.

Types of Quadrilaterals

A quadrilateral is a shape with four sides whose interior angles sum to 360˚. For the GRE Quantitative section, you should be familiar with the quadrilaterals shown below.

Square

$A = s^2$

$P = 4s$

$d = s\sqrt{2}$

Rectangle

$a = l \times w$

$p = 2l + 2w$

Parallelogram

$A = b \times h$

$P = 2w + 2l$

Trapezoid

$A = \dfrac{b_1 + b_2}{2} \times h$

$P = a + a + b_1 + b_2$

Those quadrilaterals with sloped sides—parallelograms and trapezoids—use a height measurement instead of the length of the sloped side in their area calculations. Be careful to distinguish between height and side lengths for these shapes, as the test-writers may try to trick you into treating these shapes like rectangles and squares—shapes whose heights are their side lengths.

If you find yourself needing to calculate the side length of a slanted side and are not provided with the perimeter of the shape by which to determine it using subtraction, you may be able to calculate it by considering an indicated part of the base and the height as a *right triangle*.

Sample Question #3 looks at parallelograms. A parallelogram must have two sets of congruent/parallel opposite sides; therefore, this parallelogram must have two sides with a measurement of 74 mm and two base sides each with a length of 95 mm. To solve for the missing side, work backwards using the perimeter formula:

Perimeter $= 2(A + B)$, where A and B are the measurements of adjacent sides.

$338 = 2(95 + b)$

$338 = 190 + 2b$

$2b = 338 - 190 = 148$

$b = \dfrac{148}{2} = 74$

Thus, the solution is 74 mm, and the answer is D.

> ### Sample Question #3
>
> A parallelogram has a base of 95 mm. The perimeter of the parallelogram is 338 mm. Find the length for an adjacent side to the base.
>
> **A.** 36 mm
>
> **B.** 42 mm
>
> **C.** 72 mm
>
> **D.** 74 mm

Sample Question #4 requires you not only to know two formulas, but also to relate them to one another directly. Let's diagram what's going on in this question before we start. On test day, you could quickly sketch this out.

$$s^2 = \frac{s+4}{2} \times s$$
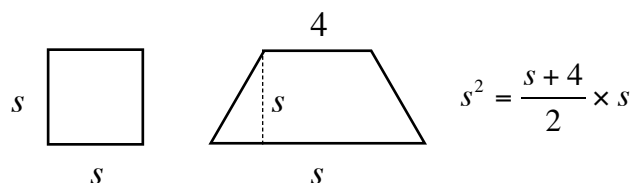
Figuring out how to approach this problem is half of the battle, if not more. Since the areas of the shapes are equal, we can write out the area formula for each shape and setting them equal to one another as shown above at right. As s forms a part of both the square and the trapezoid, it appears on each side of the equation. It's the only variable in the equation, so we can solve for it algebraically. s appears on both sides of the equation, though, so we'll need to isolate it. Taking the square root of the entire right side and then having to deal with all of its contents seems difficult, so let's isolate the variable on the right side. Before we start, though, let's rewrite s^2 as $s \times s$. This makes the next step—dividing each side by s—clearer.

Sample Question #4

A trapezoid has a base of length 4, another base of length s, and a height of length s. A square has sides of length s. What is the value of s such that the area of the trapezoid and the area of the square are equal?

   A. $\sqrt{2}$

   B. $2\sqrt{2}$

   C. 2

   D. 4

$$s^2 = \frac{s+4}{2} \times s$$

$$s \times s = \frac{s+4}{2} \times s$$

$$\frac{s \times s}{s} = \frac{\frac{s+4}{2} \times s}{s}$$

$$s = \frac{s+4}{2}$$

At this point, we can multiply each side by 2.

$$2(s) = 2\left(\frac{s+4}{2}\right)$$
$$2s = s+4$$

At this point, we can subtract s from both sides, leaving it only on one side of the equation. Since we find that $s=4$, the correct answer is D!

$$2s = s+4$$
$$(2s) - s = (s+4) - s$$
$$2s - s = 4$$
$$s = 4$$

Regular Polygons

A regular polygon is any connected figure whose side lengths are all equal and whose angles are all equal. A square for example, is considered a regular polygon. The number of side lengths is not restricted; thus, as long as a figures' sides and angles are equal, it is a regular polygon. There is a simple formula that will calculate the sum of the interior angles of any polygon. Once the sum of the interior angles is known, any and all of the angles within the regular polynomial can be found. The formula to calculate the sum of the interior angles is shown in the box on the upper-right of this page.

There are two quantities involved in the formula given above for which the GRE can ask you to solve. The straightforward approach is presented in Sample Question #6; as part of the problem, you need to use a specified number of sides to solve for the sum of the interior angles. Sample Question #7 is a similar but "inverse" problem: in this one, you're given the interior angle sum and asked to solve for the number of sides. Neither of these demand extremely complex algebra, just use of the formula. Let's tackle them now!

We're dealing with a "regular decagon" in Sample Question #5. The prefix "deca-" means *ten*—just think of "decameters" or "decaliters." So, a "regular decagon" is a shape that has ten sides all of the same length; one is shown above.

Sum of interior angles = $180(n-2)$

n = number of sides
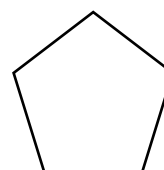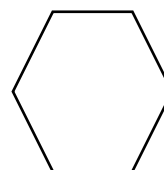
$n = 3$ $n = 4$ $n = 5$ $n = 6$
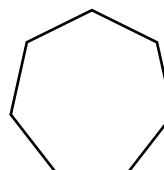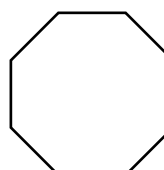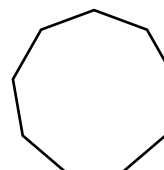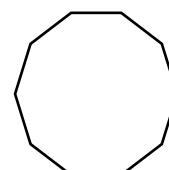
$n = 7$ $n = 8$ $n = 9$ $n = 10$

Sample Question #5

What is the measure of one interior angle in a regular decagon?

A. 144°

B. 160°

C. 172°

D. 180°

$180°(n-2)$

$180°(10-2) = 180°(8) = 1440°$

$\dfrac{1440°}{n} = \dfrac{1440°}{10} = 144°$

So, we know that $n = 10$, and we need to figure out the measure of one interior angle. Read carefully—that's a single interior angle, not the sum of all of them, which is what the formula gives; however, we know how many sides a shape has, so we know that it has that many interior angles as well. Thus, we're be able to divide the interior angle sum by the number of sides to find the correct answer: A, 144°.

Let's use that same formula to solve this next sample problem, only in reverse. In Sample Question #6, we're given the total sum of the interior angles of a regular shape and asked to calculate how many sides it has. Keep in mind that this formula only works on regular shapes! If the shape weren't specified to be regular, we'd be unable to answer the question definitively. Its sides might not be the same length, and thus its angles might be different.

A regular polygon's interior angles sum to $2160°$. How many sides does it have?

    A. 12
    B. 13
    C. 14
    D. 15

$$180°(n-2) = 2160°$$
$$\frac{180°(n-2)}{180°} = \frac{2160°}{180°}$$
$$n - 2 = 12$$
$$(n-2) + 2 = 12 + 2$$
$$n = 14$$

Don't forget to add the 2 after you divide $2160°$ by $180°$! The result if you forget this step, 12, is listed as an answer choice. Since $n = 14$, C is the correct answer.

Right Triangles

As we reviewed in the *Defining Polygons* lesson, triangles are three-sided shapes with internal angles that sum to 180°. Right triangles are a particular type of triangle in which one of the three angles is exactly 90°— a right angle. The other two interior angles will sum to 90°, and when added to the right angle, the three will sum to 180° like in any other triangle.

The Pythagorean Theorem

If you're asked for the length of a side of a right triangle, whether in the context of an area or perimeter problem or not, consider if you know or can find the lengths of the other two sides before assuming that you'll need to use trigonometry. The Pythagorean theorem, shown at right, can save you unnecessary calculations. It is used on right triangles only; a and b are the side lengths of the triangle, and c is the length of the hypotenuse, the longest side of the triangle; in a right triangle, this is always the side across from the right angle.

$$a^2 + b^2 = c^2$$

Let's take a look at Sample Question #1. We can predict that the hypotenuse of this triangle must be more than 3 cm and less than 11 cm using the law that the length of one side of a triangle cannot exceed the sum of the lengths of the other two sides. This simple rule can help you ignore answer choices very quickly should you find yourself needing to guess on a problem related to triangle side lengths. Also, it can help you make sure that a calculated answer falls in a realistic range of possibilities. Considering this, we can tell that D isn't correct, since the square root of 144 is 12, and that's above the range into which our answer can fall.

> **Sample Question #1**
>
> A right triangle has leg lengths of 4 cm and 7 cm. What is the length of its hypotenuse?
>
> **A.** $\sqrt{50}$
>
> **B.** $\sqrt{65}$
>
> **C.** $\sqrt{95}$
>
> **D.** $\sqrt{150}$

Let's get to calculating the length of that hypotenuse using the Pythagorean theorem. We're given the lengths of the triangle's legs, so $a = 4$ cm and $b = 7$ cm. (Note that it doesn't matter which leg we call a and which we call b as long as we are consistent). Having defined our variables, we can substitute them into the Pythagorean theorem and solve for the length of the hypotenuse.

Don't worry if your answer is a square root; that just means that the sum of the triangle's legs wasn't a *perfect square*. If you find it difficult or unintuitive to work with square roots in this way, you may also want to practice *solving circle problems in terms of π*. The length of this triangle's hypotenuse is $\sqrt{65}$ cm, so B is correct.

$$a^2 + b^2 = c^2$$

$$(4 \text{ cm})^2 + (7 \text{ cm})^2 = c^2$$

$$16 \text{ cm}^2 + 49 \text{ cm}^2 = c^2$$

$$65 \text{ cm}^2 = c^2$$

$$\sqrt{65 \text{ cm}^2} = \sqrt{c^2}$$

$$\sqrt{65} \text{ cm} = c$$

Special Right Triangles

Certain right triangles are referred to as "special" because of their particular combinations of angles have set side-to-length ratios. What this means is that if a triangle is a special right triangle, then you don't need to know the lengths of two of its sides to calculate the length of its third side. You just need to identify the type of special right triangle it is and the length of one of its sides. That's all the information you need!

There are two types of special right triangles: **30-60-90 triangles** and **45-45-90 triangles**. (This second type can also be called an isosceles right triangle). These two special triangles have different ratios between their sides. 30-60-90 special right triangles have a side ratio of $1:2:\sqrt{3}$ in the form short side : hypotenuse : long side. 45-45-90 special right triangles each have a side ratio of $1:1:\sqrt{2}$ in the form side : side : hypotenuse. Certain problems may be set up to require you to use these ratios. In others problems, making use of the ratios might be optional, but using them can make your calculations easier and faster.
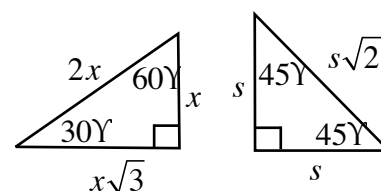
Special Right Triangles

So how exactly do you make use of these ratios? You set up a proportion. First, create a fraction using the information you have and the information you want; then, you set it equal to a fraction made up of the relevant sides' ratio. At this point, you can cross-multiply and solve. Let's practice doing this in the next two sample problems.

In Sample Question #2, we're only given the length of one side of this triangle, but look at the angles. Since this triangle contains a 90° angle and a 30° angle, it must be a 30-60-90 triangle. That means we can use our special right triangle ratios to set up a proportion. We know that one side is 10.5 cm long, and that that side must be across from the 60° angle. In the side ratio, that side is $x\sqrt{3}$. Let's calculate the length of side a, the side across from the 30° angle. That side is x in the side ratio. Let's set up our proportion with the lengths of the triangles' sides above their ratio variables.

$$\frac{10.5 \text{ cm}}{x} = \frac{a}{x\sqrt{3}}$$

Sample Question #2

Find the lengths of sides a and c of the triangle shown at right.
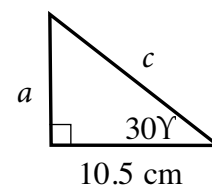
A. $a = 7.5\sqrt{3}$ cm
$c = 21$ cm

B. $a = 10.5\sqrt{3}$ cm
$c = 21$ cm

C. $a = 7.5\sqrt{3}$ cm
$c = 30$ cm

D. $a = 10.5\sqrt{3}$ cm
$c = 30$ cm

We could also set up our fraction with information about the triangle on one side and information aout the side ratios on the other.

$$\frac{10.5 \text{ cm}}{a} = \frac{x}{x\sqrt{3}}$$

No matter which way you set up your proportion, you get the same answer, as the next step is cross-multiplying, and cross-multiplying yields the same thing in both proportions.

$$10.5 \times x\sqrt{3} = c \times x$$

Now we can divide each side by x to remove that variable from the equation.

$$10.5 \times \sqrt{3} = c$$

This answer may look a bit convoluted, but it is correct! Now we need to calculate c. We could use the Pythagorean theorem at this point, since we know the lengths of two sides, but our last answer looks like it would be difficult to work with, so let's set up another proportion. We can reuse our first fraction, but need to set it equal to another. Side c is across from the 90° angle in the triangle, making it the $2x$ side in the ratio.

$$\frac{10.5 \text{ cm}}{x} = \frac{c}{2x}$$

Cross-multiply and solve for the correct answer.

$$10.5 \text{ cm} \times 2x = c \times x$$
$$10.5 \text{ cm} \times 2 = c$$
$$21 \text{ cm} = c$$

B is the correct answer!

Next, let's try a sample problem that involves an isosceles special right triangle. A similar approach can be used, but the ratios are different.

Sample Question is a little bit tricky: there's a bit of information it is subtly hiding. It gives you the length of "one leg" of the triangle and tells you that the triangle is an isosceles right triangle; isosceles triangles by definition have two legs

Sample Question #3

One leg of an isosceles right triangle is 5 m long. What is the length of this triangle's hypotenuse?

    A. 5 m

    B. 10 m

    C. $5\sqrt{2}$ m

    D. $5\sqrt{3}$ m

that are the same length, so that means that you know the length of both of the triangle's legs!

   At this point, you could take your pick of methods and use either the Pythagorean theorem or special right triangle ratios to solve. Let's practice using the ratios. Each of the triangle's legs are across from a $45°$ angle, so they'll be labeled s in the ratio. The hypotenuse for which we're solving is across from the right angle, so it will be labeled $s\sqrt{2}$ in the ratio. Let's set up our proportion.

$$\frac{5 \text{ m}}{s} = \frac{c}{s\sqrt{2}}$$

Cross-multiplying, we get the following: $5 \text{ m} \times s\sqrt{2} = s \times c$

We can divide by s to remove that variable from the equation and find our answer.

$$5 \text{ m} \times \sqrt{2} = c$$
$$5\sqrt{2} \text{ m} = c$$

C is the correct answer!

Further Application: Diagonal of a Square

If you're asked to calculate the diagonal of a square, don't scramble for a formula. Knowing how to work with special right triangles means that you can handle such a question with ease. Consider what shapes you create if you divide a square along its diagonal. You get two triangles, but these are special triangles with specific proportions. A square by definition has four right angles, so guess what's in the corner of each of the two triangles? A right angle! Furthermore, consider the side lengths of these triangles. Each side of a square is equal in length to its other three sides, so the legs of each of these triangles are identical. That means that we're dealing with isosceles right triangles. One more key point: consider the angles that are in play here. The diagonal divided two 90° angles in half, so the triangles formed are 45-45-90 triangles. We can apply the ratios associated with 45-45-90 triangles in this scenario to calculate the length of the diagonal so long as we know one side of the square, or calculate one side of the square so long as we know the length of the diagonal. Keep on the lookout for ways in which geometry problems can be presented in different ways; it may help you switch from being completely lost as to how to approach a problem to solving it as an example of familiar principles!
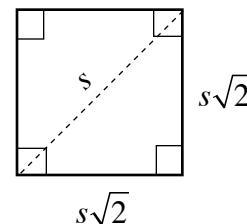
Sample Question #4 is a multi-layered word problem, but at its core, it's a diagonal of a square problem. After you find the length of the square's diagonal, you need to use Herbert's provided running speed to convert the distance into the time it takes him to dash across the garden. We're working with the diagonal of a square, so we can use the ratio of sides for a 45-45-90 triangle: $s : s : \sqrt{2}s$. Let's set up our proportion and then cross-multiply.

$$\frac{200 \text{ m}}{s} = \frac{c}{s\sqrt{2}}$$

$$200 \times s\sqrt{2} = c \times s$$

$$200 \times \sqrt{2} = c$$

We know that the hypotenuse of the triangle, and the diagonal of the square, is $200\sqrt{2}$ m. Now we need to calculate how long it took Herbert to run this distance at a pace of 7 meters per second. We can do this by multiplying our distance by the ratio of distance to time so that we end up with a result with seconds as the units. Just make sure the units cancel! In this case, we need to flip the fraction upside down to get the correct answer, and the units make this apparent. The correct answer is A.

Sample Question #4

Herbert is in charge of maintaining a square flower garden in a public park with edges of 200 m, but rabbits keep nibbling at it. Herbert chases a rabbit directly from one corner of the garden to the other while running at 7 meters per second. Approximately how long does it take Herbert to chase the rabbit out of his garden in this manner? Round your final answer only.

A. 40.41 s
B. 41.54 s
C. 42.35 s
D. 43.07 s

$$200\sqrt{2} \text{ m} \times \left(\frac{1 \text{ s}}{7 \text{ m}}\right) = \frac{200\sqrt{2}}{7} \approx 40.41 \text{ s}$$

Congruent Figures and Similar Figures

Geometric shapes can be related to one another in one of two specific ways. To compare shapes, you need to consider corresponding points and sides—those that are in the same relative location on each triangle. In the sample triangles shown below, point A corresponds with point D, point B corresponds with point E, and thus side AD corresponds with side DE.

Shapes whose corresponding sides and angles are all equivalent are called **congruent**. Congruency is delineated in mathematical shorthand by the symbol ⊕. Thus, since all of the corresponding sides of Triangle ABC are identical in length to their corresponding sides in Triangle DEF, and all of the angles in Triangle ABC are the same as their corresponding angles in Triangle DEF, the two triangles are congruent. Congruency is not specific to triangles—other shapes like squares, rectangles, and trapezoids can be spoken of as congruent to other shapes as well.
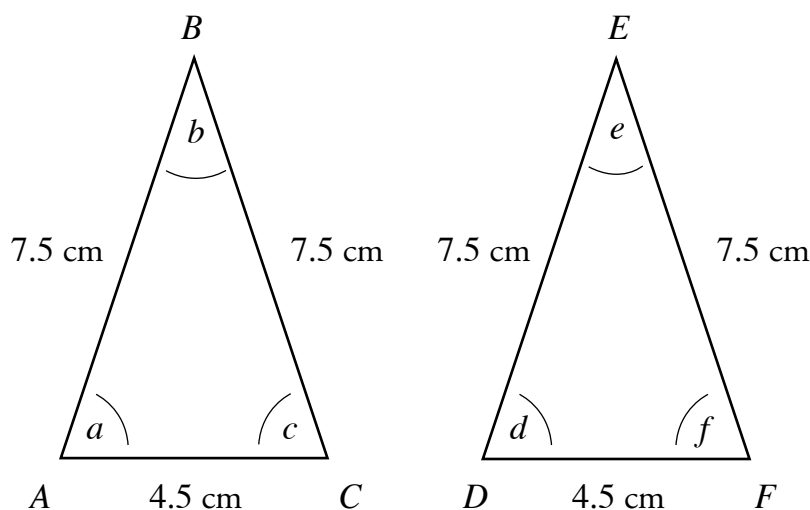
What if you're asked to figure out whether two triangles are congruent and you're not told the lengths of all their sides or the measures of all their angles? Well, it turns out that you don't need to know *all* of this information—just some of it. If you are given the information listed in any one of the points below

1.) **Side-Side-Side (SSS) Theorem**:

If you know the three side lengths of two triangles and the corresponding side lengths of each triangle are the same, the two triangles are congruent.
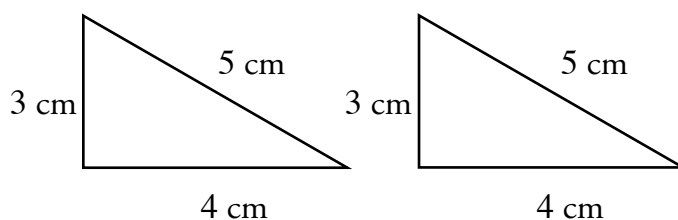
2.) **Side-Angle-Side (SAS) Theorem**

If you know the lengths of two sides and the measure of the angle between those sides and can confirm that these are identical in corresponding triangles, the two triangles are congruent.
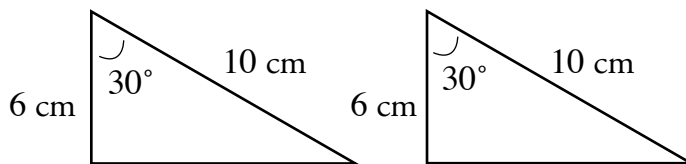
3.) **Angle-Side-Angle (ASA) Theorem**

If you know two angles in a triangle and the length of the side that falls between them and can confirm that these are identical in two triangles, the two triangles are congruent.
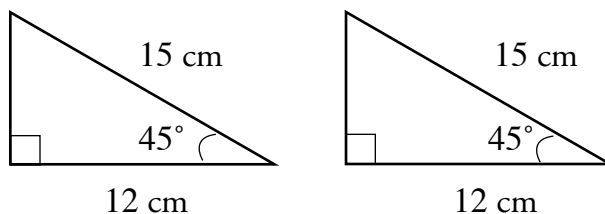
Another specific way shapes can be related is that they can be **similar** to one another. This word has a very specific mathematical definition. Shapes that are similar have the same corresponding angle measurements, but they may have different side lengths. The triangles shown below are similar!
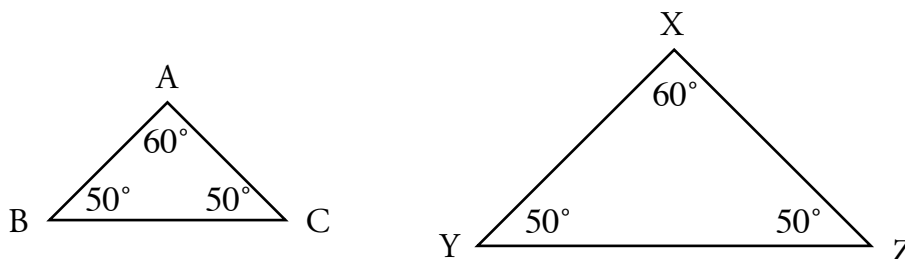
The side lengths of similar shapes are related by a specific ratio that holds constant for each of the pairs of corresponding sides. We can write out a similarity ratio for the two triangles shown above:

$$\frac{\overline{AB}}{\overline{XY}} = \frac{\overline{BC}}{\overline{YZ}} = \frac{\overline{ZX}}{\overline{CA}}$$

Say that Triangle XYZ is 2.5 times as large as Triangle ABC. We could write that as a ratio— 1 : 2.5. We can use this information to calculate side lengths. If we knew the length of $\overline{AC}$ and the ratio of the two triangles to one another, we could find the length of $\overline{XZ}$ by multiplying $\overline{AC}$ by the ratio:

$$\overline{AC} \times \frac{2.5}{1} = \overline{XZ}$$

To answer Sample Question #1, start by drawing out the light poles and their shadows and labeling the measurements that you know.

36 ft

9 ft   6 ft
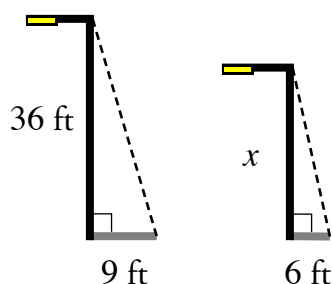
In this case, we end up with two similar triangles. We know that these are similar triangles because the question tells us that these poles are on a flat surface, meaning that the angle formed by the poles and their shadows are both right angles. Then, because the question states that the shadow cast by both poles are at the same time of day, we know that the angle formed by each pole and its hypotenuse is equivalent. Since these are similar triangles, we can set up proportions for the corresponding sides.

$$\frac{36}{x} = \frac{9}{6}$$

Now, solve for x by cross-multiplying. C is correct!

$9x = 216$

$x = 24$

Sample Question #2 deals with rectangles instead of triangles, but it requires you to apply the same principles.

$P = 36$

x

$P = 12$

12   4
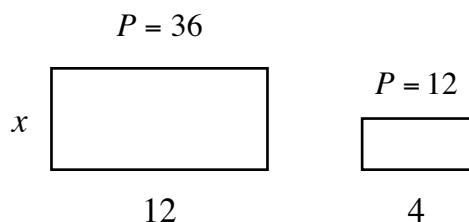
Here are the described rectangles. In this problem, we're told outright that these rectangles are similar, so we just need to construct and solve our similarity ratio—but wait: we need to figure out the height of the smaller rectangle before we can do that, since that's the dimension that will correspond with x. We're given the perimeter of the rectangle, so we can write out a quick algebraic equation to model how we'd find the perimeter and use that to solve for the unknown height of the smaller rectangle, which we'll call y.

Sample Question #1

On a flat street, a light pole 36 feet tall casts a shadow that is 9 feet long. At the same time of day, a nearby light pole casts a shadow that is 6 feet long. How many feet tall is the second light pole?

    A. 16 ft

    B. 18 ft

    C. 24 ft

    D. 30 ft

Sample Question #2

Two rectangles are similar. The perimeter of the first rectangle is 36. The perimeter of the second is 12. If the base length of the second rectangle is 4, what is the height of the first rectangle?

    A. 6

    B. 8

    C. 9

    D. 10

$$4 + 4 + y + y = 12$$
$$8 + 2y = 12$$
$$2y = 4$$
$$y = 2$$

Now that we know the height of the smaller rectangle, we can use it as part of the similarity ratio to solve for the height of the larger rectangle by cross-multiplying. A is the correct answer.

$$\frac{x}{2} = \frac{12}{4}$$
$$4x = 24$$
$$x = 6$$

Sample Question #3 also involves rectangles, but in this case, they share a side, $\overline{CF}$. To solve for the area of Rectangle ABCD, we need to calculate its length ($\overline{AE}$, or $\overline{AF} + \overline{FE}$) and its width ($\overline{BA}$ or $\overline{DE}$). $\overline{BA} = \overline{CF} = \overline{DE}$, and we're told that $\overline{CF} = 24$, so the width of Rectangle ABCD is 24. We're also told that $\overline{EF} = 12$, so we just need to calculate $\overline{AF}$ and find the sum of these two line segments.

We're told that Rectangle ABCD is similar to Rectangle CDEF, so we can write out the following similarity ratio:

$$\frac{\overline{AF}}{\overline{CF}} = \frac{\overline{CF}}{\overline{EF}}$$

We're given the length of CF and EF, so we can substitute these lengths into the proportion.

$$\frac{\overline{AF}}{24} = \frac{24}{12}$$

Cross-multiplying, we find:

$$\overline{AF} = \frac{24}{12} \times 24 = 48$$

We can find that D is correct by calculating the area of Rectangle ABCD as follows:

$$\overline{AE} \times \overline{DE}$$
$$(\overline{AF} + \overline{EF}) \times \overline{CF}$$
$$(48 + 12) \times 24$$
$$60 \times 24$$
$$1440$$

Sample Question #3
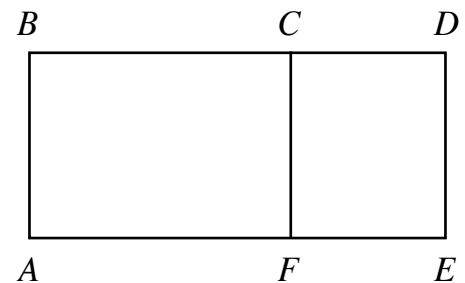
In the above diagram, Rect ABCD ~ Rect CDEF. $\overline{CF} = 24$ and $\overline{EF} = 12$. Give the area of Rect ABCD.

    A. 1360 units^2
    B. 1364 units^2
    C. 1400 units^2
    D. 1440 units^2

Area and Perimeter: Triangles, Quadrilaterals, and Circles

Some problems on the GRE Quantitative section assess whether you can calculate two common measurements of two-dimensional shapes: area (how much two-dimensional space a shape takes up) and perimeter (the distance around the edge of a shape). Any problems that deal with shapes as seemingly simple as triangles and circles may seem straightforward, but the variety of shapes you can face and the complications presented by missing information can make these problems perhaps more challenging than you initially expect. Making sure your math skills are sharp and your knowledge refreshed can make a key difference when you sit for your exam. With that in mind, let's review how to calculate area and perimeter for some familair shapes and in the process, consider a few problems in which area and perimeter are tested alongside more difficult geometric principles.

Triangles

Triangles are three-sided shapes with internal angles that sum to 180˚. Right triangles are a particular type of triangle in which one of those three angles is exactly 90˚—a "right angle." The other two interior angles in a right triangle will sum to 90˚, and when added to the right angle, the three will sum to 180˚ like in any other triangle.

$$A = \frac{1}{2}bh$$
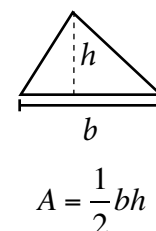
A triangle's area can be always calculated if you know the length of its base and its height: you simply multiply the base by the height and divide the result by two. (The formula writes this out as multiplying the product of the base and height by half—it gets you the exact same result). This formula holds for all triangles, so it doesn't matter if the triangle is acute, right, obtuse, isosceles, scalene, etc..

Let's try solving a question that requires the use of this formula. In Sample Question #1, we're not solving for the area directly in this instance, but the area of the triangle in question is provided, allowing us to solve for one of the other variables in the equation, the length of the triangle's base.

$$A = \frac{1}{2}bh$$

$$24 \text{ cm}^2 = \frac{1}{2}b(4 \text{ cm})$$

$$24 \text{ cm}^2 = 2 \text{ cm} \times b$$

$$\frac{24 \text{ cm}^2}{2 \text{ cm}} = b$$

$$b = 12 \text{ cm}$$

Sample Question #1

A triangle has a height of 4 cm and an area of 24 cm². What is the length of this triangle's base?

    A. 4 cm
    B. 6 cm
    C. 12 cm
    D. 20 cm

The base of the triangle is 12 cm long, so C is the correct answer. Keep in mind that you may asked to apply familiar equations in novel ways like this, and be given a variable for which you often solve. Consider how you might work algebraically with each formula you encounter to solve for each of its variables.

Rectangles and Squares

Quadrilaterals form a large family of shapes all related by the fact that they consist of four sides and four interior angles that sum to 360˚. They're each distinguished by their angles and relative side lengths. Let's examine a few distinct types of quadrilaterals, starting with the ones you're most likely to encounter on your exam and working towards more obscure examples.

We'll begin with rectangles, which are defined as any quadrilateral that has four straight sides and four right angles. Since a square is a specific type of rectangle in which all four sides are equal, the formulae we use to calculate the area and perimeter of a rectangle can also be applied to squares. These two versions of the same formula can be written differently: with a square, its length and width are equal, making the simplified equation $A = s^2$, where s is the length of one of the square's sides.

Derived from the Pythagorean theorem

$$d = c = \sqrt{a^2 + b^2}$$
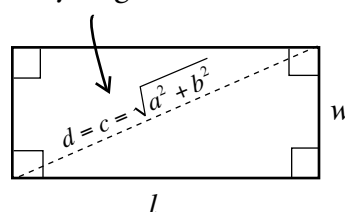
$$a = l \times w$$
$$p = 2l + 2w$$
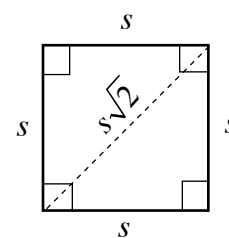
$$A = s^2$$
$$P = 4s$$
$$\text{Diagonal} = d = s\sqrt{2}$$

Sample Question #2

The area of a rectangular painting is 24.5 ft². If the painting is 3.5 ft tall, what is its width?

    A. 6.5 ft
    B. 7.0 ft
    C. 7.5 ft
    D. 8.0 ft

Now that we're reacquainted with these familiar shapes, let's try solving a simple sample problem. We're given the area and the height of the painting and asked to solve for its width. We can do this by substituting in the information we're given into the equation for the area of a rectangle and solving for the unknown variable, the width. The answer is B.

$$A = l \times w$$
$$24.5 \text{ ft}^2 = 3.5 \text{ ft} \times w$$
$$\frac{24.5 \text{ ft}^2}{3.5 \text{ ft}} = w$$
$$w = 7 \text{ ft}$$

Sample Question #3

Sam is planting a garden. Her plants need 96 square feet of fertile land on which to grow. If she is planning to use a twelve-foot wall as one of the garden's sides and wants to make it rectangular, how much fencing does she need to construct the other three sides?

A. 28 ft
B. 30 ft
C. 40 ft
D. 45 ft

Let's try another problem. Sample Question #3 may look much more complex, but it's testing the same concept of area as the last one did!

The tricky part of this problem is working out exactly how the relationship between area and perimeter is changed by the incorporation of the twelve-foot wall as one side of Sam's garden. Sketching out a diagram might be helpful in this instance—it can help you realize that while the area calculation is unaffected, one of the rectangle's dimensions (the one we labeled w) is fixed by the length of the wall. While many different rectangles might have areas of 96 square feet, this locked-in width will allow you to calculate the specific length of the rectangle that allows its area to be 96 square feet. After that, you can sum two lengths and a width to find how much fencing Sam needs. Dodge a trap by remembering not to use the regular formula for perimeter—Sam only needs three sides of fencing, not four!

This complex problem boils down to the simple calculation used to find the area of a rectangle based on the length of its sides, except while we are usually given the side lengths and asked to find the area, in this problem, we are given the area and asked to find the side lengths to find the correct answer. Let's call the side that is the 12-foot wall w and solve for the length of the other side, l.

$A = 96$ ft^2 l

Wall = w
$w = 12$ ft w

l

$$A = l \times w$$
$$96 \text{ ft}^2 = l \times 12$$
$$l = \frac{96 \text{ ft}^2}{12 \text{ ft}} = 8 \text{ ft}$$

Fence Needed = $l + l + w$

Now that we have calculated that the missing measurement of a side is 8 ft, we can solve the problem by finding the sum of two of the initially missing length sides and one of the given width sides to find how much fence Sam will need for her garden.

$l = 12$ ft
$w = 8$ ft
$w + w + l = 8 + 8 + 12 = 28$ ft

Sam will need 28 ft of fencing for her garden, so the correct answer is A.
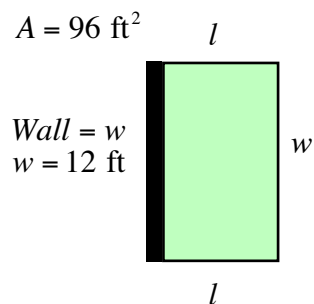
Parallelograms

If a quadrilateral has two sides of equal length but its angles are not each 90˚, it is a parallelogram. Since a parallelogram and a rectangle only differ in their angles, you can solve for the perimeter of a parallelogram the same way you solve for the perimeter of a rectangle; however, finding the area of a parallelogram is a little more involved. You do this by multiplying the base by the height. If you're not given the height, you can solve for it by creating a triangle out one of the sides of the parallelogram, the height, and part of the base. You'll need to be given part of the base, but if you need to solve for the base of a parallelogram to solve a GRE Quantitative problem, you'll be given enough information to do so. You're not likely to be asked about a parallelogram's diagonal because the two triangles it creates are not necessarily right triangles; therefore, you cannot employ the Pythagorean theorem to solve for the length of the diagonal.
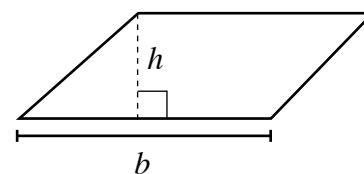
$$A = b \times h$$
$$P = 2w + 2l$$

The trick to getting Sample Question #4 correct is to remember the formula and not use the wrong numbers in it! The area of a parallelogram is calculated by multiplying its base and its height, not by multiplying the lengths of its two different sides. If you do that, you'll get answer choice C, but C is incorrect. Similarly, you don't want to multiply the base by the shorter side, which will get you answer choice A, a different incorrect answer. The height of this parallelogram is 3 cm and its base is 12 cm; multiplying these, you get 36 cm², so B is correct.

Sample Question #4

What is the area of the depicted parallelogram?

3 cm

5 cm

12 cm
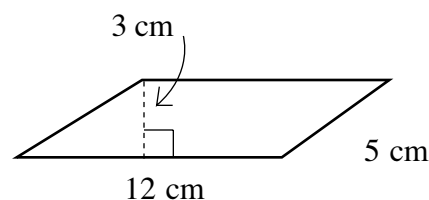
A. 15 cm²
B. 36 cm²
C. 60 cm²
D. 72 cm²

Trapezoids

Trapezoids are perhaps the most complex quadrilateral you'll have to deal with on the GRE Quantitative section. One pair of opposing sides in this quadrilateral are of equal length, but the other pair of sides—the top and the bottom of the shape—are of different lengths. The angles are determined by the relationship between the sides, and since the sides can relate to one another in different ratios, the angles are not consistent between shapes. The only thing we can confidently say about the angles of a trapezoid is what we can say about all quadrilaterals: that they sum to 360˚.

b_1

a h a

b_2
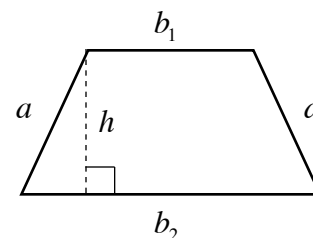
$$A = \frac{b_1 + b_2}{2} \times h$$
$$P = a + a + b_1 + b_2$$

Calculating the perimeter of a trapezoid means summing each of the sides. You may find that it's best to do this as an addition problem, not a multiplication problem, so that you don't forget to account for each of the different bases. Finding the area of a trapezoid means taking the average of the two bases and multiplying it by the height. Like when working with a parallelogram, you may need to solve for the height. The GRE Quantitative section may covertly allow you to solve for the height of a trapezoid by defining the difference between the shorter and longer base, as in the next sample problem. Watch out for this particular scenario: it's easy to miss the information you need!

Sample Question #5

The area of the trapezoid shown below is 156 cm². What is the length of side *x*? (Shape not to scale).
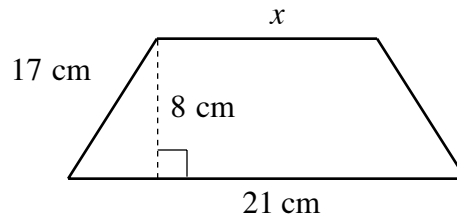
A. 15 cm

B. 16 cm

C. 17 cm

D. 18 cm

In Sample Question #5, we need to work backwards. We're given the area of the trapezoid and asked to solve for one of its bases—the shorter one. This may seem imposing, especially because calculating a trapezoid's area requires averaging the bases. Let's substitute the information we know into the equation for the area of a trapezoid and see if we can solve for the missing base.

$$A = \frac{b_1 + b_2}{2} \times h$$
$$156 \text{ cm}^2 = \frac{x + 21 \text{ cm}}{2} \times 8 \text{ cm}$$

At this point, we can use algebra to rearrange the equation and solve for *x*.

$$\frac{156 \text{ cm}^2}{8 \text{ cm}} = \frac{x + 21 \text{ cm}}{2}$$
$$19.5 \text{ cm} = \frac{x + 21 \text{ cm}}{2}$$
$$39 \text{ cm} = x + 21 \text{ cm}$$
$$x = 39 \text{ cm} - 21 \text{ cm} = 18 \text{ cm}$$

D is the correct answer.

Circles

Circles are two-dimensional shapes that lack straight edges. This feature sets circles apart from the other shapes we've considered thus far and necessitates new vocabulary to talk about their unique mathematical properties. For instance, the perimeter of a circle is called its circumference. Before we begin solving questions involving circles, we must first gain a basic understanding the measurements that are commonly referenced when manipulating them mathematically.
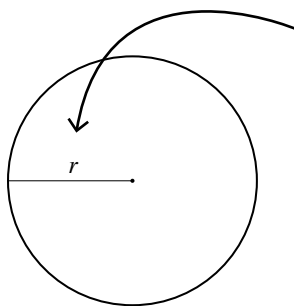
The radius (r) of a circle is any straight line spanning from the center of the circle, indicated by the point in the figure below, to any point on the circumference (edge) of the circle. A circle has infinite radii, all of which are the same length. This value is important in many calculations involving circles.
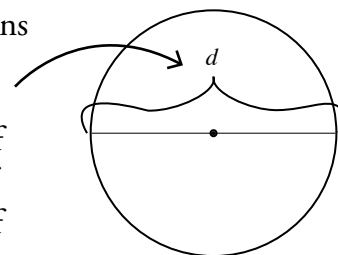
A diameter of a circle is any straight line from one end of the circle to the other end that passes through the center of the circle. Like with radii, a circle has an infinite number of diameters. This measure is also useful in circle calculations. It can be directly used when finding the circumference of a circle, and can also be used to find the radius of a circle. The length of the diameter is always twice that of the radius, so if you are given one value, you can easily calculate the other.

When working with circles mathematically, you will need to work with one more concept: π, pronounced "pie." Quite simply, π is the ratio of the circumference of a circle to its diameter. π is roughly equivalent to 3.14. While that's a nice bit of information to have, it is often just better to think of π as π, i.e. as a number just as actual as 1 or 2. This can take some getting used to, but it results in much more accurate calculations. π is the key to calculating the area or circumference of any circle.

$$\pi \approx 3.14$$
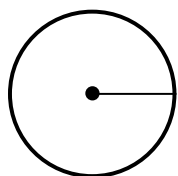
It will actually save you time in most instances to calculate answers to circle problems in terms of π. Many questions give answer choices in terms of π, so using π directly can help you streamline your problem-solving process and save valuable time. Just remember to treat π as a variable with a defined value instead of as a unit. Saying "3π" isn't like saying "3 meters" or "3 inches"—saying "3π" is similar to saying "3x," with the implication that x ≈ 3.14.

$$A = \pi r^2$$
$$C = 2\pi r$$

Now that we're familiar with circle jargon, let's look at the formulae for area and perimeter. The area of a circle is defined as $A = \pi r^2$. Since there are only two variables involved in this equation, you can calculate the area of a circle if you know the diameter or radius of that circle. The inverse of this statement is also true: if you know the area of a circle, you can calculate its diameter and radius.

Let's try a simple sample question that requires you to calculate the area of a circle and its circumference in terms of π.

Sample Question #6

Consider a circle with a radius of 5 cm. Ignoring units, what is the difference between the area and the circumference of this circle?

    A. 4π

    B. 5π

    C. 12π

    D. 15π

To figure out the correct answer to Sample Question #6, we will need to calculate both the area and the circumference of the circle using the provided formulae.

$$A = \pi r^2 \qquad C = 2\pi r$$
$$A = \pi (5)^2 \qquad C = 2\pi (5)$$
$$A = 25\pi \qquad C = 10\pi$$

At this point, we can calculate the difference between the circle's area and circumference:

$$A - C = 25\pi - 10\pi = 15\pi$$

The correct answer is D, 15π.

Now let's try a more difficult question that relies on the same equations. Sample Question #7 gives us several clues in its wording (e.g. "around," "wheel") that it concerns circumference specifically. We need to calculate the distance around the wheel of cheese. Don't get confused by the fact that a cheese wheel is a three-dimensional object. To answer this question, we will need to calculate the distance around the edge of the circle representing the top of the cheese wheel. We don't need to calculate the area, so be careful that you use the correct equation—using the area equation results in a listed wrong answer.

The correct answer is B, 24π in.

Sample Question #7

A chef wants to wrap a string around a giant wheel of cheese to support a paper label. The wheel of cheese has a diameter of 24 inches. How much string is needed to go around the wheel of cheese exactly once?

    A. 6π in

    B. 24π in

    C. 36π in

    D. 144π in

$$d = 2r = 24 \ in$$
$$2r = 24$$
$$r = 12$$
$$C = 2\pi r$$
$$C = 2(12)\pi$$
$$C = 24\pi \ in$$

Circles: Arcs, Chords, and Sectors

Circle problems on the GRE Quantitative section can move beyond the basic principles of area and perimeter and ask you to consider higher-level concepts. Test-takers who are confident in their abilities to work with circles mathematically may find that their skills are highly focused on the more basic concepts, and that their confidence wanes when they are asked to demonstrate more advanced skills. In particular, if you think of the GRE's Quantitative section as aiming to test math skills that you once mastered but may have forgotten, area and perimeter and the skills related to calculating them are likely the freshest concepts in your mind. This lesson will review some more esoteric properties of circles—variuos types of interior angles, arcs, chords, and sectors. Let's examine each in turn so that none of them presents you with any issue on test day.

Chords and Inscribed Angles

To understand what an inscribed angle is, we must first understand what a chord is. A chord is a line segment that connects any two points on the circumference of a circle. This means that a chord does not necessarily pass through the center of the circle, and thus does not necessarily represent the circle's diameter. Now, an inscribed angle is an angle whose vertex is anywhere on the circumference of the circle, and its legs are chords, intersecting two distinct points on the circumference of the circle. In the circle shown at right, $\overline{AB}$ and $\overline{BC}$ are chords, and angle $\angle ABC$ is an inscribed angle.
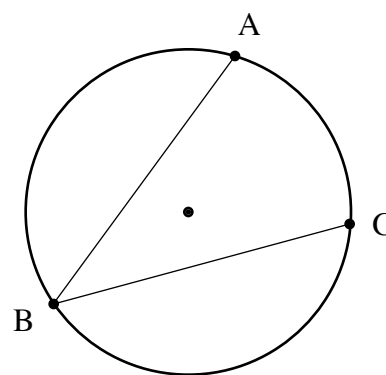
Sample Question #1 may initially appear much more complex than it actually is. With a good grasp on what each mathematical term specifies, you can breeze through this problem with ease. The trick to this problem is realizing that the largest possible chord of a circle is its diameter. This means that the diameter of the circle is 16 inches, which means an 8-inch radius.

$d = 2r$

$16 = 2r$

$\dfrac{16}{2} = \dfrac{2r}{2}$

$r = 8$

Sample Question #1

A circle has a maximum chord length of 16 inches. What is its radius in inches?

    A. 6 in

    B. 7 in

    C. 8 in

    D. 9 in

Central Angles

A central angle of a circle is an angle whose vertex is the center of the circle and whose legs are radii. Note that the sum of the measures of the central angles of a circle is always 360°. The figure at right illustrates the central angle ∠YCZ, while $\overline{CY}$ and $\overline{CZ}$ are radii.
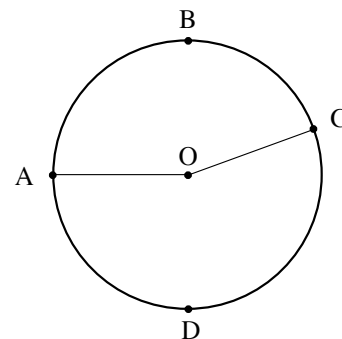
Note that the measure of the central angle is equal to the measure of the inscribed arc—that is,   YCZ = mYCZ; furthermore, note that the central angle is always one of two angles created in a circle, depending which direction you travel around the circle from the first point on the circumference to the second point on the circumference. By definition, the central angle is the smaller of the two, so it can be a maximum of 180°.

Let's go over Sample Question #2. Since the measure of a central angle is equal to the measure of the corresponding inscribed arc, all we need to do to solve this problem is calculate the circumference of the circle and subtract the given measure of ADC from it to find ABC. From there, we can create a proportion that compares arc length to central angles that will allow us to find AOC. Doing this, we find that the correct answer is C:

Sample Question #2

Consider the circle shown above. If $\overline{AO}$ = 15 cm and ABC = 18π cm, what is the measure of ∠AOC? (Figure not drawn to scale.)

    A. 125°

    B. 133°

    C. 144°

    D. 152°

$$C = d\pi = 2r\pi = 2(15)\pi = 30\pi$$
$$30\pi - 18\pi = 12\pi$$

$$\frac{12\pi}{30\pi} = \frac{AOC}{360°}$$
$$12\pi \times 360° = 30\pi \times \ AOC$$
$$AOC = \frac{12\pi \times 360°}{30\pi}$$
$$AOC = 144°$$

Arcs

An arc is a portion of a circle's circumference. Viewed independent of its source circle, an arc would be drawn as a curved line. The length of an arc is the measure of the linear distance covered by an arc. The legs of both central and inscribed angles define arcs when they intersect with the circumference of a circle.

Arcs are identified using the notation of a curved line above two letters indicating points. For example, the notation YZ is pronounced "arc Y Z" and indicates the part of the circle's circumference that lies between points Y and Z.

In the example figure, the measure of YZ is 36Y since the measure of the central angle that forms that arc is also 36Y. The length of YZ is a part of the circumference of the entire circle.
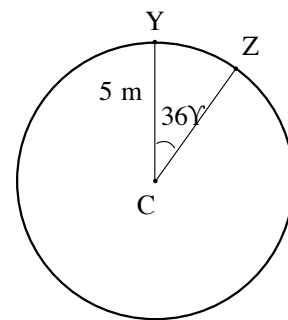
Let's look at Sample Question #3. To find the length of YZ, we need some information. The first step to finding the arc length is to find the circumference in meters:

$$C = \pi d = 2\pi r$$
$$C = 10\pi \text{ m}$$

Now that we know the circumference of the whole circle is $10\neq$ m, we use the given information regarding the measure of the central angle YCZ to find the proportion of the entire circle that makes up the arc YZ. To do this, we set up a proportion relating the following two ratios: 1) the length of arc YZ to the length of the entire circle (circumference) and 2) the measure of the inscribed arc to the total measure of the entire circle:

$$\frac{YZ}{10\pi \text{ m}} = \frac{36°}{360°}$$

We may solve by cross-multiplying, or we can notice that the right side of the equation simplifies to $\frac{1}{10}$ and then solve to make the left side of the equation also equal to $\frac{1}{10}$. No matter which method is used, our answer is YZ = π m, so answer choice B is correct. In other words, since the arc measure (which is equal to the measure of the central angle) is $\frac{1}{10}$ of the measure of entire circle (360Y), the arc length is also $\frac{1}{10}$ of the length of the circumference ($C = 10\pi$ m).

Sample Question #3

Consider the sample circle for this section of the lesson. Find the length of YZ.

A. 0.75π m

B. π m

C. 1.25π m

D. 1.50π m

Sectors

A sector is a portion of a circle enclosed by two radii and an arc. In other words, it is a "slice of the pie," if the pie were two-dimensional. In the circle below, the measure of the central angle (60˚) is equal to the measure of the intercepted arc. Sector YCZ is a portion of the entire circle.
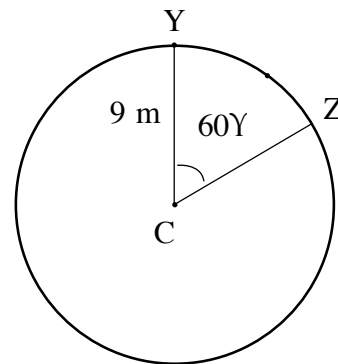
What portion of the entire circle is sector YCZ? $\frac{60°}{360°} = \frac{1}{6}$ because the central angle that makes up the sector has a measure of 60Y, and the central angles of a circle always equal 360˚. Thus, to find the area of any sector, we must set up an equation relating these two ratios: 1) central angle of the sector to the sum of all central angles in the circle, and 2) area of the sector to area of the whole circle. Mathematically:

$$\frac{60°}{360°} = \frac{\text{Area of Sector YCZ}}{\pi 9^2 \text{ m}}$$

$$\frac{1}{6} = \frac{\text{Area of Sector YCZ}}{81\pi \text{ m}^2}$$

Cross-multiply and simplify.

$$6(\text{Area of Sector YCZ}) = 81\pi \text{ m}^2$$

$$\text{Area of Sector YCZ} = \frac{81\pi \text{ m}^2}{6}$$

$$\text{Area of Sector YCZ} = 13.5\pi \text{ m}^2$$

Sample Question #4 treats the same concepts in the form of a story problem. The question asks us to find the proportion of the pie that Dave ate—the area of a sector. Since we are given radius, we can find the area of the circle.

Sample Question #4

Dave ate some of a circular apple pie that has a radius of 7.8 cm. When looking at the leftover pie from the top, it is found that 23.14π cm² of the flat plate underneath the pie is exposed in the area that Dave ate. How much of the pie did Dave eat?

A. 133.4˚

B. 136.8˚

C. 138.9˚

D. 140.3˚

$$A_{\text{Pie}} = \pi \times (7.8 \text{ cm})^2$$

$$A_{\text{Pie}} = 60.84\pi \text{ cm}^2$$

Now, we must divide the area of pie that Dave ate (which is given directly in the question) by the area of the whole pie to determine the proportion of pie he really ate.

$$\frac{23.14\pi \text{ cm}^2}{60.84\pi \text{ cm}^2} = 0.38$$

We solve this by converting the proportion of the entire pie that Dave ate (0.38) into an angle by multiplying it by 360˚. Since $360° \times 0.38 = 136.8°$, the correct answer is B.

Clock Math

The GRE may ask you to apply your knowledge of circle geometry to situations specific to clocks. The test isn't asking if you know how to tell time, but whether you are able to figure out various angles and arcs of a circle. Don't let these questions throw you off: at this point, you have all of the knowledge of circles you need to solve them, even if they seem somewhat unfamiliar. Let's look at a pair of sample clock-related questions to demonstrate how your advanced circle skills can help you solve these types of problems.

Sample Question #5

What is the measure, in degrees, of the acute angle formed by the hands of a twelve-hour clock that reads exactly 3:10?

    A. 30°
    B. 35°
    C. 40°
    D. 45°

One hand is pointing at the 2, while the other is pointing at the 3. Since there are twelve numbers on a clock face, the hands are one-twelfth of a 360° circle apart, or 30°; however, this is not the correct answer. You need to account for the ten minutes that have passed, too. Ten minutes is one-sixth of an hour, so the hour hand has also moved one-sixth of the distance between the three and the four. One-sixth of 30° is 5°, so the total measure of the angle, therefore, is B: 35°. The hour hand is moving too: five degrees every ten minutes. Forgetting to account for the movement of the hour hand is what trips up most students dealing with clock math, which is otherwise relatively easy.

Let's consider another sample question for practice. To find the degrees of a clock hand in Sample Question #6, first find the angle between each hour-long sections. Since there are twelve evenly-spaced sections, each section has an angle of 30°, as we saw in the last problem. At 11:20, the hour hand has gone one-third of the way between the 11 and the 12. Thus, there are two-thirds of 30° between the hour hand and the 12. Two-thirds of 30° is 20°. There are 60° between the 12 and the 2, where the minute hands is. Thus, there's a total of 20°+60°=80° between the hands, making D the correct answer.

Sample Question #6

On a standard analog clock, what is the angle between the hands when the clock reads 11:20? Give the smaller of the two angles.

    A. 60°
    B. 70°
    C. 75°
    D. 80°

Inscription and Circumscription

The premises of some of the GRE's geometry problems may involve combinations of several familiar shapes. You can be asked to solve for quantities related to irregularly-shaped negative spaces. The most efficient thing you can do when you find yourself facing one of these problems is to realize that you're facing one. That may sound reductive, but it's all too easy to get pulled into focusing on details at the expense of categorizing the problem properly and taking a different approach to structuring your calculations. By considering the shapes as combinations of familiar shapes instead of an unfamiliar structure, you can begin to use what you know to solve for the areas and volumes of interstitial spaces.

An "inscribed" shape is one located within another shape such that it intersects the edges of the exterior shape, whereas a "circumscribed" shape is one that encapsulates another in these specific circumstances. The detail about the intersecting edges is often subtle but exceedingly important to calculations. It can be easy to miss as important information in a diagram, and even easier to ignore when problems are presented only verbally. In the diagram at right, the circle is inscribed in the square. We could also say that the square is circumscribed around the circle. These relations can apply to both two-dimensional and three-dimensional shapes. In our three-dimensional example at right, the sphere is inscribed in the cube, and the cube circumscribes the sphere. Understanding the difference between these terms can make the difference between sketching (and solving) a problem correctly or incorrectly.

The two shapes crossed out in red beneath our correct examples display sketches in which shapes are not truly inscribed or circumscribed. At first glance, they may look fine, as one shape is entirey contained within another: a triangle in a rectangle and a sphere in a cube; however, look at the edges. Remember, part of the definition of inscription and circumscription is that the edges of the interior shape have to intersect the edges of the surrounding shape. That is not happening in either crossed-out example: the triangle has a lot of space surrounding its upper-left corner, and the sphere doesn't approach the right side of the cube. You'd need more information to solve problems about these shapes' areas and volumes (e.g. some combination of the triangles' angles and sides and/or the circle's radius) than if they were inscription/circumscription problems. You might see these on your exam, but remember: in these cases, you'll have to be given all of the information you need to solve for what you're asked for. These are crossed out here to demonstrate that inscription and circumscription are distinct mathematical arrangements that test subtle geometric analysis skills. Let's see these skills in action in a sample problem.
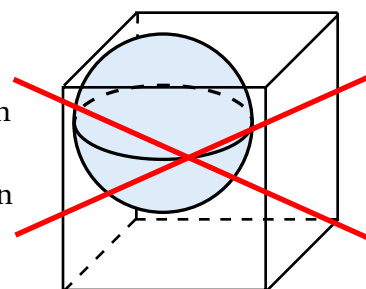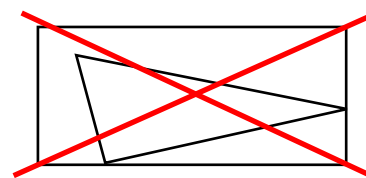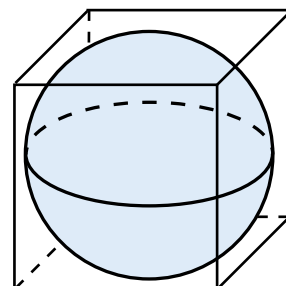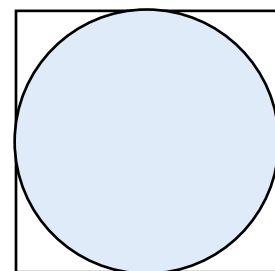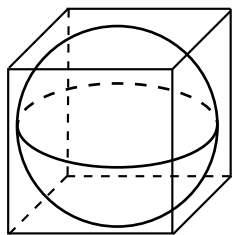
We're not given a diagram for Sample Question #1, but we can easily sketch one. To solve this problem, we're going to need to figure out how to measure the negative space outside of the sphere but inside the square. If you approach this like you approach finding the volume of a typical, regular shape, like a rectangle or a cylinder, you're not going to be very successful. One side of the negative space is represented by the curve of the sphere, whereas the other side is bounded by the lines of the box. We don't have a formula for this. If you find yourself in this situation, consider how you can create a subtraction problem such that you end up with the negative space as your answer. If we take the volume of the cube and subtract the volume of the sphere, we'll end up with all of the volume of the cube that isn't represented by the sphere's volume. Perfect! Let's write that out and expand it to see which variables we'll need.

$$V_{cube} - V_{sphere} = V_{answer}$$

$$(l_{cube} \times l_{cube} \times l_{cube}) - \frac{4}{3}\pi r_{sphere}{}^3$$

To solve this problem, we need to know two things: the length of one of the cube's edges, and the radius of the sphere. We're given the former, but what about that radius? It's not given. This is the point at which many test-takers might get frustrated and move on to another problem. But pause and consider our diagram: we *are* given the radius, albeit indirectly. If the sphere's edges intersect with the edges of the cube, what's its diameter? Exactly: the length of one of the cube's edges, or in this problem, 8 cm. The radius is simply half of this value.

Practice making this realization for different types of inscription and circumscription problems: it's the key to solving them. In placing shapes within shapes with their edges intersecting, usually a measurement given for one of the shapes will double as some other measurement for the other shape.

Let's finish solving this problem, substituting in 8 cm for *l* and 4 cm for *r*: The individual steps of the calculation are shown at right. We end up with an answer of 243.92 cm³, so the correct answer is A.

$$(8 \text{ cm} \times 8 \text{ cm} \times 8 \text{ cm}) - \frac{4}{3}\pi(4 \text{ cm})^3$$

$$512 \text{ cm}^3 - \frac{4}{3}\pi(64 \text{ cm}^3)$$

$$512 \text{ cm}^3 - \frac{4\pi(64 \text{ cm}^3)}{3}$$

$$512 \text{ cm}^3 - \frac{256\pi \text{ cm}^3}{3}$$

$$512 \text{ cm}^3 - 85.333\pi \text{ cm}^3$$

$$512 \text{ cm}^3 - (85.333 \times 3.14159) \text{ cm}^3$$

$$512 \text{ cm}^3 - 268.08 \text{ cm}^3$$

$$243.92 \text{ cm}^3$$

Sample Question #2

A rectangle is inscribed inside of a circle. If the area of the rectangle is 360 in² and the perimeter of the rectangle is 98 in, what is the area of the circle?

A. 420.25π in²
B. 600.25π in²
C. 1681π in²
D. 2401π in²

In Sample Question #2, we're dealing with two-dimensional shapes: a rectangle and a circle. Since the rectangle is inscribed in the circle, it's the interior shape, and its corners will each intersect the edge of the circle.

To find the area of the circle, it is important to know either its diameter or radius. We're not told this directly, so consider how the rectangle can help us figure this out. What part of the rectangle doubles as either the diameter or the radius of the circle? We need to identify a part of the rectangle that spans the entirety of the circle or exactly halfway. The diagonal of the rectangle stretches across the entire circle, making it the circle's diameter.
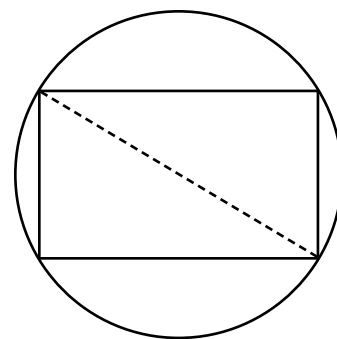
To find the diagonal of the rectangle, we need to know the lengths of its sides. We're not directly told this information either, but we are told the area and perimeter of the rectangle. That's enough information for us to calculate its side lengths. We just need to consider the area and perimeter equations and use substitution or elimination to define the overlapping variables l and w.

$$l \times w = 360$$
$$2l + 2w = 98$$

This system of equation can be solved by substitution:

$$2l = 98 - 2w$$
$$l = 49 - w$$

Now we can substitute the expression $49-w$ into the other equation for l.

$$l \times w = 360$$
$$(49 - w) \times w = 360$$
$$w(49 - w) - 360 = 0$$
$$49w - w^2 - 360 = 0$$
$$(w - 9)(w - 40) = 0$$

Note that this gives two possible values for w: $w = 9$ in or $w = 40$ in. It doesn't actually matter which one you pick to be w, because the other value will be the value for l. (Check that statement using the equation $l \times w = 360$ if you're not sure about that logic).

Knowing these two values, the diagonal can be found; it is the hypotenuse of a right triangle formed by these two lengths. Since we're dealing with a right triangle, we can break out the Pythagorean theorem.

Since the diagonal is also the diameter, we can do some substitution and set up for finding our final answer, the area of the circle. The correct answer is A!

$$a^2 + b^2 = c^2$$

$$(9 \text{ in})^2 + (40 \text{ in})^2 = d^2$$

$$81 \text{ in}^2 + 1600 \text{ in}^2 = d^2$$

$$\sqrt{1681 \text{ in}^2} = \sqrt{d^2}$$

$$d = \sqrt{1681 \text{ in}^2} = 41 \text{ in}$$

$$A_{Circle} = \pi r^2$$

$$A_{Circle} = \pi \left(\frac{d}{2}\right)^2$$

$$A_{Circle} = \pi \left(\frac{41 \text{ in}}{2}\right)^2$$

$$A_{Circle} = \pi (20.5 \text{ in})^2$$

$$A_{Circle} = 420.25\pi \text{ in}^2$$

Sample Question #3

An equilateral triangle is inscribed into a circle with radius 10 cm. What is the area of the triangle?

A. $\dfrac{100}{3} \neq \text{cm}^2$

B. $100 \neq \text{cm}^2$

C. $75\sqrt{3} \text{ cm}^2$

D. $150\sqrt{3} \text{ cm}^2$

Let's solve one more sample problem to really solidify the skill of shifting your perspective and identifying which dimensions or measurements of a given shape double with another shape's. Sample Question #3 brings triangle math into the picture. Specifically, we're dealing with an equilateral triangle—a triangle whose sides are all equal.

We need to solve for the area of this triangle, and we're given the radius of the circle. To find the area of the triangle, we can use the formula $A = \dfrac{1}{2}b \times h$. To do this, we'll need to know the length of the triangle's side (*b*) as well as its height.

How does the circle's radius relate to the triangle? Sketching it out can help you visualize this. The radius might not appear to give us any information at all at first glance; however, note that a radius—specifically a line drawn from the origin to a vertex of the equilateral triangle—bisects the angle of the vertex. Yes, this problem is going to involve angle calculations. An equilateral triangle by definition has three 60° angles, so if we bisect one, we get two 30° angles.
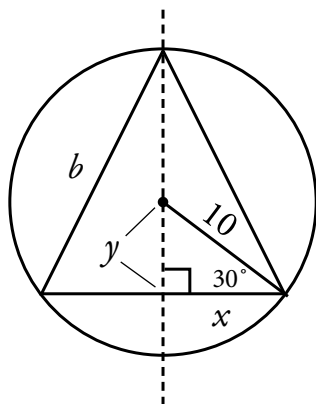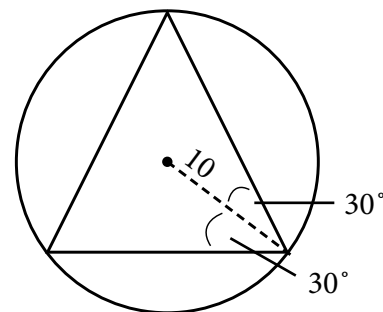
That doesn't help us find the length of a triangle's side, but it can help us find precisely half of it: just divide the triangle in half. When you do that, you create a 30-60-90 triangle from one of the bisected angles and the perpendicular line bisecting one of the triangle's edges. The 60° angle may not seem apparent, but keep in mind that that angle *has* to be 60° because of the presence of a 30° angle and a 60° angle in the triangle. Now, if we solve for the bottom edge of this little triangle, that will give us exactly half of the triangle's edge length. We can find the entire edge length by doubling that.

Recall that the ratio of the sides of a 30-60-90 triangle is given as 1 : √3 : 2. Therefore, the length of the side across from the 60˚ angle (let's call it x) can be found to be:

$$\frac{10}{2} = \frac{x}{\sqrt{3}}$$
$$10\sqrt{3} = 2x$$
$$x = \frac{10}{2}\sqrt{3}$$

This is half of the base of the triangle, so the base of the triangle can be found to be:

$$b = 2 \cdot 5\sqrt{3} = 10\sqrt{3}$$

Now we need to find the height of the triangle. To do that, we'll need to find y, the length of line between the center point and the center of one of the triangle's edges. We can solve for the length of y relatively easily because it is the side of the small 30-60-90 triangle opposite the 30° angle.

$$\frac{y}{1} = \frac{10}{2}$$
$$y = \frac{10}{2} = 5$$

The vertical line rising from the center point is the length of the circle's radius (10), which, when combined with the shorter section y (5), gives the height of the triangle as 15 cm.

Now, we can solve for the triangle's area by substituting our variable values into the area equation. The correct answer is C.

$$A = \frac{1}{2}b \times h$$
$$A = \frac{1}{2}(10\sqrt{3})(15)$$
$$A = \frac{1}{2}(150\sqrt{3}) = 75\sqrt{3}$$

Geometric Solids

You're likely to encounter a few problems testing three-dimensional geometry when you sit for your exam. Three-dimensional problems go one step farther than problems about simple flat shapes like squares and circles. They necessitate consideration of an additional dimension, so they require knowledge of different formulae while retaining all of the complexity of the two-dimensional shapes with which their three-dimensional shapes correlate.

This lesson will cover information associated with cubes, prisms, pyramids, cones, cylinders, and spheres. In doing so, it will discuss how to calculate two attributes specific to three-dimensional shapes: surface area and volume. You can picture surface area as representing the amount of wrapping paper you would need to cover the entirety of a three-dimensional shape exactly, or the flat area you'd need to paint if you were painting all sides of the shape. Volume, on the other hand, is the amount of three-dimensional space taken up by the object. You can imagine it as the amount of liquid that could fill the shape, or the amount of liquid that the shape would displace if it were entirely submerged. Being able to clearly distinguish between these measurements is paramount when working with three-dimensional shapes. Reviewing approaches to solving for these different quantities can help you confidently approach three-dimensional geometry problems on the GRE.

Cubes

Extend a square in three-dimensional space by a measurement equal to its side lengths, and you get a cube. Cubes are made up of six square faces, so it shouldn't come as a surprise that every angle in a cube measures 90°. Since a cube is made up of squares of the same size, we can infer that all of a cube's side lengths are equal. As with circles, the equations for the volume and surface area of a cube only involve one variable each, so knowing the side length means that you can solve for volume or surface area, and knowing the volume or surface area allows you to calculate the side length.

If you draw a blank on test day and forget the specific formulas, keep calm; you can quickly derive them using just common sense. Volume is always some form of multiplying three dimensional measurements together, and since a cube only has one measurement relevant to this calculation—side length—its volume equation is just the side length cubed. Need to find a cube's surface area isntead? Just find the area of one of its sides, a square, and multiply it by the shape's total number of sides—six. Approaching three-dimensional geometry problems doesn't mean solely relying on memorized formulae, though these can certainly save you valuable time. If you take time to consider the reasoning behind each formula, you can rely on simple logic if you briefly blank on test day.

$$V_{cube} = l \cdot w \cdot h = s^3 \qquad SA_{cube} = 6s^2$$
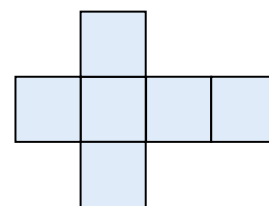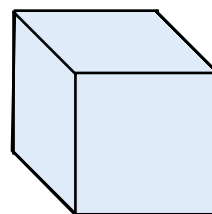
Some questions may ask you to find a percentage of a cube's volume or surface area; this requires a two-step process. The first step is to calculate the total value using the relevant equation. After the calculation is made, you can either use decimals or fractions to perform the necessary algebraic operations to find the desired percentage.

Let's try out Sample Problem #1, which uses both of these formulae. Both the volume equation and the surface area equation overlap in that they each involve one variable: the length of a cube's side. Recognizing this, we can use the formula for volume and work backwards to solve for the length of one of the cube's sides.

$$V = 64 \text{ cm}^3$$
$$V = s^3$$
$$64 \text{ cm}^3 = s^3$$
$$\sqrt[3]{64 \text{ cm}^3} = s$$
$$4 = s$$

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Sample Question #1

A cube has a volume of 64 cm³. What is the surface area of the cube?

   A. 72 cm²
   B. 80 cm²
   C. 84 cm²
   D. 96 cm²

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Knowing this, we can now switch from working "backwards" using the volume equation to working "forwards" with the surface area equation. Plugging in the value for s that we just calculated, we now have enough information to solve for the surface area of the cube.

$$SA = 6s^2$$
$$SA = 6(4)^2$$
$$SA = 6(16)$$
$$SA = 96 \text{ cm}^2$$

The correct answer is D. If you get stuck on a geometry problem, consider what formulae are related to the values you need to calculate, and look at what variables they require. Then, take another look at the problem set up with those variables in mind and consider how you might use other methods to deduce them. Doing this can help you make the important leap from one formula to another and solve the question without losing much time.

Some problems may ask you to calculate the surface area of a specified part or percent of a cube. In situations like these, don't assume that you'll be able to calculate the answer in a single step: most often, if you need to find the measurement of part of a shape, you'll need to either know or calculate the same measurement for the whole shape, and then find a given percentage or fraction of the whole

Sample Question #2

One of your friends is painting a room with square walls in his house. It takes two gallons of paint to cover one wall, and the paint cans state that one gallon of paint can cover an area of 32 square feet. If there are four walls and a ceiling in the room that Johnny is painting, approximately how many square feet does he paint?

A. 318 ft^2
B. 320 ft^2
C. 325 ft^2
D. 336 ft^2

to answer the question posed. Let's next look at one such problem.

Solving Sample Question #2 requires a few steps, so it pays off to pause and come up with a "game plan" of how you plan to attack the problem. First, consider the shape at hand. We're working with the surface area of a cube here, but the friend in the problem isn't painting the floor, so we'll need to adjust our surface area formula by considering only five sides, not six. If we were working directly with the surface area formula, we would use the following altered version:

$$SA = 5s^2$$

We're not working directly with this formula, though; this problem forces us to consider surface area in a way that avoids using the length, width, or heighth of the room and instead works by a proxy measurement of paint cans. Get ready for some unit analysis: we're told that two gallons of paint can cover one wall, and one can of paint contains enough product to cover an area of 32 square feet.

That's quite a bit of information to digest, especially in an exam setting. In situations like these, it helps to write out what information you're given. Doing this helps you slow down enough to make sure you're using the correct values for the correct equations, and in the event that you return to the problem to check your work or reconsider your answer, you have a map of your thoughts clearly delineated, allowing you an overview of your entire "game plan." Let's practice doing this now:

1 wall = 2 gallons
1 gallon = 32 ft^2

Aha—overlap, just what we need to solve this question! We need to translate from number of squares ("walls") to surface area in square feet, and since both the question stem's statements involve gallons, we can use dimensional analysis to translate number of walls to number of gallons and then number of gallons to square feet covered. You can do this in the form of two distinct equations or one multi-step equation; the multi-step equation has the advantage of allowing for very clear cross-canceling, keeping your work neat and tidy and potentially benefiting you should you review your thought process later.

Begin by listing the starting value. Here, that's the number of walls we're considering: 5. Then, list the unit equivalency as a fraction with the unit that matches that of the starting value in the

denominator and the other unit in the numerator. Continue listing units until the unit you want for your final answer appears in the numerator. Any unit that appears in both the numerator and the denominator cancels out; you can cross out units in your scratch work to make sure your equation is set up correctly.

$$5 \text{ walls} \times \frac{2 \text{ gallons}}{1 \text{ wall}} \times \frac{1 \text{ gallon}}{32 \text{ ft}^2} = ?$$

$$5 \; \cancel{\text{walls}} \times \frac{2 \; \cancel{\text{gallons}}}{1 \; \cancel{\text{wall}}} \times \frac{32 \text{ ft}^2}{1 \; \cancel{\text{gallon}}} = ?$$

At this point, we can multiply across the various fractions to arrive at our final answer. 5 times 2 times 32 is 320 ft², so the correct answer is B.

Rectangular Prisms

There are two types of prisms: rectangular and triangular. These prisms differ in the shapes that form their bases. If a prism's base is made up of a triangle, it is classified as a triangular prism; if its base is a rectangle, it is a rectangular prism.

Rectangular prisms have either a square or rectangular face on two opposite sides. Rectangular prisms are similar to cubes when it comes to the number of lines, vertices, and faces they involve; however, the key difference is that not all of the lines have the same length. You can think of a rectangular prism as a cube that has been stretched in one of its dimensions. Since one of a prism's dimensions does not equal the other two, this means that prisms have orientation that cubes do not. In other words, turning a prism on its end changes the answer you might get if considering a "painting the walls of a room"-style problem: you could consider a prism as a tall, skinny room or as a wide, shorter room.

The surface area calculation for a prism is also affected by the difference of one of its dimensions. Instead of calculating surface area as the cube of a side length that consistently reflects all sides, for rectangles, you need to calculate the area of two different shapes: two squares, and four rectangles. Taking the sum of the areas of these shapes will give you the surface area of the entire shape.
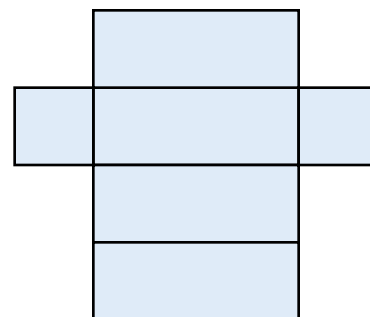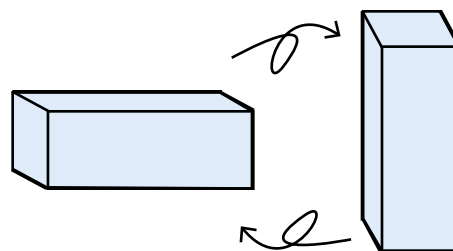
Let's next consider Sample Problem #3, where you not only need to calculate the surface area of a rectagular prism, but you also need to make use of that calculated value in an additional step.

As in the previous problem, it is exceedingly helpful to write out your given information while formulating your plan of attack.

$$V = l \cdot w \cdot h$$

$$w = 3l$$

$$w = 2h$$

$$3375 = 0.75V$$

Sample Question #3

Sally has a fish tank shaped like a rectangular prism. It has a width that is three times its length and twice its height. If the tank is 75% full when holding 3375 cm³ of water, what are the dimensions of the tank?

A. $l = 10$ cm, $w = 30$ cm, $h = 15$ cm

B. $l = 12$ cm, $w = 36$ cm, $h = 12$ cm

C. $l = 8$ cm, $w = 24$ cm, $h = 15$ cm

D. $l = 15$ cm, $w = 15$ cm, $h = 25$ cm

Look at that: all three dimensions of this prism are defined in terms of one variable, width (w). Because of this, we can rewrite the volume equation to use only two variables, volume and width, and since we're given the volume of the tank, we can then solve for the width. Knowing the width, we can then plug that into the given equations for length and height and solve for those values, thus arriving at the correct answer. Let's get going!

First, we need to solve for the total volume of the tank based on the given volume of how much water it holds when 75% full.

$$3375 \text{ cm}^3 = 0.75V$$

$$\frac{3375}{0.75} = V$$

$$V = 4500 \text{ cm}^3$$

At this point, we can substitute the total volume value into the volume equation.

$$4500 \text{ cm}^3 = l \times w \times h$$

At this point, we need to use substitution to make sure the equation uses only one variable to define each of the tank's dimensions. Let's use width, since the problem defines height and length in terms of width anyway. w will thus stay the same, but we need to rearrange the other equations so that the other variables are on one side of the equation and the other side defines each variable in terms of w.

$$w = 3l \qquad\qquad w = 2h$$

$$\frac{1}{3}(w) = \frac{1}{3}(3l) \qquad\qquad \frac{1}{2}(w) = \frac{1}{2}(2h)$$

$$\frac{w}{3} = l \qquad\qquad\qquad \frac{w}{2} = h$$

Now we can substitute those values into the volume equation and end up with a single-variable, totally solveable equation.

$$4500 \text{ cm}^3 = \frac{w}{3} \times w \times \frac{w}{2}$$

Solving for w, we get the following:

$$4500 = \frac{w^3}{6}$$

$$6(4500) = 6\,\frac{w^3}{6}$$

$$27{,}000 = w^3$$

$$\sqrt[3]{27{,}000} = \sqrt[3]{w^3}$$

$$30 = w$$

Now we can substitute the value of w into the equations for each of the other variables to solve the problem.

$w = 3l \qquad\qquad w = 2h$

$w = 30 \qquad\qquad 30 = 2h$

$30 = 3l \qquad\qquad \frac{1}{2}(30) = \frac{1}{2}(2h)$

$\frac{1}{3}(30) = \frac{1}{3}(3l) \qquad 15 = h$

$10 = l$

The correct answer is A.

Triangular Prisms

If you stretch a triangle into a third dimension, you end up with a triangular prism. Triangular prisms are made up of three rectangles and two triangles. Should you encounter a triangular prism on your GRE, you can use common sense to figure out the equations for surface area and volume. As long as you know how to calculate the area of a triangle and of a rectangle, you can simply figure out the area of each of the prism's faces and multiply by the numbers of each type of face. For volume, you can simply calculate the area of one of the triangular ends and multiply it by the height of the prism to account for the shape's third dimension.

$$V = \frac{1}{2}a \cdot c \cdot h$$

a = height of the triangular base
c = side length of triangle
h = height of the prism
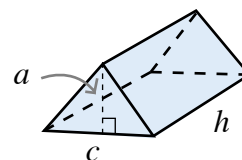
$$SA = 2(A_t) + A_{l1} + A_{l2} + A_{l3}$$

A_t = area of one triangle face
A_{l1}, A_{l2}, and A_{l3} = the areas of each of the rectangular faces
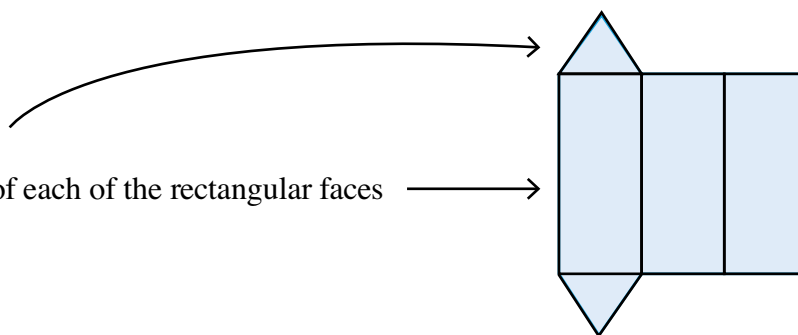
Sample Question #4 involves a triangular prism and works with volume in a way we haven't yet considered in this lesson. The last sample question considered a three-dimensional shape filled with a liquid, but this question considers a three-dimensional shape filled with many small solids: in this case, jelly beans. To solve this problem, we need to first calculate the volume of the container, and then multiply it by the number of jelly beans that fit in a given volume to cancel units and end up with a specific number of jelly beans.

Sample Question #4

Diane has a storage container shaped like a triangular prism. She wants to estimate how many jelly beans it can hold. The container's triangular base has a side length of 4 in and a height of 3 in. The container is 10 in tall. If approximately 20 jelly beans can fit in 25 in³, approximately how many jelly beans can the container hold?

    A. 48 jelly beans
    B. 52 jelly beans
    C. 60 jelly beans
    D. 64 jelly beans

The container is a triangular prism, so we'll need to use the following equation:

$$V = \frac{1}{2}a \cdot c \cdot h$$

We're given all the information we need to solve for the volume, so this part of the probelm becomes straightforward: just plug in the values to the equation and solve.

$$V = \frac{1}{2}bhH$$
$$b = 4$$
$$h = 3$$
$$H = 10$$
$$V = \frac{1}{2}(4)(3)(10)$$
$$V = \frac{1}{2}12(10)$$
$$V = \frac{1}{2}120$$
$$V = 60$$

The prism's volume is 60 in^3. Knowing the volume of the prism, we can proceed to solve for how many jelly beans will fit in it. We know that 20 jelly beans fill 25 in^3 of space, so we can set up a proportion to solve for how many jelly beans fill 60 in^3 of space:

$$\frac{20 \text{ jelly beans}}{25 \text{ in}^3} = \frac{x \text{ jelly beans}}{60 \text{ in}^3}$$

To solve, we need to cross-multiply and solve for x:

$$1200 = 25x$$
$$\frac{1200}{25} = x$$
$$x = 48$$

Approximately 48 jelly beans would fit in the prism described, so A is the correct answer.

Cones

Cones are three-dimensional objects that have a circular base and only one vertex, which occurs at the tip of the cone.

To find the volume of a cone, consider that when viewed from the side, a cone looks like a triangle, whereas a cylinder, another shape with a circular base, looks like a rectangle. Given this, it makes sense that the volume of a cone is one-third that of a cylinder. The two dimensions of a cone that are especially relevant in geometry problems are its height and its radius. (Remember, a cone's radius might be implicitly conveyed if the problem defines its diameter).
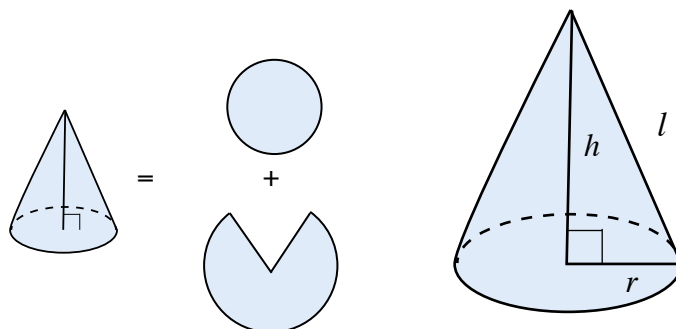
$$SA_{cone} = SA_{base\ circle} + SA_{circle\ minus\ sector}$$

$$SA_{cone} = \pi r^2 + \pi r l$$

$$V_{cone} = V_{cylinder} \times \frac{1}{3}$$

$$V_{cone} = (C \times h) \times \frac{1}{3}$$

To find the surface area of a cone, consider it as a compilation of two shapes: a circle, which forms the bottom of the cone, and a larger circle missing a sector that wraps around to form the surface of the cone's height. Our formula is composed of two parts, one that corresponds with each of these two component shapes. To solve for the shape of the "wrap-around" part, you'll need to know l, the slant height of the cone. If you're not given this information, you can solve for it using the Pythagorean theorem, using h and r as the sides of a triangle and solving for l as its hypotenuse.

$$V_{cone} = (\pi r^2 \times h) \times \frac{1}{3}$$

$$V_{cone} = \frac{\pi r^2 h}{3}$$

Let's try answering Sample Question #5. Due to the amount of information we're given, this problem is relatively straightforward: just take the equation for the volume of a cone, plug in the given volume and height, and solve for the radius.

$$V = \frac{\pi r^2 h}{3}$$

$$240 = \frac{\pi r^2 (8)}{3} = \frac{8}{3} \pi r^2$$

$$\frac{3}{8}(240) = \frac{3}{8}\left(\frac{8}{3}\pi r^2\right)$$

$$90 = \pi r^2$$

Sample Question #5

A cone has a volume of 240 cubic centimeters and a height of 7 centimeters. What is its radius?

A. $64 \neq$ cm

B. $124 \neq$ cm

C. $\sqrt{\dfrac{90}{\neq}}$ cm

D. $\sqrt{\dfrac{640}{\neq}}$ cm

$$\frac{90}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{\frac{90}{\pi}} = \sqrt{r^2}$$

$$r = \sqrt{\frac{90}{\pi}}$$

The correct answer is C.

Cylinders
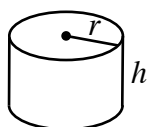
$V = \pi r^2 h$

r = radius
h = height
V = volume

If you extend a circle into three-dimensional space using straight lines at right angles to the original circle, you get a cylinder. Cylindrical volume is thus defined as the area of a circle multiplied by the shape's height.

What if you need to find the surface area of a cylinder? You can use common sense to work out that formula. To find surface area, you need to sum the areas of three shapes: the circle that forms the top of the cylinder, the rectangle that (wrapped around the circles) forms the height of the cylinder, and the circle that forms the bottom of the cylinder.

Start by finding the area of the top circle—it's the same as the area of the bottom circle. As an equation, the area of one circle would be πr^2, so the area of both circles would be $2(\pi r^2)$. Now for the rectangle. Its height is the cylinder's height, and its length is the circle's circumference, πd. We can also write its circumference as $\pi(2r)$. So, the area of the rectangle is $2\pi rh$. Summing that all together, you get the equation $2\pi rh + 2\pi r^2$, the formula for the surface area of a cylinder. If you find yourself in need of a relatively simple formula associated with geometrical shapes, you can always try to see if you can calculate it yourself by working from the familiar composite parts of the shape.

Sample Question #6 requires you to solve for a variable that isn't directly represented in the equation for the volume of a cylinder, but

Sample Question #6

A cannery wants to manufacture soup cans that are 4.5 inches tall and that hold 300 mL of soup each. 1 milliliter is equal to approximately 0.061 cubic inches. In order to meet these specifications, what must the approximate diameter of each can be? Use 3.14 for the value of π.

A. 1.10 in

B. 1.14 in

C. 2.20 in

D. 2.28 in

that's nothing you need to worry about. You're asked for diameter, but the relevant provided formula uses radius. You can simply double the radius after you're done solving for it—but don't forget! It's likely that the length of the radius will show up as one of the incorrect answer choices to trap test-takers who don't note this important distinction.

Before you can start plugging in variables, you have to take stock of your units. This problem provides a unit ratio, which is a major red flag signaling that you're going to need to do some unit conversion before you'll be able to calculate the correct answer. The volume of the can is presented in milliliters, but its height and the answer choices are given in inches. You need to convert the volume of the can from milliliters to cubic inches, and you can do that by using dimensional analysis. Starting with the given information, multiply it by different unit conversion fractions until you get a number that uses the correct units, cancelling units as you go.

$$300 \; \cancel{mL} = \frac{0.061 \; in^3}{1 \; \cancel{mL}} = 18.3 \; in^3$$

Now that all of your data is in inch-based units, you can substitute those values into the volume equation. After doing that, you'll have an equation with a single variable in it—radius, r, the variable for which you need to solve in order to find the can's diameter.

$$V = \pi r^2 h$$

$$18.3 \; in^3 = \pi r^2 (4.50 \; in)$$

$$\frac{18.3 \; in^3}{4.50 \; in \times \pi} = \frac{\cancel{4.50 \; in} \times \cancel{\pi} \times r^2}{\cancel{4.50 \; in} \times \cancel{\pi}}$$

$$\frac{18.3 \; in^3}{4.50 \; in \times \pi} = r^2$$

$$r = \sqrt{\frac{18.3 \; in^3}{4.50 \; in \times \pi}} = 1.14 \; in$$

So, the radius of the can must be 1.14 inches in order to meet the manufacturer's specifications. But wait! Don't pick B as the correct answer—that's radius, not diameter! You need to double the radius to find the correct answer:

$$d = 2r = 2(1.14 \; in) = 2.28 \; in$$

The necessary diameter of each can is 2.28 inches, so D is the correct answer.

Sample Question #7

The manufacturer from Sample Question #6 now wants to create a label for the same soup can. How much paper will be needed for each soup label, assuming that the label is as tall as the can and winds completely around it?

    A. 15.7 in²

    B. 16.1 in²

    C. 31.4 in²

    D. 32.2 in²

More cans! Specifically, the same can. Be ready for questions that run together like this when you take your exam. Let's take stock of what we know about this can:

$V = 300$ mL ≈ 18.3 in³

$h = 4.5$ in

$r = 1.14$ in

$d = 2.28$ in

The trick to solving this question involves figuring out which part of the surface area equation we need. We don't need to solve for the entire surface area of the can, just the part that the label would go around—the "rectangle" that forms the can's height.

This rectangle's height is the same as the height of the entire cylinder, which we know. Let's call that the rectangle's width. The rectangle's length, then, is the circumference of either of the circles that form the top and bottom of the cylinder. The circumference of a circle is defined as $\neq d$, and since $d = 2r$, circumference can also be written as $\neq 2r$. This we can work with—we know the radius of the can from the previous question.

$$SA = 2\pi rh + 2\pi r^2 \quad \rightarrow \quad A = Ch$$
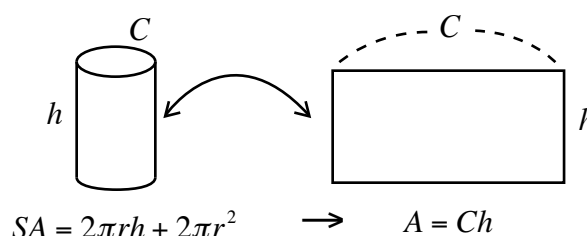
To find the area of the rectangle, then, we need to calculate the following:

$A = l \times w$

$A = \pi d \times h$

All we have to do now is substitute in the known values and solve for *A*:

$A = \pi(2.28$ in$)(4.5$ in$) = 32.2$ in²

This matches answer choice D, so D is the correct answer!

Spheres

If you spun a circle around a vertical y-axis in a third dimension, you'd end up with a sphere. While spheres may seem a bit imposing because they completely lack straight edges, you only need one piece of information—radius—to solve for either the volume or the surface area of a sphere. Keep in mind that if you are given the diameter or are able to calculate it, you can find radius from diameter by dividing it by two. So, knowing either radius or diameter is enough to find a sphere's volume.

$$V_{sphere} = \frac{4}{3}\pi r^3$$
$$SA_{sphere} = 4\pi r^2$$

Let's take a look at Sample Question #8. We're told the planetarium is in the shape of a sphere, and we're given its diameter. The given equation for volume of a sphere uses radius, so we just need to make sure that we use the diameter to find the radius before plugging that value into the equation and solving.

Sample Question #8

A spherical planetarium has a 20 m diameter. What is its volume?

    A. 750π m³
    B. 1000π m³
    C. 1333π m³
    D. 1500π m³

$$V = \frac{4}{3}\pi r^3$$
$$d = 20 \text{ m}$$
$$d = 2r$$
$$20 = 2r$$
$$r = 10 \text{ m}$$
$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi (10 \text{ m})^3$$
$$V = \frac{4}{3}\pi 1000 \text{ m}^3$$
$$V = 1333.33\pi \text{ m}^3 \approx 1333\pi \text{ m}^3$$

The planetarium's volume is 1333π m³, so the correct answer is C.

Data Analysis

Any question or group of questions accompanied by a chart, a table, or several such graphics presents you with two challenges. On the one hand, you need to understand the specific mathematical information that the format of graph you're given can convey and how this might allow you to calculate other data-related concepts; on the other hand, you need to understand how to actually obtain this information from the particular kind of graph you're given through careful reading. Failing to meet either of these challenges will prevent you from solving the problem at hand.

The following part of the book can help equip you with the knowledge you'll need to prevent the details of data analysis questions from turning into major obstacles. In the "Analyzing Data Sets" subsection, we take a look at the concepts at work in many data analysis problems, regardless of the forms their particular graphs take. We begin with an elementary examination of the details of set theory and work our way through familiar concepts like mean and median to more complex measures like standard deviation and the expected values of random variables. This part of the section focuses on the core concepts with which you'll need to be familiar regardless of the specifics of a particular graphic.

The details of graphs and charts are not insignificant, though, and so we examine them in the "Visualizing Data Sets" subsection. Here, we focus each lesson on a particular type of graph or closely related group of graphs, examining what information each kind can and can't provide and working through sample problems that ask you to use this information as the basis for a wide range of calculations you might be asked to perform on test day. We also examine the concept of normal distribution, which is closely associated with its visual depiction.

We follow our two graphics-focused sections with one about probability. It contains two lessons: one about the rules of calculating probability, and another about using the combination and permutation formulae in specific scenarios to calculate how many different ways a given number of choices can be grouped together in a particular way.

After reviewing all three subsections, you should be able to combine their distinct topics into strategies as needed to face whichever particular data analysis problem show up on your Quantitative Reasoning sections. Graphs and probability-based problems can be imposing, but they also provide a lot of information, and after this section, you'll have the basic understanding of how to recognize and make the most of all that information on test day!

Section Outline

Analyzing Data Sets

Visualizing Data Sets

Probability

Analyzing Data Sets

Data can be presented in a variety of different ways on the GRE Quantitative section, but no matter whether you're faced with a defined set of numbers, a scatter plot with a trend line, or a box-and-whisker plot, you'll need to be equipped with knowledge of a few core data interpretation concepts in order to analyze the significance of the information you're given. To begin analyzing a set of numbers in any form, you first need to understand some basic principles of set theory and set notation—concepts that apply to Venn diagrams, which we also investigate. Next, we cover the three most basic measures of central tendency: mean, median, and mode. While these concepts may be straightforward and somewhat familiar, the Quantitative section can use them in complex ways. We focus on principles of data distribution in the final two lessons in this section. First, we look at ways of measuring how disperse or clustered together a group of numeric data entries are using concepts of range, quartiles, and standard deviation. Then, we take a focused look at what you need to know about combining your data analysis skills with your probability knowledge to tackle problems involving random variables and expected values. Analyzing data sets involves a great deal of specialized language and math, but after you've mastered each piece of the puzzle, it's easy to apply your skills to data sets on the GRE, no matter the specifics of how they're presented.

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Section Outline

Sets, Set Notation, and Venn Diagrams

Central Tendency: Mean, Median, Mode

Data Distribution: Range, Quartiles, and Standard Deviation

Data Distribution: Random Variables, Sampling, and Expected Values

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Sets, Set Notation, and Venn Diagrams

Sets are formal groups of elements. When the units in a set are written out fully, the set is indicated by brackets around the elements. The "elements" in a set can be a wide variety of things: letters, numbers, words, or even people who fall into a specific category. Problems might present a set to you by writing it out entirely, or they may describe a set instead, e.g. "the set of all even numbers between 0 and 100, inclusive." Sets can be either finite or infinite. Each of the examples presented thus far has been a finite set, whereas the set of all negative numbers or the set of all even integers would each be infinite sets.

Some sample sets are shown at right; they will help illustrate a few more key rules about sets.

A = {A, A, B, B, C, D, E, E}

B = {1, 3, 5, 7, 9}

C = {3, 7, 5, 9, 1}

D = {cat, dog, mouse, snake, aardvark, tiger}

E = {A, B, C, D, E}

F = {∅}

G = {A, C, E}

Set Notation Rules

Order Doesn't Matter	Order is not significant when considering a group of numbers as a set. As a result, Set B and Set C are equal.
Duplicates Don't Count	Duplicate entries aren't significant in set notation. Thus, Set A and Set D are equal, as they each contain five elements: A, B, C, D, and E.
Empty Set	The notation used in Set F denotes the "empty set," a set with no elements in it.
Subsets	Since all of the entries in Set A are also found in Set G, we can refer to Set G as a "subset" of Set A. The empty set is a subset of every set, and every set is a subset of the universal set U.
|Set Name| Notation	The use of vertical bars when referring to sets by name indicates the number of elements in that set. For example, $|E|=5$, $|F|=0$, and $|A|=5$. (Remember, duplicate entries don't count)!

When sets overlap incompletely with other sets, we describe them using two specific terms: **union** and **intersection**. We compare the two below.

Intersection	Union
All elements in both A and B	All elements in A, B, or both A and B
A ∩ B	A ∪ B
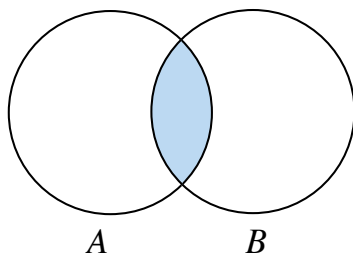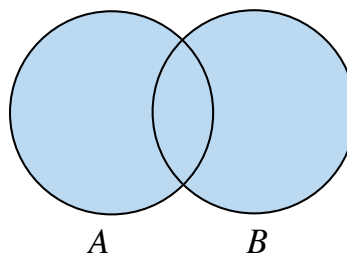
If you've read over our Calculating Probability lesson, you'll know just how significant the distinction between intersections and unions can be! Let's consider two sets, *C* and *D*, shown below.

$C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$D = \{0, 2, 6, 10, 11\}$

If we want to find the intersection of *C* and *D*, we'd need to identify the elements found both in *C* and in *D*. This would generate a set containing two elements: 2 and 6.

$C \cap D = \{2, 6\}$

If we want to find the union of C and D, we need to include any number in C, in D, or in both.

$C \cup D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

The way in which the number of elements in individual sets interacts with unions and intersections can be summed up in the following equation:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Explained verbally, this says that the number of elements in just A, just B, or both A and B is the sum of the number of elements in A and the number of elements in B minus the number of elements found in both. If we don't subtract the intersection of the sets, the elements in both A and B get counted twice: once as part of A, and once as part of B. The same concept is shown visually below.
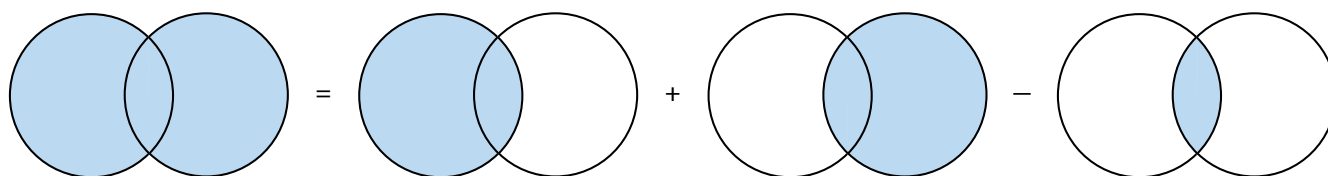
In order to find the intersection of chocolate and vanilla, it is easiest to make a venn diagram.
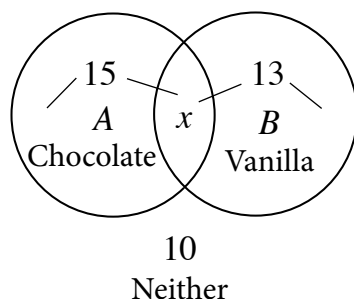
We know that $|A \cup B| = |A| + |B| - |A \cap B|$. We can treat this like an algebraic equation, filling in information that we know and solving for what we don't know. While we can't find the union of A and B using the equation, we're given more information in the probelm that can let us solve for this vaclue another way. There are 35 people in the class, and four ice-cream-opinion options: likes chocolate (15), likes vanilla (13), likes both chocolate and vanilla (?), and likes neither chocolate nor vanilla (10). The sum of all four opinions must be 35. That means that since 10 people like neither flavor, 25 people must like chocolate, vanilla, or both flavors. That's $|A \cup B|$.

Now we need to find $|A \cap B|$. Plug in A and B, we can solve for this:

$|A \cup B| = |A| + |B| - |A \cap B|$

$25 = 15 + 13 - x$
$25 = 28 - x$
$-x = -3$
$x = 3$

$|A \cap B|$, the number of people who like both chocolate and vanilla ice cream, is 3, so B is correct.

Sample Question #1

In a class, there are 15 students who like chocolate ice cream. 13 students like vanilla ice cream. 10 students like neither flavor of ice cream. If there are 35 people in the class, how many students like both ice cream flavors?

A. 2
B. 3
C. 12
D. 13

Sample Question #2

A given company has 1500 employees. Of those employees, 800 are computer science majors. 25% of those computer science majors are also mathematics majors. That group of computer science/math dual majors makes up one third of the total mathematics majors. How many employees have majors other than computer science and mathematics?

    A. 100
    B. 250
    C. 300
    D. 500

To solve Sample Question #2, we'll want to sketch out another Venn diagram. We can construct the following diagram based on the information we're given in the problem:

$$|A \cup B| = y$$

800   m

A x B

Computer Science   Math

Other Majors = z

$$|A \cup B| + z = 1500$$
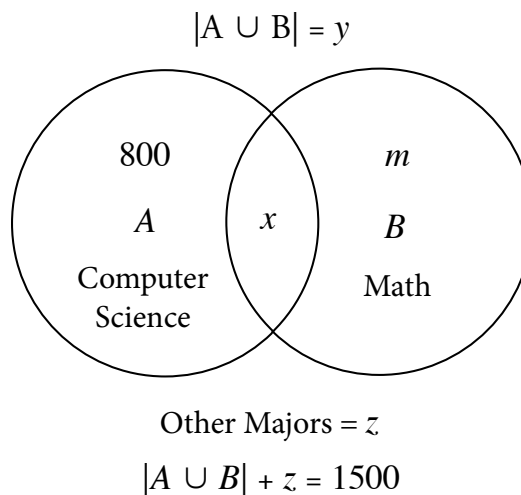
If 25% of the 800 computer science majors are also math majors, the number of people with both of these majors (x) is $800 \times 0.25 = 200$. Furthermore, this represents one third of the total of math students:

$$\frac{1}{3}m = 200$$

$$3\left(\frac{1}{3}m\right) = 3(200)$$

$$m = 600$$

So, if 600 employees majored in math, and 200 majored in both computer science and math, we can subtract the dual major group from the total number of math majors (including dual majors) to find that 400 employees must have majored only in math.

We can now fill in our diagram, but we still need to find $|A \cup B|$. Be careful not to double-add the intersection! The easiest way to do this is to take the intersection and add to it the number of computer science-only and math-only

Both Computer Science and Math

600   400

A 200 B

Computer Science Only   Math Only

Computer Science: 800   Math: 600

students: $600 + 200 + 400 = 1200$. This number represents the total number of students that have either a math or computer science major (that is, the number of students in the union of the two sets). This leaves $1500 - 1200$ or 300 students, making C the correct answer.
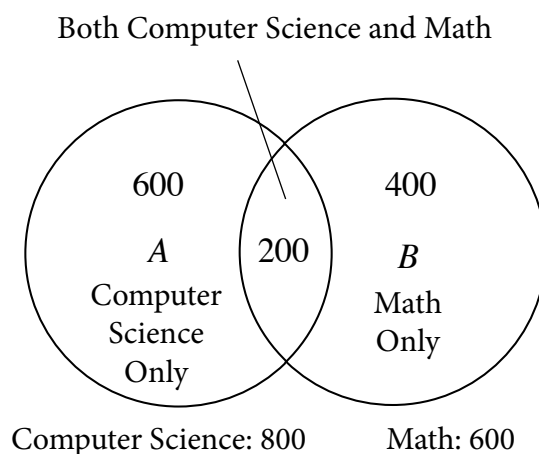
Central Tendency: Mean, Median, and Mode

Statistics is the primary field of study used to derive meaningful information from aggregate data. By applying statistical analyses to a given data set, we can easily determine trends and patterns in the data. These findings can help us draw conclusions about the applicable meanings of the data set. Basic statistical principles can help determine a "normal" value for a set, and, subsequently, can help us identify outliers. In a practical sense, these measurements are useful in recognizing changes and abnormalities (or lack thereof) in collected data, allowing us to draw meaningful conclusions about the information from a given study or sample. This lesson will look at three common statistical measures used to evaluate a central measure of a data set: mean, median, and mode.

Mean

The mean of a data set—also called the average—is used to find the exact center point of the data set. In a mathematical sense, finding the center point of the data requires you to find the sum of all data points, and then divide by the number of data points. The formula for the mean is shown using summation notation at right.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\bar{x}$ = mean
n = number of terms in set

Let's try applying this formula. For the set 3, 19, 6, 2, 9, 12, the mean will be equal to $(3 + 19 + 6 + 2 + 9 + 12) \div 6$. We divide by 6 because there are six terms in the data set. The mean of 3, 19, 6, 2, 9, 12 is 8.5.

$$\frac{3 + 19 + 6 + 2 + 9 + 12}{6} = \frac{51}{6} = 8.5$$

In a practical sense, finding the mean answers the question, "What would the value of each data point be if all of the data points were exactly equal?" In a study that is looking for consistency of data, the values in the data set should be extremely close to the mean. Values that range significant amounts around the mean demonstrate variability and inconsistency.

We can also apply this formula to data found in story problems, like the data set given in Sample Question #1.

Sample Question #1

A scientist is doing a study to determine the typical levels of carbon monoxide in the air of automotive garages. She measures the air quality at eight different garages and gets the following results: 13 ppm, 26 ppm, 4 ppm, 36 ppm, 12 ppm, 19 ppm, 5 ppm, and 9 ppm. What is the mean of her data set?

A. 14.5 ppm
B. 15.0 ppm
C. 15.5 ppm
D. 17.0 ppm

Going through the same sum-and-divide-by-total process, we can see that the correct answer is C.

$$\frac{13 + 26 + 4 + 36 + 12 + 19 + 5 + 9}{8}$$

$$\frac{124}{8} = 15.5 \text{ ppm}$$

Mean questions aren't always as simple as working from a fully provided data set, however. Just like some questions may ask you to solve for a variable in an equation that you don't usually solve for, mean questions can provide you with the average and ask you to work backwards to determine a missing value from the data set. For an example of such a problem, consider Sample Question #2. We can start by setting up a mathematical model of this problem:

$$\frac{92 + 74 + 86 + 97 + x}{5} = 88$$

x represents the missing test score. Together, the four past test scores and the final test show that there are a total of five data points in the set, so we need to divide by 5. Finally, we are told that the average must be equal to 88. Now, use the equation to solve for the missing test score, x.

$$\frac{92 + 74 + 86 + 97 + x}{5} = 88$$

$$\frac{349 + x}{5} = 88$$

$$349 + x = 5 \cdot 88$$

$$349 + x = 440$$

$$x = 440 - 349$$

$$x = 91$$

Jeanine needs to get a score of 91 on her final test in order to achieve her goal.

Let's work through a more abstract sample question to make sure your knowlege of how to work with means is cemented.

Sample Question #2

Jeanine needs to get an average of 88 in her biology class in order to maintain her GPA. If her previous test scores were 92, 74, 86, and 97, what score does she need to get on her final test in order to achieve her goal?

    A. 89

    B. 90

    C. 91

    D. 92

Sample Question #3

Lillian has a potted plant that grows exactly 3 inches each day. If the plant has a height of h on the first day, what is its average height between days 1 and 5?

    A. $h+3$

    B. $h+4$

    C. $h+5$

    D. $h+6$

Algebraically, the mean is a relatively simple representation. Just remember the formula: the mean is equal to the sum of the data, divided by the number of terms. We can apply this formula even if some of the values we're working with are unknown variables like x. We know that the plant grows 3 inches each day, and begins at height h; this allows us to write out the following:

Day 1: h

Day 2: $h+3$

Day 3: $h+3+3$

Day 4: $h+3+3+3$

Day 5: $h+3+3+3+3$

At this point, we can apply the mean formula to these expressions to find that the correct answer is $h+6$, D.

$$\frac{(h) + (h + 3) + (h + 3 + 3) + (h + 3 + 3 + 3) + (h + 3 + 3 + 3 + 3)}{5}$$

$$\frac{5h + 30}{5}$$

$$\frac{5h}{5} + \frac{30}{5}$$

$$h + 6$$

Median

While the mean is designed to identify the most central *value* of a data set, the median is used to identify the most central *term* in the given data. The mean is indiscriminate of data distribution and can be severely impacted by an outlier. In contrast, the median is much more robust to outliers, as it relies less on aggregate data and more on the individual terms present in the set.

Let's find the median of the following data set: $\{93, 45, 61, 18, 109, 22, 88\}$

To find the median, the data must first be arranged in order from smallest to largest. Next, identify the central term in the set. In the instance of a set with an even number of terms, take the mean of the two centermost terms.

First, order the terms.

$\{18, 22, 45, 61, 88, 93, 109\}$

Next, identify the centermost term.

$\{\cancel{18, 22, 45,}\ 61,\ \cancel{88, 93, 109}\}$

The median of this set is 61.

When a set contains an even number of terms, there will be two "central" terms. Here's an example of such a data set:

$\{1, 4, 4, 6, 9, 13, 17, 19, 23, 23\}$

$\{\cancel{1, 4, 4, 6,}\ 9, 13, \cancel{17, 19, 23, 23}\}$

In this case, the median is the average of the two central terms.

$$\frac{9+13}{2} = \frac{22}{2} = 11$$

The median of this set is 11.

As an illustration of how mean and median differ, let's compare these two values for the following set of data:

$\{34, 36, 39, 28, 436\}$

The average of this set is:

$$\frac{34 + 36 + 39 + 28 + 436}{5} = \frac{573}{5} = 114.6$$

The median of this set is:

$\{28, 34, 36, 39, 436\}$

$\{\cancel{28, 34,}\ 36, \cancel{39, 436}\}$

Clearly, in this case, the average is affected by the outlier (436) and becomes almost entirely meaningless. In contrast, the median of 36 is still relatively representative of the data at hand.

Let's try working through Sample Question #4, which introduces extra complexity to the median concepts we've been reviewing.

Sample Question #4

In Billy's fifth grade class, the test scores for the final math exam were, 87, 54, 63, 77, 92, 95, 91, x, and 90. The median of the test scores is 90.5.

Quantity A	Quantity B
The value of x	90

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

First, reorder the data set in ascending order. For now, let's position x last, as if it's the largest value.

$$\{54, 63, 77, 87, 90, 92, 92, 93, 95, x\}$$

Next, identify which entry should hold the median based on the number of values in the set. Since the set has an even number of data entries, the median will be the average of the two middle numbers.

If x is the largest value in the set, the two middle numbers are located at entries 5 and 6.

$$\{\cancel{54}, \cancel{63}, \cancel{77}, \cancel{87}, 90, 92, \cancel{92}, \cancel{93}, \cancel{95}, \cancel{x}\}$$

Take the average of these two numbers to see if it will be the desired median.

$$\frac{90 + 92}{2} = \frac{182}{2} = 91 \neq 90.5$$

Let's reorder our data set with the x on the other end of the set.

$$\{x, 54, 63, 77, 87, 90, 92, 92, 93, 95\}$$

$$\{\cancel{x}, \cancel{54}, \cancel{63}, \cancel{77}, 87, 90, \cancel{92}, \cancel{92}, \cancel{93}, \cancel{95}\}$$

$$\frac{87 + 90}{2} = \frac{177}{2} = 88.5 \neq 90.5$$

Now we know that the x-value will need to be one of the middle numbers that the average is being taken from; therefore, we can rewrite the set in one of two ways:

$$\{54, 63, 77, 87, x, 90, 92, 92, 93, 95\}$$
$$\{54, 63, 77, 87, 90, x, 92, 92, 93, 95\}$$

No matter which way we organize our set, our median will be the average of x and 90.

$$\{\cancel{54}, \cancel{63}, \cancel{77}, \cancel{87}, x, \cancel{90}, \cancel{92}, \cancel{92}, \cancel{93}, \cancel{95}\}$$
$$\{\cancel{54}, \cancel{63}, \cancel{77}, \cancel{87}, 90, x, \cancel{92}, \cancel{92}, \cancel{93}, \cancel{95}\}$$

We can now find the value of x: 91. $91 > 90$, so A is the correct answer.

$$\frac{x + 90}{2} = 90.5$$
$$\frac{x + 90}{2} \times 2 = 90.5 \times 2$$
$$x + 90 = 181$$
$$(x + 90) - 90 = 181 - 90$$
$$x = 91$$

Mode

The mode of a data set is arguably the easiest statistical measure to calculate: it is simply the term that appears with the greatest frequency in the data set. In other words, the mode is the number/term that appears the most. For example, the mode of the data set $\{5, 8, 12, 5, 7, 3\}$ is 5. The value 5 appears twice, while every other term appears only once.

When two terms appear with the same high frequency, the data set is said to have multiple modes. Do not average the modes! For an example of this situation, consider the following data set:

$$\{105, 119, 136, 121, 119, 133, 136\}$$

The modes are 119 and 136. Both of these values appear twice, while every other term appears only once. Thus, they must both be included in the answer.

When dealing with larger data sets, remember that the mode is the term that appears with the greatest frequency. Sometimes the mode will appear twice; other times there will be more complex distributions. Consider the following data set:

$$\{43, 45, 43, 47, 47, 44, 45, 43, 45, 44, 43, 47, 42\}$$

Rearrange the set to make the distribution more apparent.

$$\{42, 43, 43, 43, 43, 44, 44, 45, 45, 45, 47, 47, 47\}$$

The mode of this set is 43. The value 43 appears four times, the value 45 appears three times, the value 47 appears three times, and the value 44 appears two times. Despite the multiplicity of other terms, 43 is the only mode of this set because it appears the most.

In cases where every term in the set appears once, the set is said to have no mode.

Example:

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

In this set, each term appears exactly once, so there is no mode.

Let's consider Sample Question #5 to make sure that you can put this mode knowledge into practice on test day. To compare the mode to another value, we'll need to first define it, and to define it, we'll need to start by rearranging the data set:

$$\{33, 33, 33, 34, 34, 34, 34, 35, 35, 35, 36, 36, 36, 37\}$$

The value 34 appears four times. The values 33, 35, and 36 each appear three times. The mode of the data set is thus 34, and since 34 is a smaller value than 35, A is the correct answer.

Sample Question #5

Consider the following data set:

$$\{34, 33, 36, 34, 35, 33, 35, 34, 36, 36, 33, 35, 37, 34\}$$

Quantity A	Quantity B
35	The mode of the set

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Data Distribution: Range, Quartiles, Standard Deviation

When working with data sets or lists of numbers, we have numerous ways in which we can analyze the distribution of the individual data points—that is, how the points relate to one another. While we've covered mean and median in *a previous lesson*, we can calculate other measurements that focus less on giving us a clear picture of the central tendency of a data set and more on establishing how wide-ranging the data is, and the distribution of points across the set's entire range. For instance, consider the following sample data sets:

$\{-75, -42, -28, 2, 7, 25, 68, 79\}$

$\{1, 2, 3, 4, 5, 6, 7, 8\}$

$\{1, 1, 1, 1, 1, 1, 1, 31\}$

Each of these three sets has the same mean, and the first two sets have the same median.

$$\frac{1+1+2+2+10+10+10}{8} = 4.5$$

$$\frac{1+2+3+4+5+6+7+8}{8} = 4.5$$

$$\frac{1+1+1+1+1+1+1+31}{8} = 4.5$$

$\{-75, -42, -28, 2, 7, 25, 68, 79\}$

$$\frac{2+7}{2} = \frac{9}{2} = 4.5$$

$\{1, 2, 3, 4, 5, 6, 7, 8\}$

$$\frac{4+5}{2} = \frac{9}{2} = 4.5$$

While mean and median can be very useful as measures of central tendency, they don't tell us anything about how far apart the values in a set are. The data points can be clustered together or very distant from one another and still yield the same mean and median.

In the rest of this lesson, we'll investigate a few measures that tell us about the relative distance from one data point to another in a given data set. Combined with mean and median, these measurements can provide a well-rounded picture of a data set.

Range

The range of a data set is the difference between its largest value and its smallest value. The smaller the range of a set, the closer the data entries are to one another. If the range is large, then either there is significant space between the data entries or there are outliers in the data. Calculating the range is simple and just requires you to identify the largest and smallest value in a set and find the difference between them. Keep in mind that the range is a measure of distance, so it is always a positive value, regardless of whether subtracting one value from another yields a negative result. As an example, let's find the range of the following data set:

$\{2, 5, 1, 2, -7, 6, 19\}$

Arranging the set in ascending order can make it simple to pick out the least and greatest values.

$$\{2, 5, 1, 2, -7, 6, 19\}$$

$$\{-7, 1, 2, 2, 5, 6, 19\}$$

The least value in this data set is –7 and the greatest value is 19. To find the range, we just find the difference between these values. Since range is always positive, the order in which we subtract one of them from the other doesn't matter. We get the same answer either way:

$$(-7) - (19) = -26 \rightarrow 26$$

$$(19) - (-7) = 26$$

The range of this data set is 26. Let's try answering a more difficult question: Sample Question #1.

To answer this question, we'll need to test out various values of x that change its position in the data set relative to the other elements and see how this affects the range. We're told that the range is 12, so we need to pick out each of the listed values of x for which this is true of the set.

Let's start by finding the range of the set assuming x lies in the middle of the set.

$$\text{Range} = 12 - 2 = 10 \neq 12$$

This tells us that x must be either greater than 12 or less than 2. Now we need to set up arrangements of the set and equations to solve for the possible values of x.

Sample Question #1
If the range of the data set shown below is 12, what is the sum of all the possible values of x? $$\{2, 3, x, 5, 12\}$$ **A.** 14 **B.** 15 **C.** 21 **D.** 22

Possibility 1: $\{x, 2, 3, 5, 12\}$ Possibility 2: $\{2, 3, 5, 12, x\}$
Range = $12 - x = 12$ Range = $x - 2 = 12$
$x = 0$ $x - 2 + 2 = 12 + 2$
                             $x = 14$

For the range of this particular data set to be 12, x must be 0 or 14. $0 + 14 = 14$, so A is correct.

Quartiles

The spread of data in a data set can also be analyzed in terms of how it falls across the data set divided up into four parts. These four parts are called "quartiles." A "quartile" is a span of one quarter (25%) of the data when it is ordered from least to greatest in value. These four quartiles are bounded by five specific points. The second quartile, "Q2," is defined as the median of the entire data set. The first quartile, "Q1," is defined as the median of the lower half of the data set, and the third quartile, "Q3," is defined as the median of the upper half of the data set. (There's some variation in how these values are calculated, but the GRE uses these particular definitions). When you consider these three figures along with the smallest number and largest number in the data set, you have five distinct points in the data set which serve as decisive boundaries dividing it into four groups.

Perhaps confusingly, these groups are also referred to as "quartiles." When you refer to a number "as" a quartile, it means you're talking about it as the distinct value of Q1, Q2, or Q3. When you a number is described as being "in" or "below" a quartile, you're referring to the *range* of numbers between two of the distinct values.

One more measurement is associated with quartiles: the "interquartile range," or the difference between Q1 and Q3. Like other range calculations, it measures distance, so it will be positive.

	Q1		Q2		Q3	
	First Quartile	Second Quartile	Third Quartile	Fourth Quartile	
Smallest Number	Median of Lower Half of Data Set	Median of Data Set	Median of Upper Half of Data Set	Largest Number

Let's find the quartiles of the following data set for practice and use the above chart to visualize them. (Note that this chart is similar to *a box-and-whisker plot*, a type of figure specifically used to visually convey a set's data dispersion based on its quartile values).

$$\{10, 10, 11, 12, 13, 14, 21, 22, 25, 26, 29, 35, 37, 38\}$$

Q2 is the median of the data set and divides the set into the halves we use to calculate Q1 and Q3, so let's start by finding it.

$$\{\cancel{10,10,11,12,13,14}, 21, 22, \cancel{25,26,29,35,37,38}\}$$

$$Q2 = \text{Median} = \frac{21 + 22}{2} = 21.5$$

Q2 divides our data set into two halves.

$$\{10, 10, 11, 12, 13, 14, 21, \mid 22, 25, 26, 29, 35, 37, 38\}$$

Q1 is the median of the lower half, and Q3 the median of the upper half.

$$\{\cancel{10, 10, 11,} 12, \cancel{13, 14, 21}\} \qquad \{\cancel{22, 25, 26,} 29, \cancel{35, 37, 38}\}$$

$$Q1 = 12 \qquad\qquad\qquad Q3 = 29$$

We've now divided our data set into quartiles!
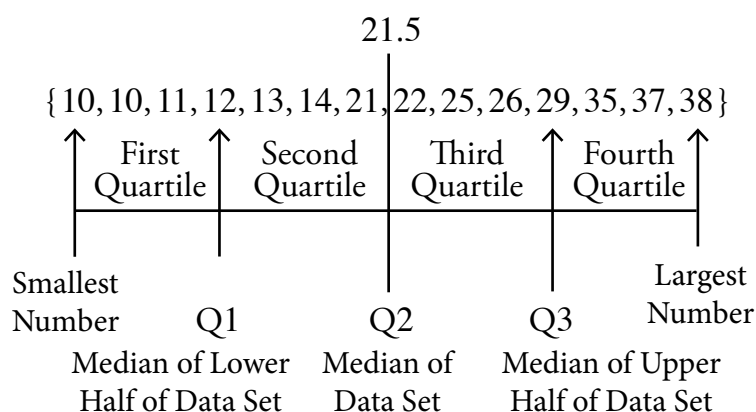
Let's also find the interquartile range of our data set. Since we've already calculated Q1 and Q3, this is as easy as doing a single subtraction problem. For our set, the interquartile range (IR) is 17.

$$Q3 - Q1 = 29 - 12 = 17$$

The GRE might also present data using "percentiles." These are similar to quartiles, but instead of dividing the data into four spans, they divide the data into 100 spans. You might be familiar with this statistical concept based on how many test scores are reported, including the GRE's. At this point, it should be apparent why quartiles are more reliable measures of scores when thousands of people take a given exam. As an example, consider a hypothetical score in the "93rd percentile." If no one does very well on the exam, the 93rd percentile might be a relatively low overall score, but if everyone does very well on the exam, the 93rd percentile would have to be a near-perfect score. The actual raw score can vary significantly depending on the relative performance of everyone who takes the exam, but percentiles still give you a good idea of how one score relates to the rest of them.

Standard Deviation

Standard deviation is a slightly more involved calculation but describes the spread or variance of the data in relation to the mean. To find the standard deviation, the mean needs to be calculated first. From there, the difference of each data value and the mean will need to be squared. After that, all the squared terms will need to be added together and then divided by the number of data entries. The final step to calculate standard deviation is taking the square root. The formula for standard deviation in mathematical terms is as follows.

$$\sigma = \sqrt{\frac{1}{N} \sum_i =1^N (x_i - \overline{x})^2}$$

σ = Standard Deviation

N = Number of Data Entries

x_i = Individual Data Values

$\overline{x}$ = Mean

Let's find the standard deviation of the set $\{2, 3, 1, 6, 7, 8\}$ for practice. If you do this on test day, make sure to keep track of your scratch work. There are a lot of distinct calculations involved that feed into one another, and it's easy to get them mixed up. First, calculate the mean.

$$\frac{2 + 3 + 1 + 6 + 7 + 8}{6} = \frac{27}{6} = 4.5$$

Next, find the difference between each value in the set and the mean, and square it.

$$(2 - 4.5)^2 = (-2.5)^2 = 6.25$$
$$(3 - 4.5)^2 = (-1.5)^2 = 2.25$$
$$(1 - 4.5)^2 = (-3.5)^2 = 12.25$$
$$(6 - 4.5)^2 = (1.5)^2 = 2.25$$
$$(7 - 4.5)^2 = (2.5)^2 = 6.25$$
$$(8 - 4.5)^2 = (3.5)^2 = 12.25$$

After that, add all those values you just found together.

$$6.25 + 2.25 + 12.25 + 2.25 + 6.25 + 12.25 = 41.5$$

Divide the sum of those values by the number of elements in the data set.

$$\frac{41.5}{6} = 6.917$$

The square root of the value you just found is the standard deviation. For our set, it's 2.63.

$$\sqrt{6.917} = 2.63$$

Even if you're not asked to calculate the standard deviation directly on the exam, certain questions could ask you to consider it conceptually, perhaps providing it and asking you to work with it along with concepts like standard deviation.

The standard deviation is the amount of variance from the mean that exists within a given data set. We're told that the population under study has a normal distribution, so we can divide the set up by adding and subtracting the standard deviation from the mean. (See calculations assocaited with graph below). Heights that are within one standard deviation of the mean will thus fall between 68.2 and 75.8 inches, and heights that are within two standard deviations of the mean will fall between 64.4 and 79.6 inches. (Note that we can apply the 68-95-99.7 rule to this data because of its normal distribution, meaning that 68% of the population would fall within one standard deviation of the mean, 95% within two standard deviations of the mean, and 99.7% within three).

Our question asks us to identify the values between one and two standard deviations away from the mean. At this point, we can tell that we need to pick out answer choices between 64.4 and 68.2 or between 75.8 and 79.6. This means that A, C, and D are correct!

Sample Question #2

Multiple answers may be correct. Select all that apply.

In the United States, a study measured the average height of males. The study found that in a representative, randomly selected population the average height of men was 72 inches. Further tests were conducted to find that the data set had a normal distribution and standard deviation of 3.8 inches. Which of the following heights could fall between one and two standard devitions away from the mean?

A. 65.3 in
B. 68.7 in
C. 76.2 in
D. 78.4 in

$\mu - 2\sigma$	$\mu - \sigma$	μ	$\mu + \sigma$	$\mu + 2\sigma$
72 in – 2(3.8 in)	72 in – 3.8 in	72 in	72 in + 3.8 in	72 in + 2(3.8 in)
64.4 in	68.2 in		75.8 in	79.6 in
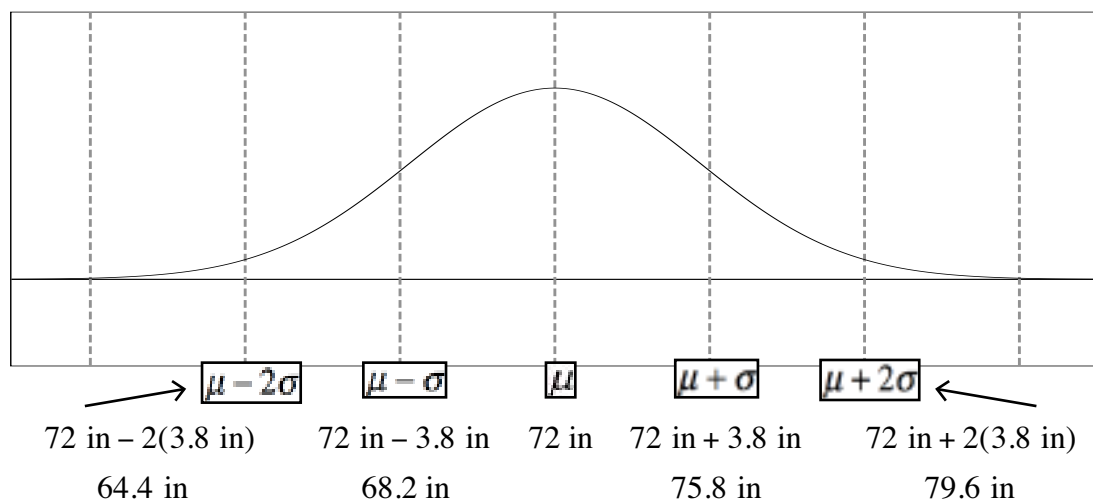
Data Distribution:
Random Variables, Sampling, and Expected Values

You're already familiar with the concept of finding the mean of a presented data set, and also with the concept of probability. Certain GRE Quantitative problems ask you to combine your skills and knowledge related to these two fields of mathematics in ways that might not initially seem intuitive. In this lesson, we're going to introduce you to some of the specific vocabulary such questions might employ and demonstrate algebraic procedures you can use to solve common question setups.

Before we get to the questions, though, we need to introduce a new mindset of approaching a presented data set, no matter whether it is given in the form of a set of numbers in brackets, a table of data, as a histogram, or in any other form. Consider the following list, a collection of entries showing how many days various survey respondents exercised for at least half an hour:

    0, 0, 1, 2, 3, 3, 3, 3, 4, 4, 7, 7

If you were asked to find the probability that a survey respondent exercised three days per week, how would you find it? You'd need to consider each of the list's numbers as one possible result of the action of picking a number. After this paradigm shift, the problem might not look so difficult, as you could apply your standard rules of finding the probability of a given event. You could create a fraction: the number of particular outcomes over number of possible outcomes. There are four "3" elements in this list, so four (not three!) would be the numerator, and there are twelve total numbers in the list, so twelve would form the denominator.

$$P(3) = \frac{4 \text{ 3's}}{12 \text{ Total Numbers}} = \frac{4}{12} = \frac{1}{3}$$

Let's take a moment to analyze the perspectival shift that you had to make in order to solve that problem. Instead of viewing the numbers in the list for their numerical value, you viewed them in terms of **frequency** as entries that happened to have different numerical values. From this perspective, the list might have been composed of the names of different animals or of colors—you'd still just be interested in how many times it showed up in the list, not in its value. We could display our data about how many times each number appeared in the list in a type of table called a **frequency distribution**, shown at right.

Days Exercised	Frequency
0	2
1	1
2	1
3	4
4	1
7	2

A question might define z as the number of times a survey respondent exercised per week. z would be a **random variable**—one with a value determined by picking from a group of numbers at random. Above, we found the probability that $z = 3$. We could call this probability "P(3)" or "P($z = 3$)"—it would represent the same quantity. We could calculate probabilities based on inequalities with z in them as well—if asked to find P($z < 2$), we'd find the probability that a survey respondent exercised zero days per week and the probability that respondent exercised one day per week and add them together.

If we calculate the probability of *z* equaling each number in our list, we can generate a new set of data related to that initial list, shown at right. If we display our probabilities of picking each individual number from the data set in the form of a table, it's called a **probability distribution**. Note that if we add up all of the probabilities of drawing each number from the list, we get 1:

$$P(0) + P(1) + P(2) + P(3) + P(4) + P(7) = 1$$

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{12} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = 1$$

$$P(0) = \frac{2}{12} = \frac{1}{6}$$

$$P(1) = \frac{1}{12}$$

$$P(2) = \frac{1}{12}$$

$$P(3) = \frac{4}{12} = \frac{1}{3}$$

$$P(4) = \frac{2}{12} = \frac{1}{6}$$

$$P(7) = \frac{2}{12} = \frac{1}{6}$$

z	P(z)
0	0.167
1	0.083
2	0.083
3	0.333
4	0.167
7	0.167

Even without doing the fraction math yourself, you can expect this, because the probability of drawing one of the numbers in the list from all of the numbers in the list is certain—1, in the form of probability math. This makes it easy for us to display probability-data-about-data as a histogram, specifically referred to as a **relative frequency distribution**. Note that the word "relative" is added here because we consider the entire set to be 100% and look at each entry as having a weight relative to the set as a whole.

Here's the takeaway for this entire type of data analysis math: **relative frequency and probability are interchangeable ways of looking at the same numeric value from different perspectives**. How much of the data set is composed of 4's? 0.167, or 16.7%. What's the probability of drawing a 4 out of all the numbers in the data set? 0.167, or 16.7%. How many of the survey respondents exercise for exactly four days per week? 0.167, or 16.7%. What's the probability that if *z* equals the number of survey respondents, that *z* = 3? 0.167, or 16.7%. What area of a histogram will the bar representing survey responders who exercised four days per week take up relative to the are of the rest of the bars? 0.167, or 16.7%.

The GRE Quantitative section might ask you to find a specific value on your exam. It might refer to it as the **mean of the distribution**, **the mean of the random variable**, or the **expected value** for a given set of data. These are all calculated in the exact same way. Finding the mean of the distribution requires some sophisticated analysis combining data entries' numerical values with their frequency. Just remember that the mean of the distribution, by any of its names, indicates the mean *with consideration of frequency*—in other words, giving each entry its proper weight in the calculation based on how frequently it appears in the data set. If you've ever done some algebra to calculate your grade in a class based on your grades on assignments and exams that are worth different percentages of your grade, this math should seem pretty familiar, if employed in a new context.

Let's find the mean of the distibution for our data set. We need to take each numerical value (number of days exercised) and multiply it by its relative frequency. Remember, you calculate relative frequency in the same way as you calulate probability of drawing that value from the entire set!

A general formula you can use to calculate these quantities is

$$P(\text{Entry 1})\text{Value}_{\text{Entry 1}} + P(\text{Entry 2}) \times \text{Value}_{\text{Entry 2}} \ldots$$

For our data set, that would be

$$\frac{1}{6}(0) + \frac{1}{12}(1) + \frac{1}{12}(2) + \frac{1}{3}(3) + \frac{1}{6}(4) + \frac{1}{6}(7)$$

Luckily, there's a simpler way to perform this calculation. It requires using each of the frequency fractions with a common denominator of 12.

$$\frac{2}{12}(0) + \frac{1}{12}(1) + \frac{1}{12}(2) + \frac{4}{12}(3) + \frac{2}{12}(4) + \frac{2}{12}(7)$$

We can simplify this expression by combining the common denominators and making one large fraction:

$$\frac{2(0) + 1(1) + 1(2) + 4(3) + 2(4) + 2(7)}{12}$$

This is easier to solve, so let's get to it!

$$\frac{0 + 1 + 2 + 12 + 8 + 14}{12} = \frac{37}{12} = 3.083$$

The mean of the distribution is 3.083. This is a weighted average of all of the values in the data set.

Much of the difficulty in solving these types of data distribution problems comes in the wide vareity of forms that such problems can take. Consider that you could be asked to calculate the mean of the distribution given any of the following information.

1. A directly presented numerical data set
2. A frequency distribution table
3. A table showing relative frequency
4. A relative frequency distribution (histogram of relative frequency)
5. A probability distribution table
6. A probability distribution (histogram of probability)

Remember, if you are asked to calculate the "mean of the distribution," "mean of the random variable," or the "expected value," you need to know two things about each entry: its numerical value and its relative frequency. **Relative frequency is the same as the probability of picking that entry out of all of them at random.** Questions about relative frequency might be asked in terms of a random variable (e.g. "z represents the number of times a survey respondent exercised per week. What is the probability that $z < 0$?"), but don't let that throw you off.

Let's try out some sample questions so that you can practice using this vocabuary and interpreting data presented in different ways.

Keep in mind that in answering Sample Question #1, we'll need to calculate two different means for the given data set: one unweighted (Quantity A) and one weighted by frequency (Quantity B). These are distinct values and could be very different depending on how the data is distributed, so don't immediately pick (C) just because you see the word "mean" twice!

Let's start by finding Quantity A. There are 11 elements in this data set, so we'll sum them and divide that sum by 11:

$$\frac{2 + 2 + 8 + 11 + 11 + 11 + 12 + 15 + 15 + 18 + 36}{11} = \frac{141}{11} = 12.82$$

To find Quantity B, we'll need to find the relative frequency of each data point's number and mulitply it by that point's value.

$$\frac{2}{11}(2) + \frac{1}{11}(8) + \frac{3}{11}(11) + \frac{1}{11}(12) + \frac{2}{11}(15) + \frac{1}{11}(18) + \frac{1}{11}(36)$$

Remember, if you don't reduce your relative frequency fractions, you can bundle them together using a single denominator for the entire expression, which will save a lot of fraction math!

$$\frac{2(2) + 1(8) + 3(11) + 1(12) + 2(15) + 1(18) + 1(36)}{11}$$

$$\frac{4 + 8 + 33 + 12 + 30 + 18 + 36}{11}$$

$$\frac{141}{11} = 12.82$$

Look at that! The weighted mean is exactly the same as the unweighted mean, so it turns out that C is the correct answer after all! Keep in mind we needed to do that calculation to confirm this, though, because as we've seen, it's possbile that the mean and the mean of the distribution are very different values.

Sample Question #1

{2, 2, 8, 11, 11, 11, 12, 15, 15, 18, 36}

Quantity A	Quantity B
The mean of the data set	The mean of the distribution for the data set

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

Sample Question #2
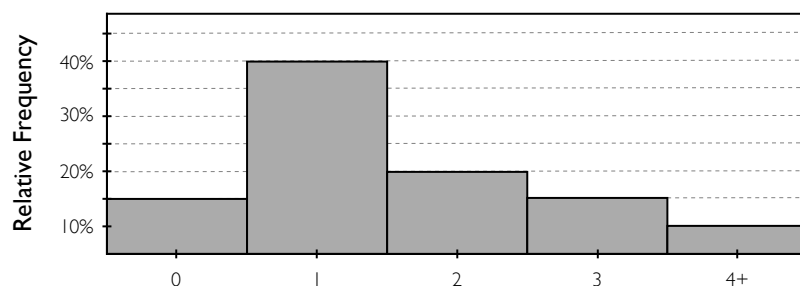
If this relative frequency distribution was calculated from the results of a survey in which 14 of the participants answered "1," how many people were surveyed in total?

A. 35 people
B. 37 people
C. 41 people
D. 42 people

This next sample question does not directly provide a data set for us to analyze; instead, it gives us a relative frequency distribution to work with. Before we can answer this problem or even plan out our calculations, we need to figure out what, exactly, this histogram is conveying. "Relative" tells us that the data are being considered as they relate to the group as a whole, considered as 100%. The actual data being shown is the frequency of each type of response to the survey. 40% of the responses received were "1" responses since the "1" column lines up with a relative frequency of 40% on the y-axis.

Now we can start our calculations. We need to reverse engineer the total number of people surveyed, and we're given the relative frequencies of each response as well as one more piece of information: 14 people responded "1" to the survey. That's exactly what we need—a definite number of people to associate with a response and its relative frequency. The "1" bar indicates 14 people, then, and those 14 people must represent 40% of the entire data set, based on the relative frequency distribution. Knowing that, we can proceed in a few different ways to arrive at the same information.

We can set up a proportion to solve this problem. We know that 14 is 40% and we need to find 100%. By treating these percents as whole numbers, we can set up a proportion and cross-multiply to solve for the total number of people surveyed. Alternatively, we can just ask, "14 is 40% of what?" and translate this question into algebra: Either way, we arrive at the correct answer, 35—A.

$$\frac{14}{40} = \frac{x}{100}$$
$$40x = 1400$$
$$x = \frac{1400}{40} = 35$$

$$14 = 0.40 \times x$$
$$x = \frac{14}{0.40} = 35$$

The last sample question we'll look at in this lesson gives us a probability distribution table and asks us to solve for the "expected value" of *s*. This is the same thing as the "mean of the distribution" or the "mean of the random variable *s*." We're given probabilities in the form of decimals, but that works well in our formula: since the denominator would be 1 if we turned these values into fractions, we don't need to write our formula as a fraction at all. C is the correct answer!

$$\left(P(0) \times 0\right) + \left(P(1) \times 1\right) + \left(P(2) \times 2\right) + \left(P(3) \times 3\right)$$

$$0.245(0) + 0.485(1) + 0.175(2) + 0.095(3)$$

$$0 + 0.485 + 0.350 + 0.285$$

$$1.12$$

s	*P(s)*
0	0.245
1	0.485
2	0.175
3	0.095

What is the expected value of *s*?

A. 0.955
B. 1
C. 1.12
D. 1.16

Visualizing Data Sets

Data can be presented visually in a panoply of different formats, many of which the GRE Quantitative Reasoning section can feature in its Data Analysis questions. In this next section, we look at different types of graphs and charts, explaining what particular type of data each one is suited to display and how to read each kind of graphic for all of the information that it conveys. Reviewing how to work with each of these graphs and charts can help you identify their distinctions and similarities, characterizing each one as a unique context in which data can be shown and preparing you to face with confidence whichever one might appear on your particular Quantitative sections. In addition to discussing box-and-whisker plots, scatterplots, line graphs, bar graphs, histograms, and pie charts, we also include a lesson on the normal distribution: what it is, what it signifies, and what you need to know to understand it at the GRE level. Questions involving complex graphics should seem a lot less scary after reading through this section!

Section Outline

Box-and-Whisker Plots

Scatterplots, Trends, and Line Graphs

Bar Graphs and Histograms

Circle Graphs and Pie Charts

Visualizing Normal Distributions

Box-and-Whisker Plots

Sets of data can be imposing when they're presented as long lists of numbers. Furthermore, when they're presented in this fashion, the list format gives us very little information about the structure of the data. Do these numbers cover a large span of values? Are most of them close to one value, with a few outliers, or are they evenly spread out? It takes more than a quick glance at a set to figure out these qualities: mathematical calculations are involved.

Much like how a graph can bring a visual dimension to data in a table that allows it to be easily read and interpreted, box-and-whisker plots help convey a great deal of information about numerical sets with no calculations required on the part of the observer. Instead, box-and-whisker plots incorporate the results of common calculations like mean, median, and range into their structure.

While the GRE Quantitative section won't ask you to come up with a box-and-whisker plot on your own from scratch, seeing how these plots are constructed can help cement your knowledge of what each aspect of their structure conveys. Let's walk through constructing one for this data set:

$$\{24, 36, 27, 42, 39, 40, 61, 57, 19, 22, 34, 37, 38, 40, 24, 20\}$$

That's quite a few numbers, and they're all jumbled up in no particular order. Let's start by arranging them in increasing order.

$$\{19, 20, 22, 24, 24, 27, 34, 36, 37, 38, 39, 40, 40, 42, 57, 61\}$$

That's a bit better. Now we can see that the lowest value in this set is 19 and the highest is 61. To construct a box-and-whisker plot, we're going to need to calculate a few benchmark values, including the mean and median, so let's start with those as they are likely most familiar to you.

To find the mean of our data set, we need to find the sum of each entry and divide by the number of entries. We have 16 numbers in our set, so 16 will form the denominator of our fraction:

$$\frac{19 + 20 + 22 + 24 + 24 + 27 + 34 + 36 + 37 + 38 + 39 + 40 + 40 + 42 + 57 + 61}{16} = \frac{560}{16} = 35$$

Now for the median. Our set has an even number of values in it, so we'll need to find an average:

$$\{\cancel{19}, \cancel{20}, \cancel{22}, \cancel{24}, \cancel{24}, \cancel{27}, \cancel{34}, 36, 37, \cancel{38}, \cancel{39}, \cancel{40}, \cancel{40}, \cancel{42}, \cancel{57}, \cancel{61}\}$$

$$\frac{36 + 37}{2} = 36.5$$

Next, we need to find the edges of the quartiles of our data set. Remember, a "quartile" is a span of one quarter (25%) of the data when it is ordered from least to greatest in value. Box-and-whisker plots display Q1, Q2, and Q3 to divide a data set into these four parts and show the relative dispersion of data present in each part.

$$\{19, 20, 22, 24, 24, 27, 34, 36, 37, 38, 39, 40, 40, 42, 57, 61\}$$

Q1   Q2   Q3

| Smallest Number | Median of Lower Half of Data Set | Median of Data Set | Median of Upper Half of Data Set | Largest Number |
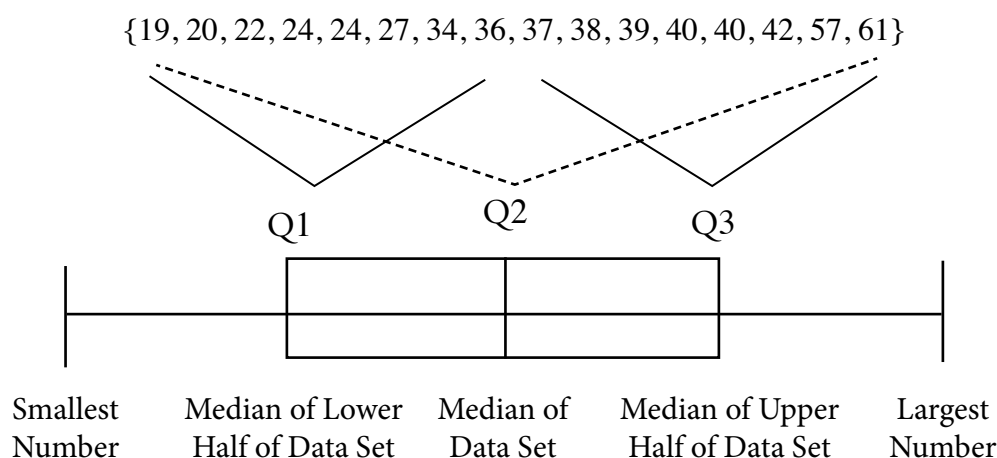
So how does all of this data division end up being displayed as a box-and-whisker plot? It's the box-and-whisker plot's job to display these specific ranges and boundaries in a visually convenient way. The "box" of a box-and-whisker plot is formed by the Q1, Q2, and Q3 values: Q1 is its left border, Q2 is the line in the middle of it, and Q3 is the right border. The left "whisker" denotes the smallest number in the data set and the right "whisker" the largest number.

Let's calculate the quartile values for our data set. Q2 is the median, and we already calculated that: 36.5. Q1 is the median of the lower eight numbers in the set, which is the average of 24 and 24—24. Q3 is the median of the top eight numbers in the set, which is the average of 40 and 40—40. The "whiskers" of our plot will be 19 and 61. This is enough information to draw out a box-and-whisker plot for our set.

5   10   15   20   25   30   35   40   45   50   55   60   65
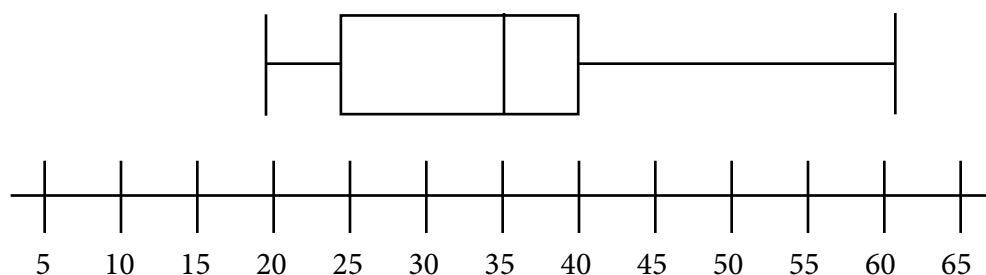
One thing you might be asked to calculate given a box-and-whisker plot is a value called the "interquartile range." Just like it sounds like, it is the difference between two quartiles: specifically, the result of subtracting Q1 from Q3. For our data set, this is $40 - 25 = 15$.

Multiple box-and-whisker plots can be displayed using the same number line scale in order to facilitate comparison between them. Consider the plots on the next page. How does the new set shown above the box-and-whisker plot for our set compare to it? What can we tell about the structure of these data sets?

Sample Question #1

Data Set A

Data Set B

5   10   15   20   25   30   35   40   45   50   55   60   65
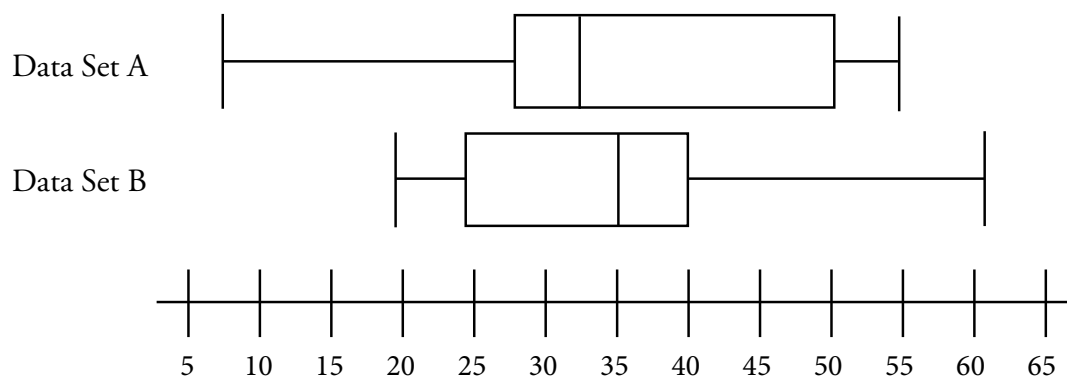
Quantity A

The interquartile range of Data Set A

Quantity B

The median of Data Set B

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

We can find the interquartile range of Data Set A by subtracting Q1 from Q3. Remember, Q1 is the value conveyed by the left edge of the box and Q3 is the one conveyed by the right side of the box. Thus, the interquartile range is: $50 - 27.5 = 22.5$.

The median of a data set is conveyed by the middle line in the box-and-whisker plot, so for Data Set B, this value is 35. B is the correct answer! Note that you could be asked to compare interquartile ranges based on visual estimation, without being given a numeric scale. As long as you're told that both data sets are being measured on the same scale, it doesn't matter what that scale actually is if you just need to pick out which is larger.

Scatterplots, Trends, and Line Graphs

Line Graphs

A line graph is a type of graph in which data is presented as pairs of numbers mapped onto the x- and y-axes of the Cartesian plane. Line graphs are suited for displaying the same type of data as bar graphs, so it is not surprising that the two types of graphs are similar in some respects. Unlike a bar graph, a line graph displays its values as coordinate points instead of bars, and it connects them to one another with a straight or curved line. This makes it easy to trace the rise and fall of whatever's being measured, and arguably puts more focus on the overall trend than on the individual points.

Unlike bar graphs, line graphs are reserved for data in which the x-variable has some definitive order to it and is chronological and/or sequential. Convention has it that line graphs proceed from left to right, so randomly-arranged variables wouldn't work for this system. For example, if we rearranged the order of the variables on the x-axis in our sample graph, it would be very difficult to see the chronological trends in the data, like how voter turnout generally increased from day 11 to day 16.

Voters Reporting by Day for Local Election

Number of Votes Cast vs *Day of Election*
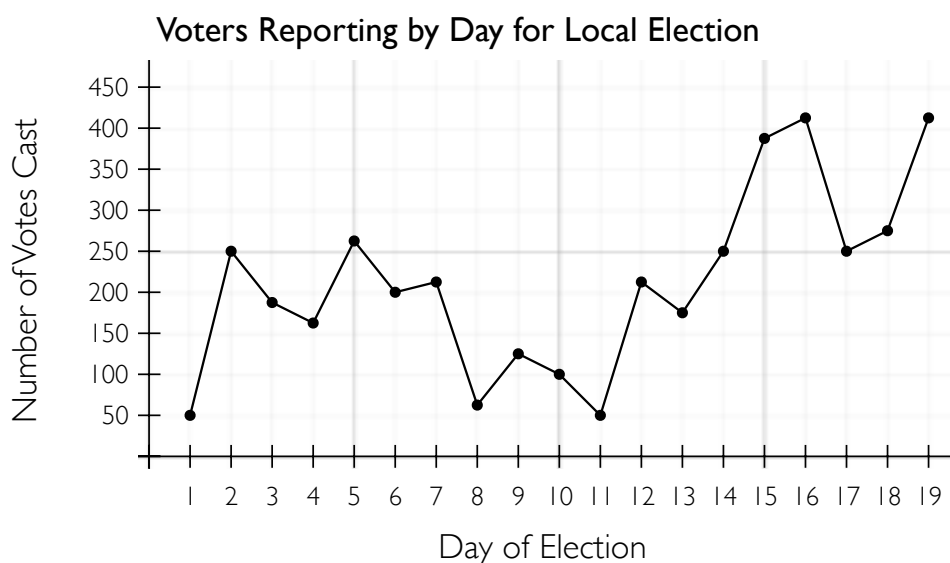
Because line graphs are designed to demonstrate trends, GRE Quantitative questions about line graphs might reasonably be trend-focused as well. Questions about trends are just asking you about the patterns that emerge when you view the data as an overall picture. They may ask you about the entire graph, but you just need to look at the big picture, not at each individual point.

One thing to consider about each line graph you look at is whether or not the graph is tracking a single quantity's change over time, or lots of individual quantities. For instance, our sample line graph above depicts individual quantities of voters who voted on particular days. If instead our line graph tracked the total number of voters who had voted in the election thus far, it would be constantly increasing and not feature any dips. Be sure to thoughtfully consider how your variables relate, especially before you start answering questions about a graph!

It's pretty easy to pick out the day on which the most voters turned out to participate in the election: we just have to identify the highest point in our line graph. The graph generally increases

Which day(s) of the election saw the highest voter turnout?

    A. Days 2 and 5
    B. Day 10
    C. Days 7 and 12
    D. Days 16 and 19

as it moves to the right, and we find that it peaks at two points: Day 16 and Day 19, making D the correct answer.

Note that if our graph were tracking the total number of voters who had voted over time, we would not be looking for the highest point; we'd be looking for the greatest change between two days.

Sample Question #2 has us looking for a date range during which the voter turnout decreased. Because the question says "consistently," we want to make sure that the voter turnout *only* decreased—we want to avoid any answer choice date ranges where it increased and then decreased, or decreased and then increased. The only date range during which voter turnout only decreased is from Day 2 to Day 4, so A is the correct answer.

Just as bar graphs can feature multiple bars, line graphs can feature multiple lines. The graph below shows the average temperature each month in three different cities, and each city's temperatures are shown on one line. A key is provided so that you can tell which line is associated with which city. Consider the graph briefly before we answer some quesitons about it.

Sample Question #2

When did voter turnout consistently decrease?

    A. Day 2–Day 4
    B. Day 5–Day 7
    C. Day 7–Day 9
    D. Day 11–Day 13

Average Monthly Temperatures
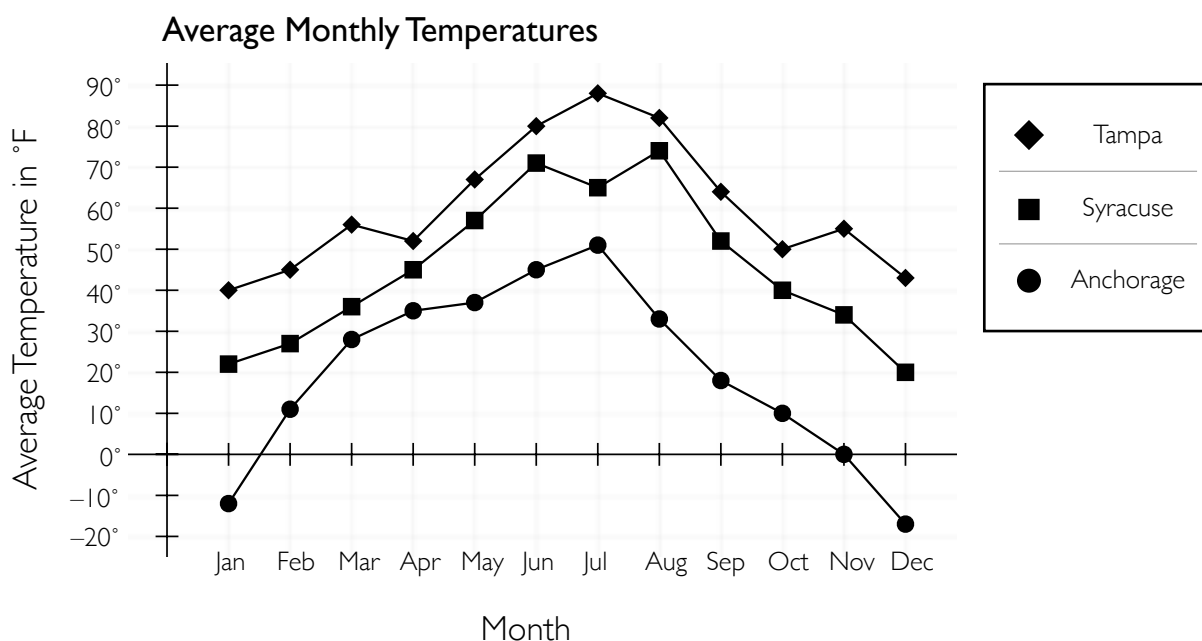
Sample Question #3

When and where did the greatest increase in monthly average temperature shown on the graph occur?

    A. From January to February in Anchorage
    B. From February to March in Tampa
    C. From February to March in Syracuse
    D. From October to November in Tampa

In looking for the "greatest increase" in monthly average temperatures, we're looking for the steepest positive slope between two points, not for the greatest values the points represent. While all of the answer choices feature two points between which the average monthly temperature increases, the greatest increase occurs from January to February in Anchorage, so A is the correct answer.

Sample Question #4

What was the average temperature in October across all three cities?

    A. 30°F
    B. 33°F
    C. 37°F
    D. 41°F

When answering Sample Question #4, don't let the wording confuse you: we need to find the average of certain points, which are themselves averages. Looking at the recorded values for October, we find values of 50°F in Tampa, 40°F in Syracuse, and 10°F in Anchorage. To find our answer, we need to add these points and divide by three. Doing so, we find that the correct answer is B.

$$\frac{50°+40°+10°}{3} = \frac{100°}{3} = 33.3° \approx 33°$$

Sample Question #5

What was Syracuse's average tempearture in June?

    A. 45°F
    B. 65°F
    C. 72°F
    D. 80°F

Sample Question #5 is quite direct: it's just asking us to identify a specified point on the graph. Be careful, though: the incorrect answers all relate to other points on the graph, including those you might pick if you mix up which line refers to which city. The average tempearature listed for Syracuse in June is 72°F, so C is the correct answer.

Scatterplots and Trend Lines

If the points graphed on a coordinate plane are not connected with a line and don't proceed chronologically or sequentially from right to left, the graph is called a scatterplot. Scatterplots measure data that involves two variables, and each variable is measured on one axis. Scatterplots often involve a large number of points, as the more points presented, the easier it is to see how the two variables relate to one another. For instance, on the graph shown at right, as the x-axis variable increases, the y-axis variable generally increases as well, though there are exceptions to this trend.

Gymnast Routine Scores vs. Routine Practice Time
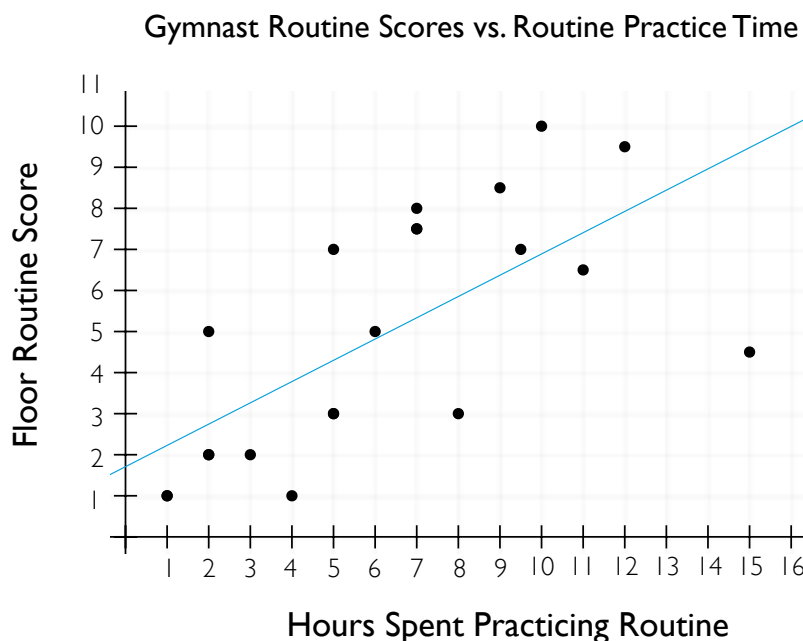
Sometimes, a line is included on a scatter plot to help you see the relationship between the variables—the trend in the data—in an estimated, linear fashion. Such a line is called a "trend line." If one is not provided and you need to estimate one, you can do so by finding the slope of the line between a point with a low x-value and a point with a high x-value. The closer a value is to a trend line, the closer it is to the expected value for that particular x-variable. Thus, because it allows you to spot expected values vs. surprising ones, trend lines can help you pinpoint "outliers"—points of data that are relatively far away from the trend line and thus don't do a good job of representing the general trend that holds true for the data set as a whole.

Trend lines also allow us to make specific estimates about our data. We can use the trend line to estimate precisely what the y-value will be for a point with a given x-value; we just identify this x-coordinate and trace upwards until that point intersects the trend line, then we identify the y-coordinate of this point on the trend line. Similarly, a trend line allows us to estimate exactly how much change in the y-variable a given change in the x-variable can be expected to produce. To do this, we'd need to find the rate of change (the slope) of the trend line by picking two points on the line and solving for the change in *y* over the change in *x* between them. (Note that we don't have to pick points in our scatter plot, but can just identify two points on the trend line, regardless of whether or not they're part of our graphed data set that the line is estimating).

For instance, our line crosses the y-axis at about 1.75, so one point we could consider is (0, 1.75). Another point is (12, 8); we can tell because the trend line intersects the corner of the crossing x-axis and y-axis guidelines. The slope between these points is rise over run, or $\frac{\Delta y}{\Delta x}$. This equals $\frac{8 - 1.75}{12 - 0}$, or $\frac{6.25}{12}$. Reduced, our trend line has a slope of about $\frac{1}{2}$. What does that mean in the context of our graph? It means that for every increase in the x-variable by 2—that is, for each additional two hours a gymnast practices his or her routine—they can reasonably expect an increase in their routine score of about one point. Note that this trend line is linear, but we have evidence that this relationship doesn't hold for our data above a certain point; the scores for gymnasts who practiced between ten and twelve hours are reasonably close to the trend line, but there is one point representing a gymnast who practiced his or her routine for 15 hours and received a score of 4.5, which is very far from the trend line. While this point is an outlier in relation to the trend line, it also suggests that the payoff of practicing a routine decreases after a certain point. Let's try answering some sample questions about the scatter plot shown below.

Final Exam Grade and Extra Credit Correlation

Score on Final Exam (y-axis) vs. Number of Extra Credit Points Earned During the Semester (x-axis)
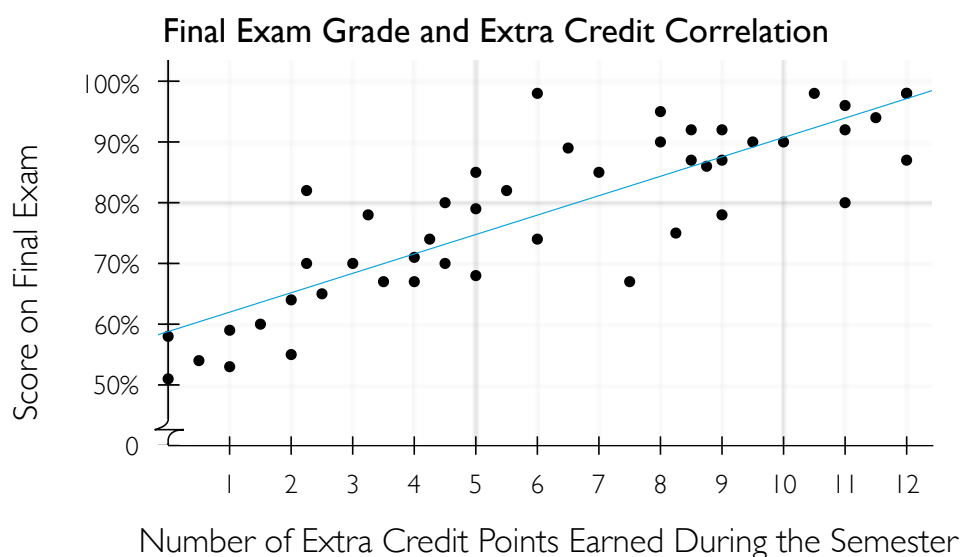
Sample Question #6

If a student earned 8.5 extra credit points, we could reasonably expect that student to get which of the following scores on the final?

A. 70%
B. 75%
C. 80%
D. 85%

Sample Question #6 gives us an x-value and asks us to estimate the y-value with which it would be reasonably associated. Our graph includes a trend line, so we can answer this question by finding the y-value on the trend line at the given x-value. Finding 8.5 on the horizontal axis, we trace upwards until we hit the trend line, then trace left and see what y-value that point has. The y-value is about halfway between the 80% and 90% tick marks, so a student who earned 8.5 extra credit points could be predicted to score about an 85% on the final. That means that D is correct.

Sample Question #7 is asking us to quantify the relationship between the rise and run of the trend line; in other words, it's asking for its slope. We can find the slope by identifying two points and finding $\frac{\Delta y}{\Delta x}$. This trend line doesn't pass through any whole-number points exactly, so we'll need to estimate. It crosses the y-axis at about 58 units up, so our first point will be (0, 58). Then, at 5 units over on the x-axis, it is at a value of approximately 75. So, our second point will be (5, 75). Now for our formula:

$$\frac{75-58}{5-0} = \frac{17}{5} = \frac{3.4}{1}$$

This means that for every extra credit point a student completes, his or her final exam score can be reasonably expected to go up by about 3.4%. The closest answer is C, 3%

Sample Question #7

For every extra credit point earned, about how much did a student's score on the final exam increase?

  A. 1%
  B. 2%
  C. 3%
  D. 5%

Sample Question #8

A scatter plot graphs hours of weekly exercise on the x-axis and the weights of the exercisers on the y-axis. If every two hours of weekly exercise translates to about 0.75 lbs lost for the participants, which of the following could be the equation of the data set's trend line?

  A. $y = -\frac{8}{3}x + 9$

  B. $y = -\frac{3}{8}x + 4$

  C. $y = 2x - \frac{8}{3}$

  D. $y = 2x + \frac{3}{8}$

Sample Question #8 takes us into the realm of the hypothetical but asks you to apply the same skills you've practiced in answering previous questions in this lesson. What might the described graph look like? Sketching out the general shape of this graph's data might be helpful. If we're plotting hours of weekly exercise increasing on the x-axis and weight of exercisers on the y-axis, and we're definitively told that weekly exercise translates to some amount of weight lost in exercisers, we're going to have a negative slope, because the more the participants exercise (the larger the x-value), the less they will weigh (the lower the y-value). That knocks out two answer choices right away: C and D, those with positive slopes. Now we need to figure out the actual slope. Change in y here would be the loss of 0.75 lbs, or –0.75. Change in x would be a gain of two hours of exercise, or 2. So, our slope would be $\frac{-0.75}{2}$, or, simplified, $-\frac{3}{8}$. Only answer choice B has this slope, so B is correct.

Bar Graphs and Histograms

You're likely to encounter a few types of graphs and charts on GRE Quantitative section, and bar graphs are perhaps the most familiar. If you haven't brushed up on your graph-interpreting skills lately, the next few pages' exercises can serve as a beneficial skill check before you head into your exam. Make sure that you can solve each type of bar-graph-related question with confidence. Before we get to the review, however, we should briefly consider what bar graphs are, and in doing so, pay careful attention to the different types of information that might be provided or withheld in a given question setup.

Voters Reporting Per District

A bar graph is a visual representation of bivariate data, or data in the form of pairs of numbers. Just like on a line graph, these number pairs are mapped onto the x- and y-axes of the Cartesian plane; however, a bar is drawn between each point and and one of the axes to help viewers see at a glance how the values compare. The length of bar that sits on the axis is irrelevant, and should be the same for each bar; the length of a bar that extends into the Cartestian plane conveys its value. The axes are labeled to tell readers which dimension stands for which measurement, and the measurements the bars represent may or may not be sequential or numerical in nature. For example, bars could represent the numbered consecutive weeks in a semester (numerical and sequential), the days of the week (sequential but not numerical), or the number of votes different candidates received in an election (numerical but not sequential). Some types of data, like months of the year, are discrete and offer no "in-between" points, whereas others, like a numerical scale from 1 to 100, offer the chance for bars to fall between the labeled tick marks. In these latter instances, it is important to read the graph especially carefully.

Bar graphs may utilize multiple bars that represent different subsets of data. For example, we could create a bar graph that shows the responses to a survey about the favorite food of each person in school. We could further break down this data by assigning it into subsets such as age, grade level, or gender of responders. Since the visual nature of bar graphs is conducive to trend-spotting and comparison-making, these subsets would make it easy to spot any trends that exist amongst different groups of responders. Such a graph would make it easy to see if eighth graders liked pizza more than freshmen, for instance. If we didn't have subsets, we'd only see how many students liked pizza in total. Multiple subsets—and multiple bars—adds useful complexity.
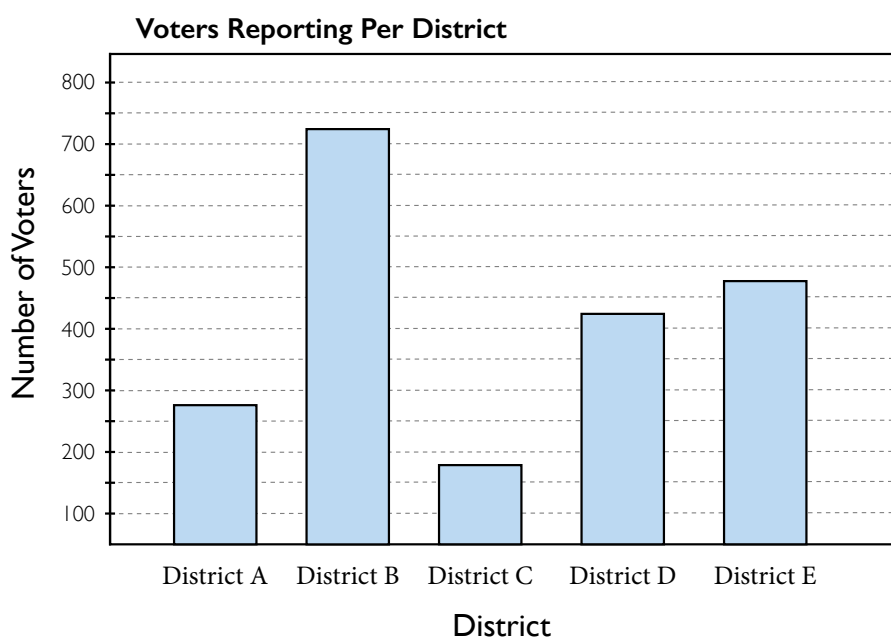
For now, let's just consider the basic bar graph. This may seem simple, but a single-bar graph conveys a great deal of data. The GRE's questions about bar graphs span several different types of questions: you might be asked to retrieve specific details from the graph, identify a trend, or make a prediction based on the big picture a graph portrays. Graph-related questions present an opportunity to test you on reading, interpretation, or both.

Theater Club Monthly Net Expenses and Earnings
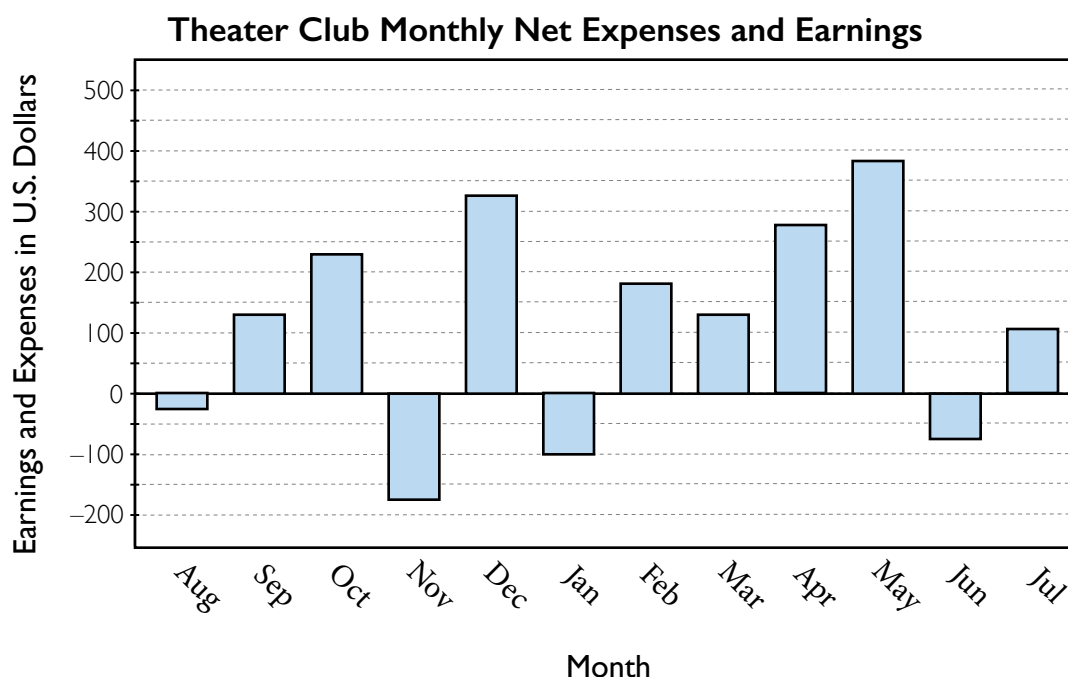
Before you start analyzing trends on a bar graph, make certain that you understand what each axis stands for. On the bar graph shown above, the x-axis shows different months of a year, and the y-axis shows an amount of money. The x-axis does not form the "floor" of this particular bar graph; negative values are present. These are represented by the bars extending below the x-axis line.

At first glance, Sample Question #1 might look difficult, but take another look: it's just describing what it's asking you to pinpoint in financial language instead of directly stating it. Take a moment to consider what exactly constitutes a month in which earnings do not exceed expenses. Earnings are positive on our graph; they're the money that the theater club takes in; conversely, expenses are negative; they're the money that the club spends. So, if we were looking for a month where earnings exceed expenses, we would be looking for a month in which the positive values must have been larger than the negative values, leaving us with a net positive value; however, this

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Sample Question #1

During which of the following months did the theater club's earnings NOT exceed its expenses?

    A. August
    B. September
    C. February
    D. May

question inserts one more logical shift: it includes the word "not." So, we're looking for a month where the positive values were not greater than the negative ones—in other words, a net negative month on the graph. So, this question is just asking us to identify which of the months has a negative bar on the graph! Put that way, the problem looks much easier. Of the months given as answer choices, September, February, and May all have positive bars on the graph; August is the only month that has a net negative value, so A is the correct answer.

Sample Question #2 turns our attention from identifying one particular bar on the graph based on given criteria to working with the data represented by each of them in a batch group. Whether working with one bar or all of them, remember that the information you need to is given somewhere in the combination of the question stem and the graph itself. Here, we're being asked for the "net change in the threatre club's finances for the entire year." Let's translate that: we need to figure out how much money the theater club gained or lost for the entire year shown in the graph. Each bar in the graph represents the changes in the club's finances for a given month, so to find the total change for the year, we just need to sum the bars' values.

> ### Sample Question #2
>
> Which of the following most closely approximates the net change in the theater club's finances for the entire year?
>
> **A.** $1175
> **B.** $1200
> **C.** $1350
> **D.** $1425

This question requires you to take the visual bar graph and extract estimated values from it for each point on the y-axis (each month). If you're worried about how accurate your numbers will be based on trying to figure out exactly where the height of a bar hits on the x-axis, you'll be happy to note that the question includes the word "approximates." This means that our numbers may not be 100% accurate, but a good estimate will be sufficient to solve the problem. What does "a good estimate" mean in the context of reading graphs? It all depends on how the numbers on the graph's axes are labeled. On our graph, each labeled number on the y-axis is associated with a tick mark, and these alternate with unlabeled tick marks. So, the unlabeled tick mark between 0 and 100 stands for 50. We can very reasonably estimate the height of a bar halfway between each tick mark, meaning that we can differentiate bar heights by $25. We could presumably note when a bar is three-quarters of the way to the next tick; this would allow us to divide each space between tick marks into four equal units, allowing our estimates to differ by 50 divided by 4, or $12.50; however, the values of each bar in this graph can be estimated as being on or exactly in between two of the tick marks, so we won't have to get that specific. Let's get estimating! At this point, you'd definitely want to jot down your estimates on scrap paper as you make them; keeping a dozen specific numbers each correlated with a specific month in your short-term memory in a test environment just isn't advisable.

As you're estimating, take particular care not to get switched around when working with negative values. If you're in the habit of assuming that half a tick mark up from a labeled one correlates to the labeled value plus or minus an addition $25 based on the direction of the axis, it can be easy

to misread a value of –175 as –225! Taking your time in estimating can save you lots of hassle. You don't want to have to recheck your estimations later; you need that time to answer or review other questions.

Aug	–25
Sept	+125
Oct	+225
Nov	–175
Dec	+325
Jan	–100
Feb	+175
Mar	+125
Apr	+275
May	+375
Jun	–75
Jul	+100

Your notes on scrap paper might look something like the table shown at left. Make sure you clearly designate which values are associated with which months. Messy work here can cost you time. Horizontal lines dividing each row in your table make sure that everything stays clearly delineated, and your success on this problem doesn't come down to guessing which month you meant to align a given number with.

Now we have all the numbers we need to solve the problem; we just need to add them together. Again, pay careful attention to which numbers are negative; incorrect answer choices may feature the results you get if you mistakenly treat one or more negative values as positive ones.

$$-25 + 125 + 225 + (-175) + 325 + (-100) + 175 + 125 + 275 + 375 + (-75) + 100 = 1350$$

The theater club made $1350 during the year, so the correct answer is C.

Sample Question #3 goes one step further: it asks you to work with the individual values not of all the bars, but of specified ones. Specifically, it asks you to calculate an "average net gain or loss per month." Since our data is presented as monthly gains and losses, all we need to do is average the specified months. Which months are those? We're told that the average should cover the time period "from the beginning of August to the end of December." That's just another way of saying August through December, including August and December. We already estimated the values for these months while answering the previous sample question, so we can just grab those specific months' values and average them. Remember, to average a set of numbers, you sum them together and divide by how many numbers are in the set.

Sample Question #3

What was the average net gain or loss per month for the theatre club from the beginning of August to the end of December?

- **A.** –$25.00
- **B.** $0.00
- **C.** +$60.00
- **D.** +$95.00

$$\frac{-25 + 125 + 225 - 175 + 325}{5} = \frac{475}{5} = 95$$

We got a positive number, which in the context of our graph translates to a an average net gain. The theatre club gained ninety-five dollars a month on average during the specified months, so the correct answer is D.

Horizontal Bar Graphs

The bars on a bar graph don't necessarily need to rest on the x-axis; they can rest on the y-axis instead. Just like in the previous bar graph we considered, bivariate data is being displayed in the Cartesian plane. The only difference is that in this orientation, the bars extend to the right, and their lengths, not their heights, are the relevant dimension that correlates with the data on the x-axis. If you can read a vertical bar graph, you can read a horizontal one, but you may find it slightly less intuitive. We'll go over a few sample questions based on the horizontal bar graph shown at the right so you can practice reading this type of bar graph accurately.

Votes for School Mascot

Mascot Animal: Frog, Jaguar, Porcupine, Hawk, Wolf, Lion, Bear

Number of Votes Received (x-axis: 25 50 75 100 125 150 175 200 225 250 275 300)
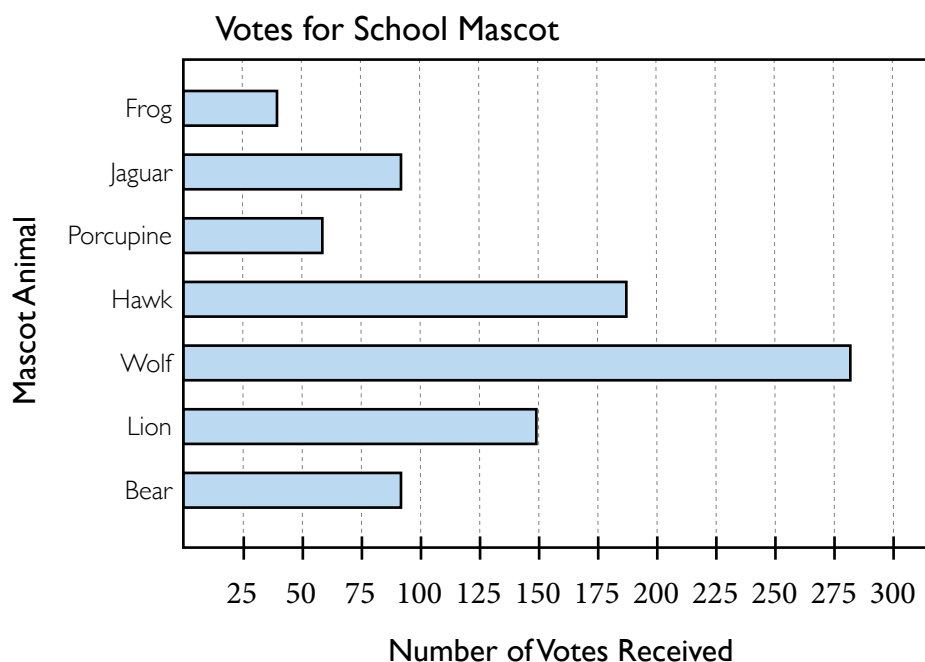
Sample Question #4

By approximately how many votes is the hawk mascot beating out the next most popular mascot?

A. 25
B. 38
C. 55
D. 64

Sample Question #4 asks us about two mascots: the hawk mascot, and the next most popular mascot. Which is that? Looking at the relative lengths of the bars, we find that the lion mascot is the next most popular. Now we need to apply numbers to this scenario. The hawk mascot received approximately 188 votes, since its bar extends halfway between the 175-vote and 200-vote tick marks. The lion mascot received 150 votes. To find the amount by which the hawk mascot beat the lion mascot, we just need to subtract these values. Doing so, we find that the correct answer is B:

$$188 - 150 = 38$$

To solve the sample question presented on the next page, you'll need to calculate what percentage of the votes a given mascot (porcupine) received. If this seems challenging, consider how you would calculate a percent of a whole if there were no graph involved. For this scenario, you'd need to know the total number of votes and the number of votes that the porcupine mascot received. You can use

the graph to figure out that information: to find the total number of votes, you'll need to sum the votes each mascot received, and you can figure out how many votes the porcupine mascot received by looking at that particular bar. After that, you just need to divide the porcupine mascot votes by the total number of votes and multiply the resulting decimal by 100. The correct answer is B!

Frog	38
Jaguar	90
Porcupine	60
Hawk	188
Wolf	280
Lion	150
Bear	90

$$\frac{60}{38 + 80 + 60 + 188 + 280 + 150 + 90} = \frac{60}{886}$$

$$60 + 886 = 0.06772009 \approx 0.0677$$

$$0.0677 \times 100 = 6.77\% \approx 7\%$$

We're going to need those estimates again for Sample Question #6, which has us adding various categories' votes to those of other categories and seeing if that upsets the election. Let's first add the jaguar mascot votes to the lion mascot votes:

$$90 + 150 = 240$$

We have to compare this result to the graph's given winning mascot tell if this new total would cause the lion mascot to be the winner instead. The wolf mascot won the poll with 280 votes, so the combined lion and jaguar mascot votes wouldn't be enough to displace it, and B can't be the correct answer. Let's next consider the second scenario and add the porcupine and frog mascot votes to the bear mascot votes:

$$60 + 38 + 90 = 188$$

188 is again less than 280, so the wolf mascot would still be the winner if both of these voting shifts occurred, making A the correct answer.

Sample Question #5

Approximately what percentage of votes did the porcupine mascot receive?

    A. 3%

    B. 7%

    C. 12%

    D. 15%

Sample Question #6

If the students voting for the lion mascot convince all the students voting for the jaguar mascot to vote for the lion mascot and all the students voting for the porcupine and frog mascots vote for the bear mascot, which mascot will win the election?

    A. Wolf

    B. Lion

    C. Bear

    D. Hawk

Multi-Bar Graphs

As previously mentioned, some bar graphs might have subcategories of data to display. One way that graphs can present such data is to use multiple bars per category on the axis on which the bars are resting.

You can think of the bars as dividing out from main categories almost like a branch diagram like the one shown at right. Note that the categories that each of the grade levels is split into are the same categories for each; this allows the graph to have some consistency represented in its key. Instead of branches, though, the graph uses multiple different-colored bars for each labeled category of bar on the axis.

Poll Responders

Freshmen
- Cat-owning freshmen
- Dog-owning freshmen
- Freshmen with other pets

Sophomores
- Cat-owning sophomores
- Dog-owning sophomores
- Sophomores with other pets

Juniors
- Cat-owning juniors
- Dog-owning juniors
- Juniors with other pets

Seniors
- Cat-owning seniors
- Dog-owning seniors
- Seniors with other pets

Faculty and Staff
- Cat-owning faculty and staff
- Dog-owning faculty and staff
- Faculty and staff with other pets

Categories of bars on axis

Differing bars in each category

School Survey: How Many People Own Pets?

Key:
- Cat owners
- Dog owners
- Owners of other types of pets

Axis: 100, 200, 300, 400, 500, 600, 700, 800

Categories: Freshmen, Sophomores, Juniors, Seniors, Faculty and Staff

To answer Sample Question #7, we'll need to quantify each of the amounts presented in the answer choices. Some answer choices require us to sum groups of numbers:

A: 460

B: $210 + 300 = 510$

C: $225 + 375 = 600$

D: $225 + 200 + 125 + 90 + 145 = 785$

Once we've done this, we can see that owners of other types of pets (D) represents the greatest number of people.

Sample Question #8 requires us to find the percent that faculty and staff cat owners represent of all cat owners that responded to the survey. Consider how to find a percent: we'll need to find the number of faculty and staff cat owners, and that will form our numerator. Then, we'll need to find the total number of cat owners who responded to the survey. That will require the addition of five different bars on the graph, and the resulting sum will form our denominator.

$$\frac{480}{225 + 375 + 200 + 340 + 480} = \frac{480}{1620}$$

$480 \div 1620 = 0.296296296 \approx 0.296$

$0.296 \times 100 = 29.6\%$

Note that the question says "approximately" and that our answer choice is quite far away from any of the other options; thus, it's reasonable to pick the closest estimate. The correct answer is C, 30%.

Sample Question #7

Which of the following represents the greatest number of people?

- **A.** Faculty and staff who own dogs
- **B.** Juniors and seniors who own dogs
- **C.** Freshmen and sophomores who own cats
- **D.** Owners of other types of pets

Sample Question #8

Approximately what percentage of cat owners surveyed were faculty or staff?

- **A.** 15%
- **B.** 20%
- **C.** 30%
- **D.** 35%

Segmented Bar Graphs

There are multiple ways that a bar graph can present subcategories of data. Instead of providing multiple bars per category, it might provide one bar, but divide it into multiple sections. This sort of graph may look imposing, but you can treat each bar as a shorthand way of presenting multiple bars. Each different "stripe" represents a subcategory, just like each different bar in one category would.
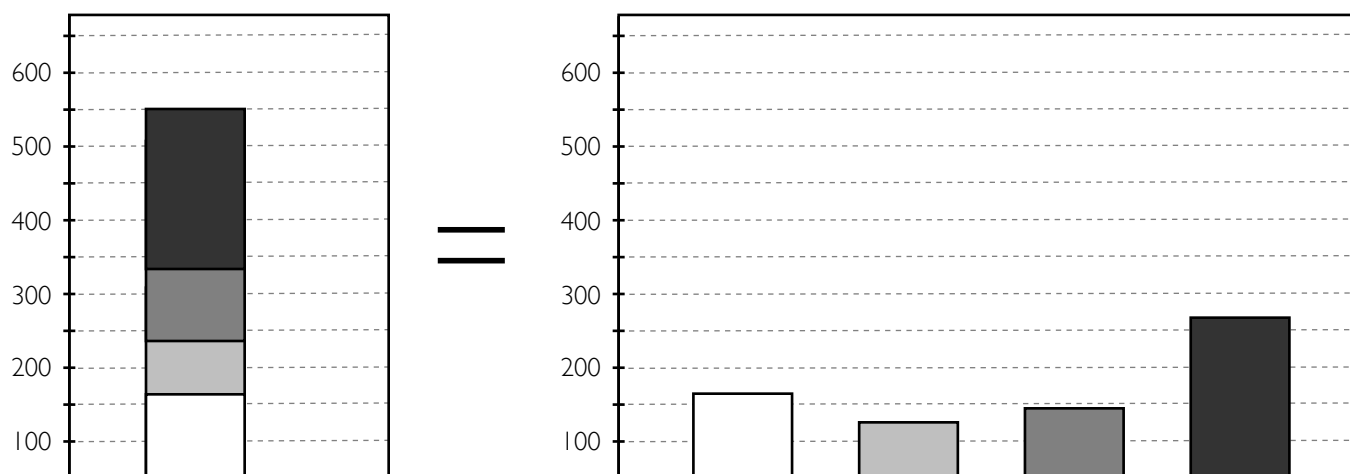
One advantage that these stacked-bar graphs have over those that place multiple bars in a category on an axis is that it is very easy to see the total value of a all the subcategories in a category at a glance. For the stacked bar shown above on the left, it's easy to see that its four component parts sum to 550. When the data is presented as multiple bars as it is on the right, you have to estimate and then sum them to find this value.

Poll Results: "Do you support the plan to remodel the dining hall?"
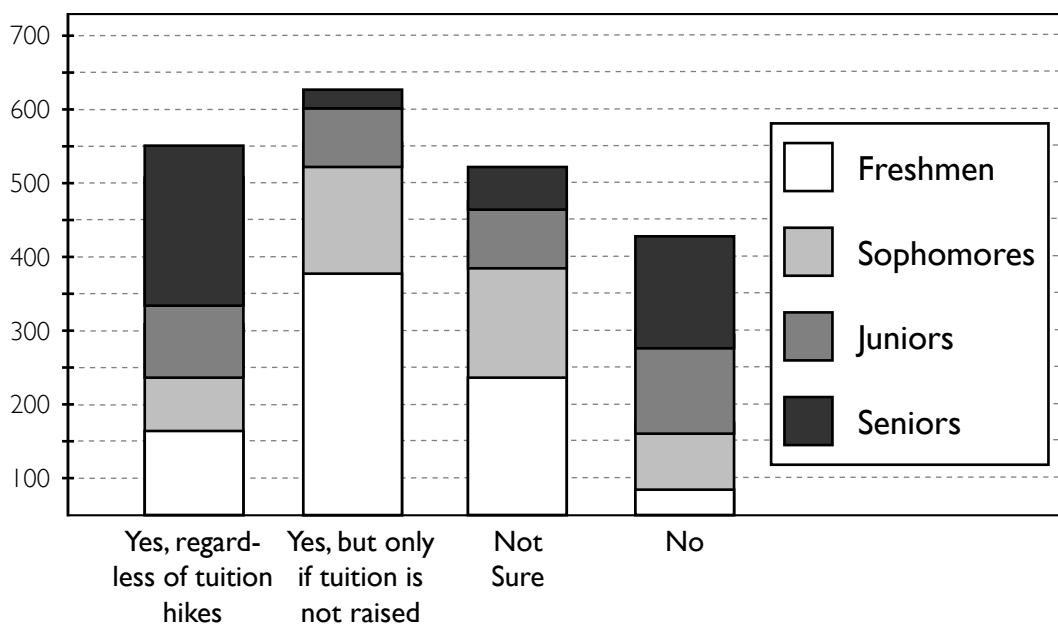
Sample Question #9 presents us with a hypothetical situation that ties in with the categories presented in our graph. We're asked to identify how many seniors will no longer support the dining hall renovation project after it's decided that tuition will be increased. Well, one of the survey responses and thus our categories on the x-axis is "Yes, but only if tuition is not raised." The number of seniors who gave this response will be the answer to this question. The senior part of the bar extends from 600 to midway between the 600 and 650 tick marks—that's 25 seniors, making A the correct answer.

Sample Question #10 is a bit trickier, as it presents claims about the graph's data that we need to analyze. We need to pick out the claim that the graph supports, meaning that three of them will *not* be supported—false statements. Let's go through them in order. For questions like this, it's important to analyze each answer choice individually.

A. Which bar makes up the most of the "No" category? It's not freshmen—it's seniors. This isn't the correct answer.

B. If tuition is raised, the students who responded "Yes, but only if tuition is not raised" will not support the renovation, leaving only those who responded "Yes, regardless of tuition hikes" in favor of it. The number of students who comprise the "Yes, but only if tuition is not raised," "No," and "Not Sure" responses is far greater than the number of students who responded "Yes, regardless of tuition hikes," so this isn't the correct answer.

C. For this statement to be true, more than half of the junior bars' total length would need to be in the "Not sure" column. By the looks of it, a little less than 25% of it is. That's nowhere near half, much less "more than half."

D. Which class is most likely to support the plan to remodel the dining hall no matter how it affects tuition? We need to check the "Yes, regardless of tuition hikes" category and see which section of the bar is largest. That's seniors, so this is the correct answer!

Sample Question #9

The university's board of trustees approves the project on the condition that tuition will be raised 4% the following year for all students. How many seniors that support the project, upon learning this, can be assumed to no longer support the project?

A. 25
B. 50
C. 220
D. 245

Sample Question #10

Which of the following claims is supported by the data in the chart?

A. Freshmen are most likely to oppose the plan to remodel the dining hall.
B. Even if tuition is raised, the majority of students will still support the dining hall renovation.
C. More than half of the junior class is not sure whether or not they support the dining hall renovation project.
D. Seniors are most likely to support the plan to remodel the dining hall regardless of its potential effect on tuition.

Histograms

A histogram is a special type of bar graph that is used to represent the frequency of data. **Frequency** is the number of times a specific value occurs in a data set, or put another way, the number of times a response occurs in the total set of responses.

A histogram differs visually from a bar graph in a few subtle ways. Most obviously, there is no space between a histogram's bars or between its edge bars and its edges. This helps to convey that its bars represent all possibilities in a given range, leaving out no possibilities. All histograms are set up so that the data collected is categorized on the x-axis. The y-axis is labeled with increasing percents and is labeled "**Relative Frequency**." Why "relative?" Histogram data is calculated so that the frequency of each response is calculated relative to all the responses collected. So, if we asked ten people how many days per week they eat breakfast, and three of them said every day, the relative frequency of that response would be three responses out of the ten total respones we received, or 30%. Because a histogram shows the frequency of each possible category for a given context, all of the bars in a histogram sum to 100%. This also affects the categories that are used. For our "how many days a week do you eat breakfast" example, a response of 0 makes sense if you never eat breakfast. But if we were asking how many different foods were featured in the last meal you ate, a response of zero wouldn't make sense; a meal has to involve at least one type of food! Histogram responses may be bundled into a range so that a few individual responses comprise a single category. For our "breakfast" example, we might condense our responses into 0–1 times a week, 2–3 times a week, 4–5 times a week, and 6–7 times a week. While there are eight different responses we could have gotten (0, 1, 2, 3, 4, 5, 6, or 7), our histogram would only have four bars. If the parameter a histogram is measuring has an open-ended context, the end categories might use the phrases "or fewer" or "or more." These don't make sense for every sitaution, though. For instance, a histogram measuring how many cookies people ate at a party could reasonably end with a "four cookies or more" category.

Consider the sample histogram below. We'll try some questions about it on the next page.

How Many Houseplants Do Students Own?
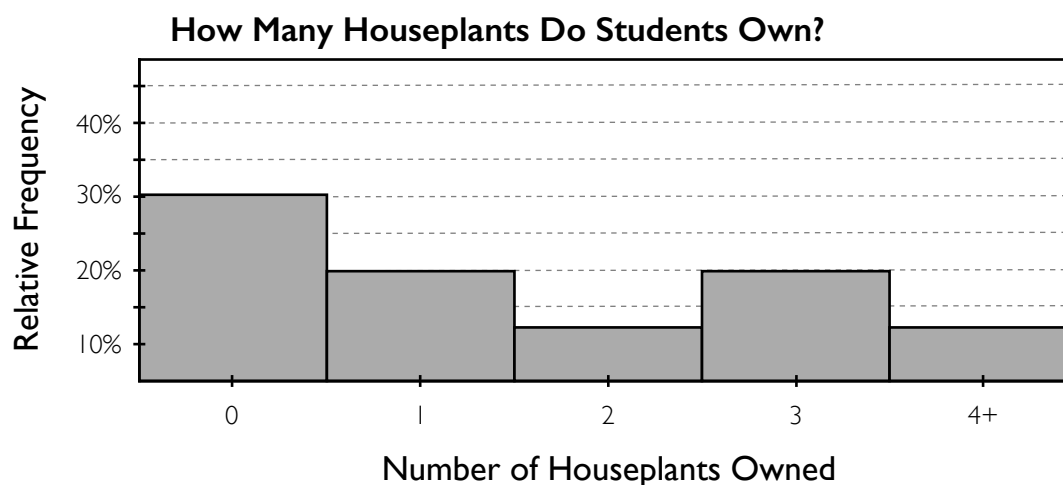
To answer Sample Question #11, we'll need to convert the percent data our histogram gives into the number of actual students that fall into a particular category: that of owning no houseplants. To do this, we'll need to know the total number of responses collected; after all, we can't find out what a certain percent of a total is if we don't know how many responses make up the total! The histogram tells us that 30% of the students polled said that they don't own any houseplants. The question stem tells us that 20 students were surveyed in total, so finding the correct answer is just a matter of calculating 30% of 20.

$$30\% \text{ of } 20 = 0.30 \times 20 = 6$$

6 students said that they don't own any houseplants, so B is the correct answer.

In answering Sample Question #12, we only need to work with the presented percentages. Since we're asked about probability—the chances of picking a student who owns at least two houseplants—we already have all the data we need. We can treat relative frequencies as probabilities because they are derived the same way as probabilities are. Think about it: if 3 out of 10 students provide a given response to a question, that's a relative frequency of 30%, and your probability of picking one of these three students from the ten that were questioned is 3 out of 10, or 30%. So, to solve this problem, we can simply add together the relative frequencies of the bars included in the statement "owns two or more houseplants": 2, 3, and 4+. The 175 total response information is completely extraneous.

$$12.5\% + 20.0\% + 12.5\% = 45\%$$

The correct answer is C.

Sample Question #11

If twenty students were surveyed to gather the data for this histogram, how many of them responded that they own no houseplants?

    A. 5
    B. 6
    C. 7
    D. 8

Sample Question #12

If this data was derived from the responses of a group of 175 students and you pick a student at random from this group, how likely is it that the student you pick owns at least two houseplants?

    A. 35%
    B. 40%
    C. 45%
    D. 50%

Circle Graphs and Pie Charts

Circle graphs and pie charts are two names for the same type of graph. Such a graph is divided up into various sectors, and each sector represents one data measurement—much like the bars on a bar graph or the points on a scatter plot. Circle graphs make it easy to instantly see how the data measurements compare to one another. One glance at a pie chart displaying the results of a poll should be enough to convey what the most and least popular options are, and even allow you to estimate by how much of the population those options differ.

Unlike the other types of charts we've looked at, pie charts don't use axes. Instead, they rely on *circle math*. In doing so, they bring together a few types of data you might be asked about: the central angle of the sector, the percentage of the information collected that sector represents, and the number of responses that contributed to that sector. The GRE Quantitative section may present questions that give you some but not all of this information and ask you to calculate the rest. After a simple example, we'll walk through some sample problems that ask you to calculate various measures.

Imagine you're tasked with creating a circle graph to represent the results of a poll about how a group of people plans to vote on a ballot issue. When you see the results of the poll, you find that the results are split 50/50: exactly half of the responders said that they plan to vote yes, and half the responders are planning on voting no. Because there are two possible responses in your survey, yes or no, the circle graph will need to have two sectors. And because each option received half of the responses, each sector will need to take up half of the circle, or 180° of it. Your completed circle graph might look something like the one shown at right.

Plan to Vote Yes   Plan to Vote No
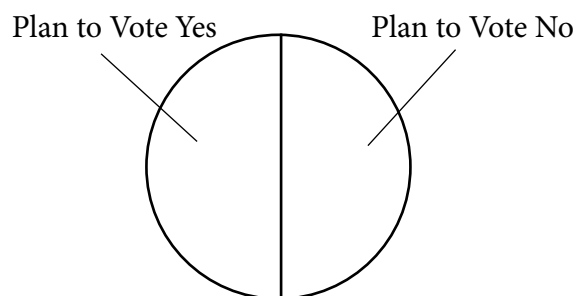
Response to survey: "Which proposal do you support?"	
Total Responses Collected: 4,850	
Response	Percent of Responses
Proposal 1 only	40%
Proposal 2 only	15%
Both proposals	25%
Neither proposal	20%

Now imagine that you need to model the results of a more complex poll, provided in the table to the left. How do you figure out how much of the circle each sector needs to take up? You use the same logic you employed for the simpler situation: let's examine what you did. First, you tallied up how many sectors your circle graph will need. This poll has four possible responses, so your graph will need four sectors. Next, you correlated the percent of each response to a graphic representation of 180°. For this more complex poll, you need to calculate how much of the total 360° of the circle each option's percentage represents. Let's use "Proposal 1 only," 40%, as an example:

$$40\% \text{ of } 360° = 0.40 \times 360° = 144°$$

The sector representing "Proposal 1 only" will need to be 144° of the total circle. What if you're also asked to calculate how many responders voted for Proposal 1 only? This may seem much more difficult, but you only need to find 40% of the number of total responses, which you're given.

40% of 4850 = 0.40 × 4850 = 1940

That means that 1940 people responded "Proposal 1 only" in the poll.

The GRE Quantitative section might take these problem-solving workflows and reverse them on you. For instance, consider Sample Question #1. Can you use the given information to reverse-engineer the number of responses that were collected?

Favorite Food Survey

Sample Question #1

Consider the graph shown at right. If 46 people said that hamburgers were their favorite food, approximately how many people were surveyed in total?

    A. 149
    B. 158
    C. 160
    D. 171
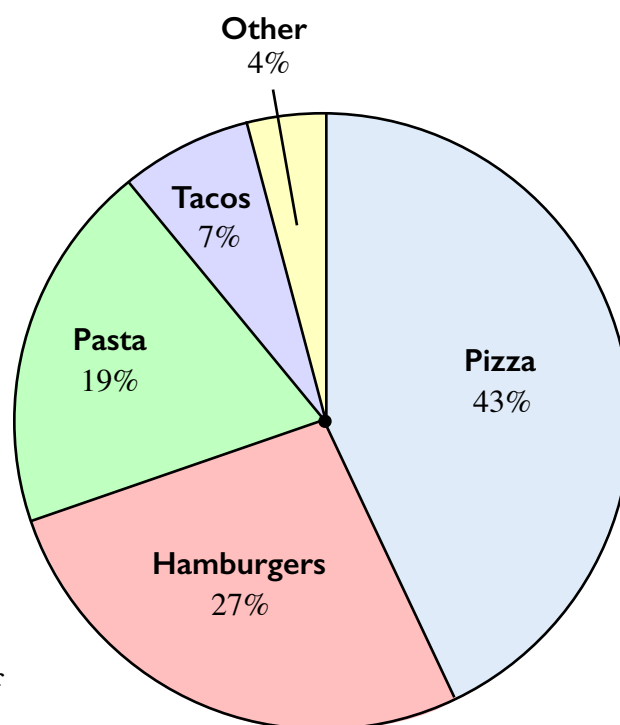
It may not be readily apparent, but you have all of the information you need to solve Sample Question #1 with a single calculation. You know that 46 people prefer hamburgers, and that those 46 people represent 27% of the total people surveyed. So, you can ask yourself: 46 is 27% of what number? You can represent this question using algebra and solve:

$46 = 0.27x$

$x = \dfrac{46}{0.27} = 170.37037037 \approx 171$

Keep in mind that you need your answer to be larger than 46—about four times as large, since 27% is close to 25%. Also, if you need to round, rounding up makes the most sense, since presumably the poll wasn't asking half-people for their opinions. D is the correct answer. Let's try a slightly more challenging sample question next!

Sample Question #2 takes circle graph principles and asks you to apply them without actually giving you a graph to use. That's ok—you don't need a graph to answer this question. You're being asked for the central angle of the sector for a certain response. To figure out how much of the pie chart the "cat owners" slice of the pie should take up, you need to know what portion of the total responses that group represents. You can figure that out easily, as you're given the number of cat-owning responders and and the total number of responses collected. So, you can create a fraction and then multiply it by 360° to find that the correct answer is A.

$$\frac{35 \text{ cat owners}}{110 \text{ total responses}} \times 360° = 114.5°$$

Easy as pie! Let's try another one.

Sample Question #2

You need to construct a circle graph for a recent poll. Of the 110 people surveyed, 35 of them said they had pet cats. What should the central angle be of the sector representing cat owners in your circle graph?

    A. 114.5°
    B. 116.5°
    C. 118.0°
    D. 119.4°

Sample Question #3

Consider the circle graph at right that displays the results of an election for the president of a chess club. The rules of the club state that if the two presidential candidates each receive over 35% of the vote, they will be elected as co-presidents. The next runner up will serve as vice president. What are the results of the chess club election?

    A. President: Jasmine
         Vice President: Bill

    B. President: Bill
         Vice President: Harvey

    C. Co-Presidents: Bill and Jasmine
         Vice President: Harvey

    D. President: Jasmine and Harvey
         Vice President: Bill

Chess Club Voting Results

Harvey 164 votes

Jasmine 345 votes

Bill 313 votes
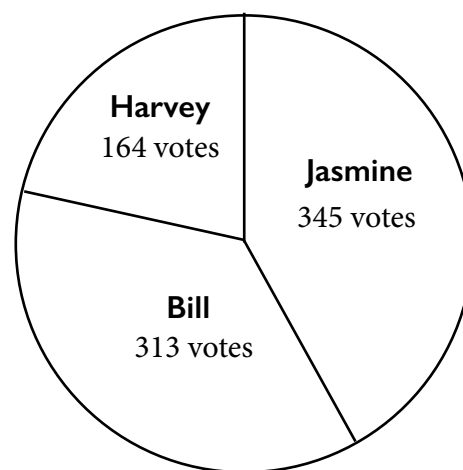
Sample Problem #3 is asking you to find the percent of the vote each candidate received, and then to check it against the chess club's rules to figure out the results of the election. First, we need to calculate the percent of the vote that Jasmine, Bill, and Harvey each recieved. We can do that because we're given the pie chart, which indirectly tells us how many people voted in total. We just have to sum together the votes the three candidates received to figure that out. Then, we can create fractions of the total for each candiadate.

$$\text{Votes}_{\text{Jasmine}} + \text{Votes}_{\text{Bill}} + \text{Votes}_{\text{Harvey}} = \text{Votes}_{\text{Total}}$$

$$345 + 313 + 164 = 822$$

Now we have a denominator for our fractions as we figure out the percent of the vote each candidate received.

$$\text{Percent}_{\text{Jasmine}} = \frac{345}{822} = 0.4197 \times 100 = 41.97\%$$

$$\text{Percent}_{\text{Bill}} = \frac{313}{822} = 0.3808 \times 100 = 38.08\%$$

$$\text{Percent}_{\text{Harvey}} = \frac{164}{822} = 0.1995 \times 100 = 19.95\%$$

According to the club's rules, Jasmine and Bill will serve as co-presidents since each candidate received over 35% of the vote. That means that the runner-up is Harvey, who will serve as vice president. C is the correct answer.

The next sample question requires you to work around a missing piece of information that may initially seem critical: the total number of responses collected.

Sample Question #4

The graph at right depicts the results of a recent poll. If 33 people said that lavender was their favorite color, how many people said that green was their favorite color?

A. 110
B. 112
C. 115
D. 118

Poll Results: Favorite Colors

Lavender 7%
Blue 20%
Yellow 9%
Orange 8%
Green 25%
Red 31%
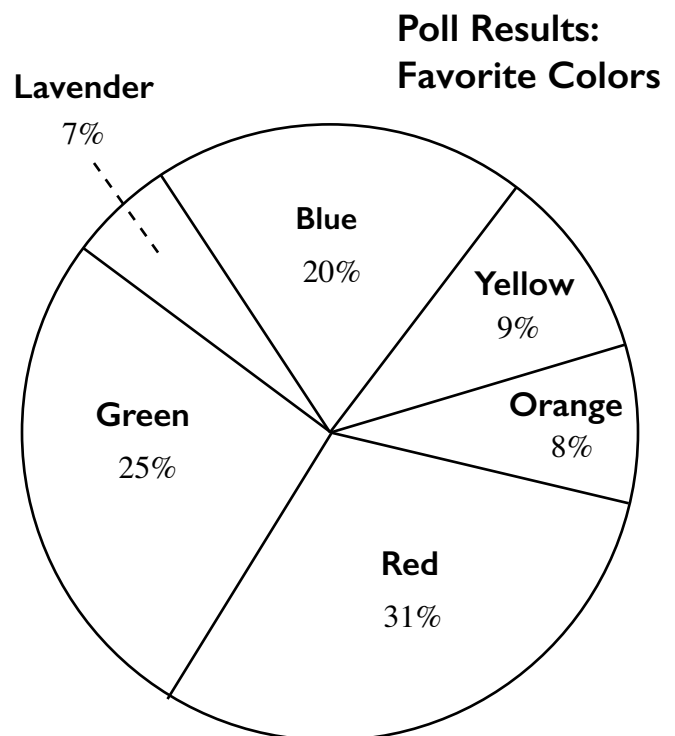
This question doesn't provide you with the total number of responses collected, but it does provide you with a particular number of responses that correlates with a particular percentage of responses. That's all you need to be able to figure out how

many responses correlate with any of the other listed percentages. The relationship between the percent and actual number of responses has to be the same for all of the different responses graphed in the pie chart, so you can set up a proportion.

$$\frac{\% \text{ of Type 1 Responses}}{\# \text{ of Type 1 responses}} = \frac{\% \text{ of Type 2 Responses}}{\# \text{ of Type 2 Responses}}$$

It doesn't matter which response you designate as "Type 1" and which you designate as "Type 2" as long as you keep them consistent and make sure your proportion works out. Let's call "Type 1" lavender-related responses and "Type 2" green-related responses. Also, to simplify the calculation, we can use whole numbers for the percents.

$$\frac{0.07}{33} = \frac{0.25}{x \text{ people}}$$

Now, we just need to cross-multiply and solve for x to figure out the correct answer.

$$0.07 \times x = 33 \times 0.25$$

$$x = \frac{33 \times 0.25}{0.07} = \frac{8.25}{0.07} = 117.857 \approx 118$$

118 people answered that green was their favorite color, so the correct answer is D.

The multiple-answer format of Sample Question #5 may make it seem as if it will require five different proportion calculations to solve, but we can take a shortcut by changing our perspective. Instead of performing five calculations, one for each answer choice, to convert percentages into number of responders, let's convert the number of responders that serves as the cut-off point for correct answers in this problem to a percentage. Then all we have to do is compare the percent cut-off point to the percentages given in the pie chart!

Sample Question #5

Refer to the pie chart "Poll Results: Favorite Colors." If 145 people responded to the survey, for which of the following colors did 20 or more people vote?

Multiple answers may be correct. Select all that apply.

    A. Red
    B. Yellow
    C. Green
    D. Blue
    E. Lavender

$$\frac{20}{145} = 0.137931034 \times 100 = 13.79\%$$

Red (31%), Green (25%), Blue (20%) each received more than 13.79% of the vote, so more than 20 people voted for each of these colors, but the same isn't true of Yellow (9%) and Lavender (7%). That makes the correct answers A, C, and D.

Visualizing Normal Distributions

The bell-shaped distribution, or normal distribution, is often observed upon drawing a smooth line connecting the tops of the bars in histogram. The classic bell-shaped curve represents data from many natural phenomena. The graph below shows a normal distribution for a generic set of data.
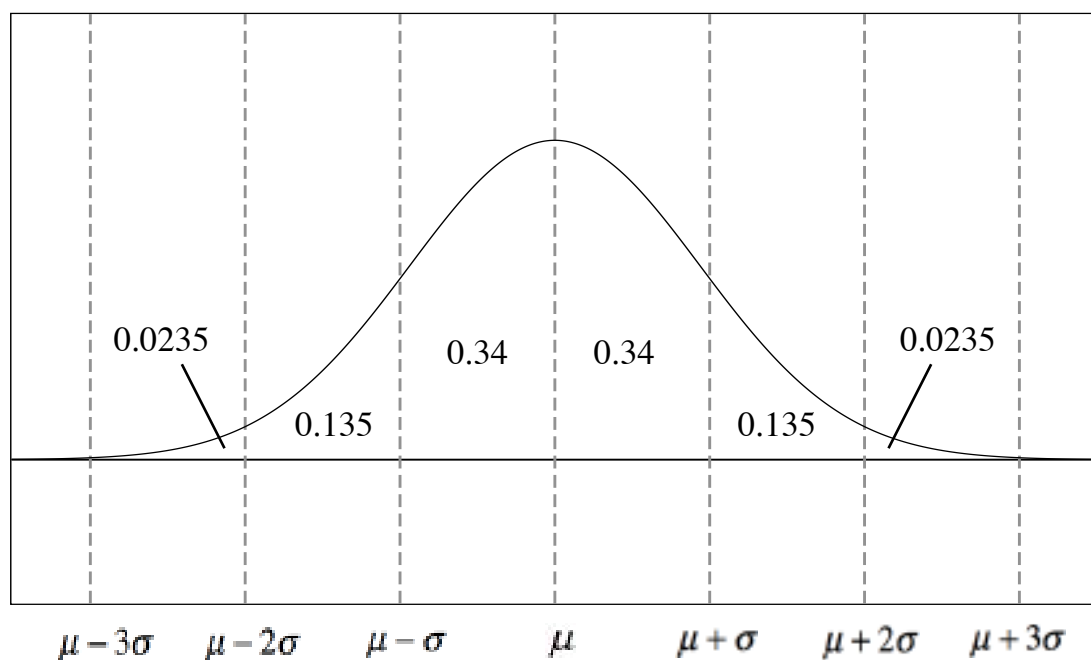
In the above graph, μ is the mean (average) of the data and σ is the standard deviation (relative measure of the spread) of the data. In a normal distribution, the two tails of the graph never actually touch the x-axis, which means that the normal distribution is a continuous probability distribution. Thus, there is no theoretical maximum or minimum value in this data model. Note the percentages and their association with the standard deviation bars. That is, data following a normal distribution can be closely approximated by **the 68-95-99.7 rule**. This rule estimates the percentage of the total data (area underneath the curve) that is bound by each standard deviation from the mean. About 68% of the data (and area underneath the curve) is within one standard deviation of the mean. Similarly, about 95% of the data is within two standard deviations of the mean, and about 99.7% of the data lies within three standard deviations of the mean. Note that the mean, median, and mode of a set of data that follows a normal distribution are usually very close together.

While the normal distribution of data always is bell-shaped, it's mean and standard deviation are variable. Normal distributions can be drawn with the mean in the middle of the graph or on either end of the graph. If the standard deviation of the data set increases, so does the width of the curve; if the standard deviation of the set decreases, the curve becomes more narrow. Regardless of the shape of the curve or the location of its mean on the x-axis, the 68-95-99.7 rule holds true for any normal distribution.

As mentioned earlier, the normal distribution follows a continuous probability distribution. This means that the total area underneath the graph is 1 or 100%. Like with histograms, we almost always consider intervals of variables rather than single variables themselves. This makes sense graphically. Consider the normal distribution below.

-15 -5 5 15 25
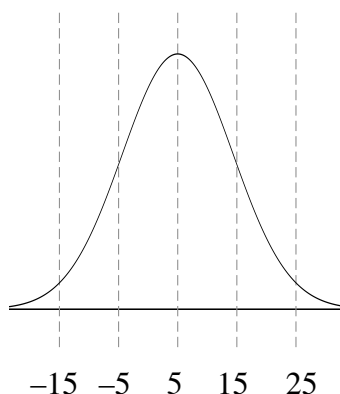
Even though there are no units and we are not told what data is being presented, we can use this distribution to deduce some key information that will be useful when quantifying the data in specific ways. Right away, we know that the mean for this data is 5 and the standard deviation is 10. When working with normal distritbutions, keep in mind that the variable (like the distribution as a whole) is continuous. In other words, rather than asking ourselves the probability of randomly choosing a 20 from our distritbution, we need to ask ourselves the probability of randomly choosing any number within a range of numbers that contains 20. This should make more sense when we recall that the probability of choosing any number on the graph is equal to the area under the curve. 20 is a single point that is 1.5 standard deviations above the average. When we draw a vertical line segment up from 20 until it intersects with the curve, we do not have an area since a line segment is one-dimensional! Conversely, if we ask ourselves to find the probability of of choosing a range of numbers that contains 20, say 15–25, then we *do* have a two-dimensional area underneath the curve (indicated at the right), which we can calculate as the probability of its occurrence.

-15 -5 5 15 25
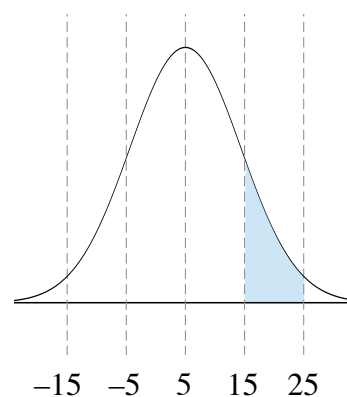
If you are familiar with calculus, you know that finding the area underneath the curve from 15 to 25 is the same as taking its integral. Finding the integral of a function with a normal distribution involves some pretty messy calculations, and is not expected of you on the GRE Quantitative section. Instead, we can use the 68-95-99.7 rule to get our answer. We are looking for a range of numbers (15–25), which is between one and two standard deviations above the mean. Referring to the first image in this lesson, we find that the probability of randomly selecting a number between 15 and 25 is about 0.135. We can say that about 13.5% of any numbers chosen at random will be between 15 and 25.

x is a random variable that is normally distributed with a mean of 9 and a standard deviation of 3.

0   3   6   9   12   15   18
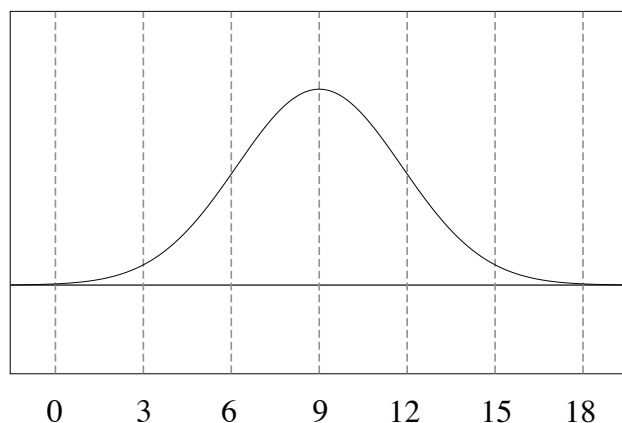
For Sample Question #1, we are looking for the probability of *x* being greater than 15, which is two standard deviations away from the mean. We know that approximately 95% of the area underneath the curve is within the first two standard deviations of the mean. Thus, there is 5% beyond two standard deviations in either direction. To find the probability of *x* being greater than 15, we simply divide this by two to get 2.5%. A is correct.

For Sample Question #2, we are dealing with two values that are a different number of standard deviations away from the mean, in opposite directions. 3 is two standard deviations below the mean and 12 is one standard deviation above the mean. We know that all the data within one standard deviation from the mean will be included in the overall probability, so our answer has to be greater than 0.68. How much greater? To answer this question, we have to add the probability that $3 < x < 6$. This is the probability that *x* is between one and two standard deviations below the mean.

Mathematically, this is equal to $\frac{1}{2} \times (0.95 - 0.68) = 0.135$.

Thus, our final answer is $0.68 + 0.135 = 0.815$. The

correct answer is C. Note that to get 0.135, we had to multiply the probability by one half as to specifically include those values on one side of the mean, namely below. This is mathematically sound since the normal distribution is symmetric about the mean.

Sample Question #1

What is $P(x > 15)$?

  A. 2.5%
  B. 5.0%
  C. 10.0%
  D. 12.5%

Sample Question #2

What is $P(3 < x < 12)$?

  A. 0.725
  B. 0.775
  C. 0.815
  D. 0.925

In solving Sample Question #3, we are looking for $P(9 < x < 12)$. This is the probability of an individual having a monthly sandiwch consumption between the mean and one standard deviation *above* that mean. We know that about 0.68 of the data fall within one standard deviation of the mean. Since we are only considering one standard deviation above the mean, we will divide this number in half to get our answer, which is approximately 0.34. To figure out how many people this refers to, we need to multiply it by the total number of individuals in the population, 2500:

$$0.34 \times 2500 = 850$$

The correct answer is D!

If the above distribution corresponds to the number of sandwiches eaten per month by a particular population of 2500 individuals, approximately how many people eat between 9 and 12 sandwiches per month?

    A. 725
    B. 750
    C. 800
    D. 850

Probability

The ability to use math to model the chances of certain events occurring is a crucial skill tested by the GRE Quantitative section. In this next part of the book, we investigate the different rules used in this branch of mathematics to obtain accurate results. In "Calculating Probability," we look in-depth at how to calculate the odds of a single event occurring or multiple events occurring in tandem, paying particular attention to the concepts of independent and dependent probability as well as the multiplication rule. After this, we cover how to calculate the number of possibilities in a scenario with a specific number of options in our "Factors and Permutations" and "Combinations" lessons. Odds are, after reviewing this next section, you'll be more confident in your understanding of these important topics!

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Section Outline

Calculating Probability

Factorials, Combinations, and Permutations

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Calculating Probability

Probability is the branch of mathematics that models the chances of specified events or groups of events occurring. Fractions or decimals with values between 0 and 1 are used to represent the chances of a specified event happening. 0 and 1 form the boundaries of the probability system: a probability of 1 means that an event is certain to happen, while a probability of 0 means that an event is certain not to happen. The probability of an event can be represented with the shorthand P_{event} or $P(event)$.

Probability representations are partial values generated by placing the number of desired options over the total number of options possible. Put so abstractly, this may be a bit confusing, so let's consider an example: a fair coin. When flipping such a coin, there are two possible outcomes: the coin can land heads-up or tails-up. If we want to model the probability (the "chances") of a fair coin landing on heads, we could do so by counting the number of outcomes we want and dividing it by the number of total outcomes. In this case, there are two total outcomes possible (heads or tails) and we want to calculate the odds of one of them occurring (P_{heads}), so we could model the situation using either the fraction $\frac{1}{2}$ or the decimal 0.5.

The probabilities of all possible options in a given situation need to sum to 1, because it's certain you'll get some result. As a result of this, you can also easily calculate the probability of an option *not* occurring. In cases in which there are only two options presented, you can simply subtract the chances of the other option occurring from 1. For our coin-flipping example, say we wanted to calculate the chances of the coin landing tails-up. Well, if the coin lands heads-up one out of every two coin flips, then it must land tails-up on the other one out of two coin flips. This logic is supported mathematically:

$P(\text{all options}) - P(\text{heads}) = P(\text{tails})$

$1 - \frac{1}{2} = \frac{1}{2}$

Let's try a slightly more complex example: Sample Question #1. To start, we need to take stock of the situation described. To calculate probability, we need to know the total number of possible outcomes. In this case, that's the total number of sandwiches the restaurant serves: 25. This will form the denominator of our fraction. But what do we put in the numerator? The only other number mentioned in the question

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Sample Question #1

Of the twenty-five sandwiches on a restaurant's menu, six of them have mayonaise on them. If you order a sandwich at random, what is the probability of you ordering a sandwich that does *not* have mayonaise on it?

A. $\frac{6}{25}$

B. $\frac{9}{25}$

C. $\frac{16}{25}$

D. $\frac{19}{25}$

is six, but before jumping to conclusions, be careful to look at the designation. The menu lists six sandwiches that have mayonaise on them; we want to calculate the probability of ordering a sandwich that does *not* have mayo on it. We can do this in one of two ways:

1.) Take the total number of sandwiches (25) and subtract the options that have mayonaise on them (6). Then, make the resulting number the numerator in a fraction with 25 in the denominator.

$$25 - 6 = 19$$

$$\frac{19}{25}$$

2.) Since we only have two options—a given sandwich either does or does not have mayo on it— we can subtract the probability of ordering a mayonaise-containing sandwich from 1 to find the probability of ordering a sandwich that does not have mayo on it. Six out of the twenty-five possible sandwiches have mayo on them, so the probability of ordering one of these is $\frac{6}{25}$.

$$1 - \frac{6}{25} = \frac{19}{25}$$

No matter which method you use, you get the same answer: if you order a sandwich at random, you have a $\frac{19}{25}$ chance of ordering a sandwich that does not have mayonaise on it.

Before we consider the next few sample problems, let's talk about decks of cards. Playing cards may very well make an appearance on your GRE Quantitative section, as they provide a moderately complex, consistently structured data set to which probability problems can refer. It would behoove you to brush up on your general knowledge of playing card structure before taking your exam! If you get stuck on a playing card-related problem, make sure to keep the basics in mind: standard decks consist of four suits: hearts, diamonds, spades, and clubs. The heart and diamond cards are traditionally red suits, while the spade and club cards are traditionally black suits. Each suit contains a card with a value of one (the "ace"), one card each for values two through ten, and then the "face cards": jack, queen, and king.

With that in mind, let's consider Sample Question #2. How many queens are in a standard deck of cards? Well, there are four suits, and each suit has one queen, so there are four queens. 52 will be the denominator of our probability fraction since there are 52 possible cards you could draw. We're looking for the probability of drawing a queen, so 4 becomes the numerator, and the answer is $\frac{4}{52}$.

Sample Question #2

If you pick a card from a standard deck of playing cards at random, what is the probability of drawing a queen?

A. $\frac{3}{52}$

B. $\frac{4}{52}$

C. $\frac{5}{52}$

D. $\frac{7}{52}$

Dependent vs. Independent Events

When calculating the probability of multiple events occurring in specific groups, the logic involved—and the math representing it—gets more complex. Before we go over different ways to calculate the probability of multiple events, we need to begin by cementing an important distinction. When considering multiple events, it is imperative for you to determine whether the probability of one event occuring affects the probability of the second event occurring. If one event's outcome can or does affect the other, the events are **dependent**. If no matter what happens, the two probabilities will not affect one another regardless of outcome, the two events are **independent**. If it's not possible for the events to occur simultaneously, they are independent, and furthermore, they are described as **mutually exclusive**. The table below provides some examples of dependent and independent events.

Dependent Events	Independent Events	Mutually Exclusive Events
Pick one, then another from the same group without replacement Drawing a card from a deck of cards and then drawing a second card without replacing the first	*Pick one, replace it, then pick another from the same group* Drawing a card from a deck of cards, replacing it, and then drawing a second card	Picking a heart, picking a diamond, picking a club, and picking a spade are all mutually exclusive events, since each card only belongs to one suit.
Conditional scenarios: do A, then B, where B depends on the results of A P(winning a prize) where if you draw a face card, you're allowed to flip a coin, and if that coin lands heads-up, you win a prize. Whether or not you get the opportunity to flip the coin is determined by the results of the card draw, so these events are dependent.	*Do A, then B, where the two have no bearing on one another* Drawing a card and rolling two dice. The result of the card draw doesn't affect the dice roll and vice versa.	In contrast to the scenario above, picking a red card and picking a face card are *not* mutually exclusive, since picking a face card that is also a red card is a possibility.

Probability of Multiple Events: "And" or "Or"

If a problem asks about the probability of one event happening **AND** another event happening, and the events are independent of one another, calculate the probability of each event happening and then multiply them together. The events *must* be independent, or your calculation will be incorrect because some possibilities will get counted twice. Multiplying a number by a fraction or decimal between 0 and 1 makes that number smaller. A smaller probability indicates a less likely result. If you want to calculate the probability of two events happening together, that is going to be less likely than either of them happening alone. If we model the probabilities of each event happening as a Venn diagram, the probabilitiy of them both happening would be represented by the union of the two events—the overlapping part.

On the other hand, if a problem asks about the probability of one event happening **OR** the other event happening, calculate the probability of each event happening separately and then add them together. This is logical: the probability of either event happening should be larger than the probability of one of the events in particular occuring.

There's one detail to watch out for, though: whether or not you include the possibility of both events occurring. This can change your answer dramatically, as we'll see in the next pair of sample questions. Pay attention to the wording of the questions you're given: if you're asked for the probability of "Event 1 or Event 2" occuring, that means that you are not allowed to include the probability of both occuring in your answer and must subtract the probability of this result occurring from your summed probabilities.

Thus, two things are crucial when considering multiple events:

- -
1.) Whether the events are independent or dependent

2.) Whether the problem is asking about one event "and" another or one event "or" another
- -

Assume Event 1 and Event 2 are independent events.

$$P_{Event\ 1} \text{ and } P_{Event\ 2}$$
$$P_{Event\ 1} \qquad P_{Event\ 2}$$
$$P_{Event\ 1} \times P_{Event\ 2}$$
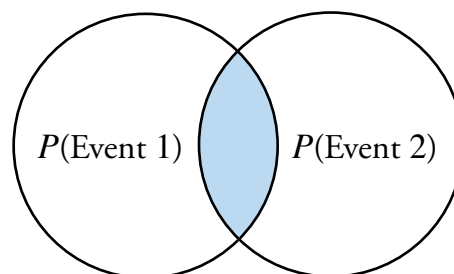
$$P_{Event\ 1} \text{ or } P_{Event\ 2} \text{ or } P_{Event\ 1\ and\ Event\ 2}$$
$$P_{Event\ 1} \qquad P_{Event\ 2} \qquad P_{Event\ 1\ and\ Event\ 2}$$
$$P_{Event\ 1} + P_{Event\ 2} + (P_{Event\ 1} \times P_{Event\ 2})$$
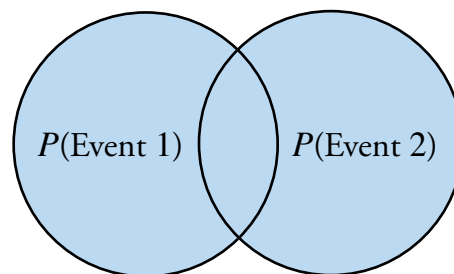
$$P_{Event\ 1} \text{ or } P_{Event\ 2} \text{ but not } P_{Event\ 1\ and\ Event\ 2}$$
$$P_{Event\ 1} \qquad P_{Event\ 2} \qquad P_{Event\ 1\ and\ Event\ 2}$$
$$P_{Event\ 1} + P_{Event\ 2} - (P_{Event\ 1} \times P_{Event\ 2})$$
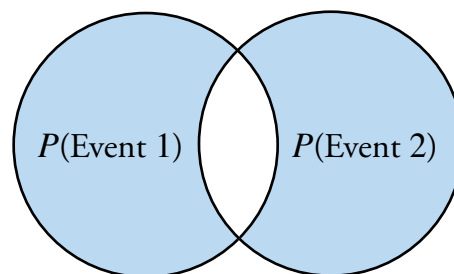
Sample Question #3 asks about two events that have no bearing on each other: flipping a coin and rolling a die. No matter how the coin flip turns out, it won't affect the roll of the die, and vice versa. These events are independent. We're being asked about the probability of them both occurring ("and") so we can simply find the probability of both and multiply them. The coin flip has two possible results—heads or tails—and the dice roll has six possible results, one for each of its faces. We're looking for one particular result for each action, so the numerator will be 1 in each of our probability fractions. Remember, when multiplying fractions, you can just "multiply across"—there's no need for common denominators. The correct answer is B.

$$P_{\text{Tails and Rolling a 4}} = P_{\text{Tails}} \times P_{\text{Rolling a 4}} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Sample Question #4 is a bit more difficult, as it's a card-drawing question. All card-drawing questions ask an implied question: are the cards being drawn being replaced, making the events independent, or are they not being replaced, making the events dependent? Aha, this problem is one of the latter: no mention is made of cards being replaced, so we need to treat the events as dependent on one another. After all, if we're not replacing the cards as we draw them, every card drawn slightly changes the odds of drawing a given card afterward.

The next implicit question we need to answer is "How many aces are there in a deck of cards?" The answer is four: each suit contains one ace. So, the odds of drawing an ace on our first draw are $\frac{4}{52}$. How many cards are left in the deck after that? 51, since we've drawn one of them. How many aces are left? We've drawn one of them at this point, so there are only three left. Since we're being asked about the two events occurring together, it's an "and" question, not an "or" question, and we need to multiply these probabilities. After we multiply and simplify, we're left with a fraction, whereas our answer choices are decimals. Dividing the numerator by the denominator and rounding allows us to pick out C as the correct answer.

$$P_{\text{Ace and then another ace}} = P_{\text{Ace \#1}} \times P_{\text{Ace \#2}} = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$$

$1 \div 221 = 0.004524887 \approx 0.005$

Sample Question #3

What is the probability of both getting tails on a fair coin flip and rolling a 4 on a fair six-sided die?

- **A.** $\frac{1}{20}$
- **B.** $\frac{1}{12}$
- **C.** $\frac{1}{10}$
- **D.** $\frac{1}{6}$

Sample Question #4

What is the approximate probability of drawing two consecutive aces from a standard deck of 52 playing cards?

- **A.** 0.4
- **B.** 0.05
- **C.** 0.005
- **D.** 0.008

Sample Question #5 applies the same principles in a different context. This time, we're working with calendars instead of dice rolls or coin flips. We're also working with 31 different birthdays instead of just a few events, which can easily throw you off.

Let's calculate Quantity A first, starting with the probability that one student has his or her birthday in January. That probability will be the number of days in January placed over the number of days in a year. That's $\frac{31}{365}$. Now we need to incorporate the fact that we're calculating the probability of this occurring for all 31 students in the library. If we were doing so for just two students, what would that problem look like? It's an "and" problem dealing with independent events, so it would just be the fraction multiplied by itself. After all, one student's birthday doesn't affect the others, and each student has the same odds of having a birthday in January.

$$\frac{31}{365} \times \frac{31}{365}$$

For 31 students, we just need to extend this pattern 31 times, since each student would have a probability fraction representing the odds of his or her birthday falling in January. We can shorthand this by raising the fraction $\frac{31}{365}$ to the thirty-first power: $\left(\frac{31}{365}\right)^{31}$. That's a pretty imposing figure, but don't calculate it out yet. Let's look at Quantity B instead. After all, 31 students are involved in that calculation as well—maybe something will cancel. We're told that there are 52 Saturdays in this particular year, so we can repeat the same logic we used in determining Quantity A and get $\left(\frac{52}{365}\right)^{31}$.

Look at that! The power appears for both quantities, so in comparing their values, we can effectively discard it. (This is the same maneuver as omitting something that appears on each side of an equals sign). We can simply look at the base to determine which is larger and which is smaller. Since the probability of having a birthday on a Saturday is greater than having a birthday in January, Quantity B is greater and B is the correct answer.

Sample Question #5

There are 31 students in the library. It is not a leap year, and this year has 52 Saturdays in it.

Quantity A	Quantity B
The probability that all students have their birthdays in January.	The probability that all students have their birthdays on a Saturday this year.

- **A.** Quantity A is greater.
- **B.** Quantity B is greater.
- **C.** The two quantities are equal.
- **D.** The relationship cannot be determined from the information given.

Dealing with More Than Two Events

Should you be asked to calculate the probability of more than two events, don't stress out about the added complexity. You simply need to apply the logic you use when working with two-event probabilty problems and extend it a bit. It such circumstances, it can be very helpful to sketch out a Venn diagram visually detailing exactly the unions and intersections for which you need to solve. Such a diagram can translate to a set theory equation, which can be adjusted to contain variables instead of sets.

Let's practice this step-by-step method in answering Sample Question #6. This is about as difficult as probability problems get at this level—not only do we have three different conditions to contend with, but only two of the three conditions are mutually exclusive. As a result, there are also a lot of options to consider since you could draw a diamond, an even diamond, or an diamond divisible by three; however, you can't draw an even diamond divisible by three, since by definition an even positive number won't be divisible by three. A Venn diagram showing all the different possible conditions is shown at right.

Now it's time to calculate the values that our diagram will help us organize. Don't jump to counting how many of the 52 cards that fit into each of the "overlapping" categories—those that fulfill multiple conditions. We'll be able to figure out the probabilities of drawing a card that falls into either of those using the probabilities of the main three conditions. First, consider the probability of each individual event: drawing a diamond, an even number, or a number divisible by three.

Sample Question #6

A card is drawn at random from a deck of fifty-two cards. What is the probability of drawing a diamond, a card with an even number on it, or a card with a number divisible by three on it, but not a card that falls into more than one of these categories?

A. $\dfrac{17}{26}$

B. $\dfrac{19}{26}$

C. $\dfrac{8}{13}$

D. $\dfrac{11}{13}$

P(Diamond and Even #)

P(Diamond) P(Even #)

P(Diamond and Divisible by 3) P(Divisible by 3) P(Even number divisible by 3)
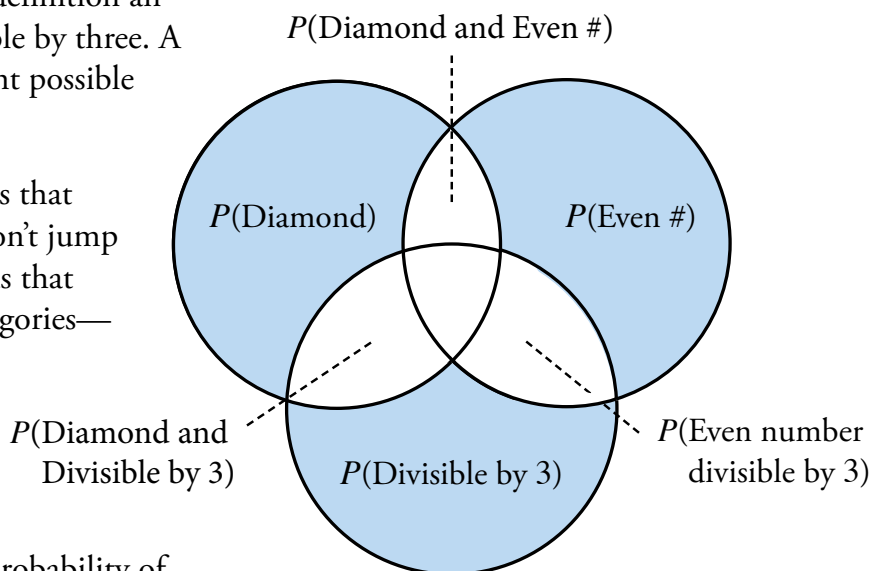
When drawing a diamond, there are four suits, and one suit is entirely diamonds. If you know that each suit contains 13 cards, great! If not, you can count cards, or you can go about this calculation an indirect way: since the entire deck is made of four suits, diamonds must make up one quarter of the deck. Whether you use this information or put 13 over 52 and reduce, $P_{\text{Diamond}} = \dfrac{1}{4}$. When drawing an even number, there are five possible even numbers to draw in each suit—2, 4, 6, 8, and 10. Since a suit contains 13 cards, $P_{\text{Even number}} = \dfrac{5}{13}$. If you want to draw a number divisible by three, there are three such values present in each thirteen-card suit—3, 6, and 9. So, $P_{\text{Number divisible by 3}} = \dfrac{3}{13}$.

Now that we have some probabilities, we need to figure out how to use them to get the correct answer. Consider the Venn diagram on the previous page: we're only looking for the probability representing the shaded sections. Since our problem uses the conjunction "or," we need to add these sections together. We specifically need to not include the unshaded sections, so any probabilities associated with these sections will need to be subtracted from our sum. Using the diagram to construct a formula, we get something that follows this general form:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$$

Defining the conditions based on the specific context of this problem, we get:

P(Diamond OR Even Number OR Number Divisble by 3) = P(Diamond) + P(Even Number) + P(Number Divisble by 3) – P(Diamond AND Even) – P(Diamond AND Divisible by 3) – P(Even AND Divisible by 3) – P(Diamond AND Even AND Divisible by 3)

Phew! That's a lot, but now all we have to do is calculate a probability for each combination, substitute those values into this equation, and solve. The probability of drawing an even number that is also a diamond is an "and" combination, so we need to multiply these values together:

$$P_{\text{Diamond}} \times P_{\text{Even}} = \dfrac{1}{4} \times \dfrac{5}{13} = \dfrac{5}{52}$$

We can do the same for the probability of drawing a diamond that is divisible by three:

$$P_{\text{Diamond}} \times P_{\text{Divisible by 3}} = \dfrac{1}{4} \times \dfrac{3}{13} = \dfrac{3}{52}$$

Finally, the probability of drawing an even number that is divisible by three is . . . more difficult to calculate than you might expect! If you try finding the product of the probabilities involved here, you get an answer that doesn't make a lot of sense, especially compared to our other answers.

$$P_{\text{Even}} \times P_{\text{Divisible by 3}} = \dfrac{5}{13} \times \dfrac{3}{13} = \dfrac{15 \div 13}{169 \div 13} \approx \dfrac{1.15}{52}$$

This is because the two sets of possible numbers we're examining here are *not* mutually exclusive. There's a number that fulfills both categories—6—so it appears in both and throws off our calculation. Suddenly, we need to account for that 6—it would be in the overlapping part of a Venn diagram where the circles are labeled "Even" and "Divisible by 3."

When you're trying to calculate the probability of one event AND another event, finding the product will only be correct if the two events don't overlap at all, or in other words, are independent. In cases like these, it's easier to just count the number of options that fall into the category at hand.

Case in point: how many even numbers in the set {2, 4, 6, 8, 10} are divisible by 3? One: 6. There will be one six in every suit, so the probability of drawing a card with a number on it that is both even and divisible by 3 is $\frac{4}{52}$, or $\frac{1}{13}$.

The same problem arises when we try to find the probability of drawing a number that is a diamond, even, and divisble by three. If we just find the product of these probabilities, it's not accurate:

$$P_{\text{Diamond}} \times P_{\text{Even}} \times P_{\text{Divisible by 3}} = \frac{1}{4} \times \frac{5}{13} \times \frac{3}{13} = \frac{15 \div 13}{676 \div 13} \approx \frac{1.15}{52}$$

We can tell this is incorrect because it's the same answer we got when we tried to calculate the chances of drawing an even numbers divisible by three. There are four of those cards and only one is a diamond, so this answer should definitely not be the same! Again, our math is off because the 6 is being counted twice: once as part of the "Even" category and once again as part of the "Divisible by 3" category. Thinking about this logically, we've already figured out that there's only one card in the deck that fits into this particular category: the six of diamonds. The probability of drawing that card is $\frac{1}{52}$, so we have our probability.

At this point, we can go ahead and substitute in all of our different probabilities to the equation we came up with earlier and solve:

P(Diamond OR Even Number OR Number Divisble by 3) = P(Diamond) + P(Even Number) + P(Number Divisble by 3) – P(Diamond AND Even) – P(Diamond AND Divisible by 3) – P(Even AND Divisible by 3) – P(Diamond AND Even AND Divisible by 3)

$$P_{\text{Diamond}} \; OR \; P_{\text{Even}} \; OR \; P_{\text{Divisible by 3}} = \frac{1}{4} + \frac{5}{13} + \frac{3}{13} - \frac{5}{52} - \frac{3}{52} - \frac{1}{13} - \frac{1}{52}$$

Creating a common denominator for our fractions makes adding and subtracting them much easier. Reducing gets us to our correct answer, C: $\frac{32}{52} = \frac{16}{26} = \frac{8}{13}$.

Factorials, Combinations, and Permutations

Probabilities are useful in determining the likelihood of an event. A probability is generally defined as the chances or likelihood of an event occurring. It is calculated by identifying two components: the event and the sample space. The event is defined as the favorable outcome or success that we wish to observe. On the other hand, the sample space is defined as the set of all possible outcomes for the event. But what happens if we do not know how many outcomes we have? Outcomes are not always easily identified or calculated; however, mathematical operations associated with permutations and combinations can make these processes easier.

In this lesson, we will discuss permutations and combinations, as well as factorials, a major component of the formula for each type of calculation. In doing so, we will provide you the information needed to calculate the outcomes of events under given circumstances.

Permutations

Permutations provide the number of outcomes *when the order of events matters*. Permutations are calculated using the following formula:

$$_nP_r = \frac{n!}{(n-r)!}$$

In this formula, *n* refers to the number of things or items in the model and the variable *r* refers to the number of ways that items can be ordered (i.e. the number of "bins" or "slots" present in the model). Put a different way, *n* represents how many choices we have to pick from and *r* represents how many elements we're picking.

Let's look at an example in order to better illustrate permutations. Suppose there are four marbles in a bag—a red one, a blue one, a white one, and a black one—and a researcher wants to know how many outcomes are possible if a person picks two of the marbles when the order of the marbles matters. If order matters, we need to treat picking a red marble and then a blue marble as a different result than picking a blue marble and then a red marble, even though in each instance we end up with the same colors of marbles after picking two. Thus, we need to use the permutation formula. To do this, we need to assign numbers to each of the variables we have: we have 4 marbles from which to choose, so $n=4$, and we want to pick two from that group of four, so $r=2$. Therefore, we can write our equation out like this:

$$_4P_2 = \frac{4!}{(4-2)!}$$

We need to calculate the number of permutations present in this model; however, we need to understand how to perform calculations involving factorials. Factorials are denoted with an

exclamation point (!). For every non-negative integer *n*, we can define its factorial by calculating the product of all of the integers less than or equal to *n*. For example, let's observe the following operations:

$$2! = 2 \times 1$$
$$4! = 4 \times 3 \times 2 \times 1$$
$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

A helpful hint: when working with factorials in both the numerator and the denominator of a fraction, it is often easy to cancel out much of the factorials and simplify the calculation to finding the product of only a few numbers instead of a great many of them. This can help you solve these problems more efficiently. For example, consider our equation:

$$_4P_2 = \frac{4!}{(4-2)!}$$

Solving the denominator's subtraction problem, we get:

$$_4P_2 = \frac{4!}{2!}$$

At first glance, the factorials might obscure the fact that this fraction can be reduced, but consider it with the multiplication written out: 2 and 1 appear in both the numerator and the denominator, so they can cancel.

$$_4P_2 = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}}$$

Keep in mind that you can simplify factorials even if their base numbers wouldn't normally cancel out. For example, picking two elements from nine:

$$_9P_2 = \frac{9!}{(9-2)!} = \frac{9!}{7!} = \frac{9 \times 8 \times \cancel{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}$$

In addition, you don't need to write out the denominator's factorial; just observe how it cancels.

$$_9P_2 = \frac{9 \times 8 \times \cancel{7!}}{\cancel{7!}}$$

The larger the base numbers in factorials in fractions, the more difference simplifying them can make. Use this technique to solve our marble example:

$$_4P_2 = \frac{4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} = 12$$

There are twelve possible permutations, shown in the table shown at right.

Event	Combination
1	Red, Blue
2	Blue, Red
3	Red, White
4	White, Red
5	Red, Black
6	Black, Red
7	Blue, White
8	White, Blue
9	Blue, Black
10	Black, Blue
11	Black, White
12	White, Black

Let's use our skills to solve Sample Question #1. We know that the researcher is testing four items or things—in this case the hormones: auxin, gibberellin, abscisic acid, and cytokinin. (Keep in mind that problems might not tell you directly how many things are in play, but might instead give you a full list of them and require you to count the elements yourself). Next, we know that there we can have three hormones in each treatment or three slots/bins. Since the order of the hormones matters to the researcher, we must use the permutation formula.

$$_nP_r = \frac{n!}{(n-r)!}$$

Substitute in our known values, expand the factorials, simplify them, and solve. There are 24 possible outcomes, so A is correct.

$$_4P_3 = \frac{4!}{(4-3)!} = \frac{4 \times 3 \times 2 \times \cancel{1}}{\cancel{1}} = 24$$

Sample Question #2 asks us to consider two situations. Quantity A looks like it will follow the same logic as our last two examples, so let's calculate it now. Order matters, so we're using the permutation formula, and we're selecting 8 people from a group of 20:

$$_{20}P_8 = \frac{20!}{(20-8)!} = \frac{20!}{12!}$$

$$_{20}P_8 = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13 \times \cancel{12!}}{\cancel{12!}}$$

Let's hold off a moment on actually finding that product and consider Quantity B; we may be able to compare the two quantities without needing to perform that calculation. For Quantity B, we are finding the number of unique ways we can order these doughnuts, so we're using factorials, but we're not selecting them from a larger group: we're using all 12. That means that Quantity B is simply 12!, which must be less than the product of 20 through 13. Quantity A is much larger, so A is correct.

Sample Question #1

A researcher is studying the effects of hormone treatments on coleoptile and radicle growth in corn sprouts. The researcher wants to test the effects of four hormones: auxin, gibberellin, abscisic acid, and cytokinin. The researcher can only give each sprout a combination of three hormones and the order in which they are given matters for the study. How many outcomes are present in this study?

    A. 24

    B. 36

    C. 48

    D. 64

Sample Question #2

Quantity A	Quantity B
The number of unique ways eight people from a group of twenty can be seated at a dinner table	The number of unique ways a dozen doughnuts can be ordered on a long buffet plate

    A. Quantity A is greater.

    B. Quantity B is greater.

    C. The two quantities are equal.

    D. The relationship cannot be determined from the given data.

Combinations

Now that we have discussed factorials and permutations, we need to discuss combinations. In doing so, we will provide you with the information needed to dissect a problem and identify whether to calculate permutations or combinations in order to identify the proper number of outcomes of a particular scenario. This information will help you to quickly and accurately solve problems and clear up confusion that may arise on test day.

Combinations help us to calculate the number of outcomes in a given model when *order does not matter*. In other words, pulling a red marble out of a bag and then pulling out a blue marble from the bag is considered the same result as pulling a blue marble and then a red marble. Combinations are calculated using the following formula:

$$_nC_r = \frac{n!}{(n-r)!r!}$$

The combination formula is similar to the permutation formula. In the combination formula the variable, n, refers to the number of things or items in the model and the variable, r, refers to the number of ways that items can be ordered. Let's use the previous example to calculate the number of combinations in the the marble model. Suppose there are four different colored marbles—red, blue, white, and black—and a researcher wants to know how many outcomes are possible if a person picks two of the marbles when the order of the marbles does not matter. In this particular model we can assume that the orders of the marbles pulled does not matter to the researcher. In cases where we do not care how combinations are ordered, we can use the combination formula.

$$_4C_2 = \frac{4!}{(4-2)!2!}$$

Let's start by expanding the factorials.

$$_4C_2 = \frac{4!}{(2)!2!}$$
$$_4C_2 = \frac{4\times3\times2\times1}{(2\times1)2\times1}$$
$$_4C_2 = \frac{24}{4} = 6$$

Event	Combination
1	Red, Blue
2	Red, White
3	Red, Black
4	Blue, White
5	Blue, Black
6	Black, White

The only difference between this and the permutation formula—that was used in the previous section—is an additional term in the denominator. We have the factorial of the number of bins or slots multiplied by the factorial of the number of things minus the number of slots. We know that there are six different combinations of outcomes for this model, shown in the table at right. In the combination example where order doesn't matter, we have half as many possible outcomes. If the only thing differing between

two described scenarios is whether or not the order of selected elements matters, then the scenario in which order does matter will have more potential combinations than the scenario in which order does not matter. Remembering this fact can help you breeze through any Quantitative Comparison questions that are testing the difference between combinations and permutations.

Let's tackle a few more sample questions to make sure that the distinction between these mathematical concepts is cemented.

Sample Question #3 likely looks imposing due to the amount of text that it involves. Once we break it down, we can see that we're just being provided with a question stem listing different options for different aspects of cupcakes and two different scenarios where certain options have been pre-decided. This narrows the number of potential choices for each scenario in a potentially unique way.

Let's start by counting up how many different categories of options we have, and how many options appear in each category.

Cake flavor: pick 1 or 2 out of 3

Frosting type: pick 1 out of 3

Frosting color: pick 1, 2, or 3 out of 8

Now that we've listed out all of the options, let's consider how we would calculate the total number of combiantions. Order doesn't matter here, so we'll use the combination formula. We'll actually need to use this formula three times: once for cake flavor, once for frosting type, and once for frosting color. We'll then need to muliply the results of each formula together to find the total number of options. For categories that involve multiple potential numbers of choices (e.g. pick 1 or 2), we'll need to run the formula twice, once for each number of picks. Let's get started.

> ### Sample Question #3
>
> When ordering cupcakes at a bakery, customers have the options of picking up to two cake flavors from vanilla, chocolate, or red velvet; one frosting type from buttercream, cream cheese, or brown sugar; and up to three colors from red, orange, yellow, green, blue, purple, black, or white. How many unique options does a customer ordering cupcakes have at this bakery?
>
> **A.** 1464 options
> **B.** 1510 options
> **C.** 1542 options
> **D.** 1656 options

Cake flavor : pick 1 or 2 out of 3

Pick 1 out of 3: $_3C_1 = \dfrac{3!}{(3-1)!1!} = \dfrac{3!}{(2)!1!} = \dfrac{3 \times \cancel{2!}}{\cancel{2!} \times 1} = \dfrac{3}{1} = 3$

When picking one out of a number of options, the combination formula shows that you just end up with that number of options. We can use this to save time on later calculations.

Pick 2 out of 3: $_3C_2 = \dfrac{3!}{(3-2)!2!} = \dfrac{3!}{(1 \times 1)2 \times 1} = \dfrac{3 \times 2 \times 1}{2} = \dfrac{3}{1} = 3$

So, customers have in total six different options when selecting cake flavor(s).

Frosting type is just pick one out of three, and we already found that that equals three, so customers have three options when selecting frosting type.

When selecting frosting color, customers can select one, two, or three types of frosting out of eight options. Picking one out of eight will result in eight options, so we just need to calculate the number of options picking two out of eight and three out of eight produce, respectively.

Pick 2 out of 8: $_8C_2 = \dfrac{8!}{(8-2)!2!} = \dfrac{8!}{(6)!2!} = \dfrac{8 \times 7 \times \cancel{6!}}{\cancel{6!} \times 2 \times 1} = \dfrac{56}{2} = 28$

Pick 3 out of 8: $_8C_3 = \dfrac{8!}{(8-3)!3!} = \dfrac{8!}{(5)!3!} = \dfrac{8 \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!} \times \cancel{3 \times 2} \times 1} = \dfrac{56}{1} = 56$

So, when picking frosting colors, customers have $8 + 28 + 56 = 92 =$ options from which to choose.

Now we can calculate the total number of options by finding the product of each category's total options. The correct answer is D!

6 cake type options × 3 frosting type options × 92 frosting color options = 1656 total options

Question Strategies: Quantitative Reasoning

Perhaps the first thing that jumps to mind when you consider the GRE's Quantitative Reasoning section is the wide variety of mathematical concepts on which you might be tested; however, the test also employs an impressive array of widely varied question formats in testing you on these concepts. For many questions, knowing how to handle the format is just as important as familiarity with the content being tested. With that in mind, this next section examines the four different Quantitative Reasoning question formats and outlines strategies for tackling each one as efficiently as possible. Much of the challenge of the GRE is created by its strict time limits, so many of our recommended tactics focus on ways to save time and avoid unnecessary calculations wherever possible while still remaining as confident in your response as if you had methodically performed every relevant calculation. We conclude the section with an examination of Data Interpretation questions which, while not technically a specific question type, are a predictable grouping of material that you can expect to see somewhere on your exam. Don't make the mistake of prioritizing content knowledge at the expense of familiarity with question form—after working through this next section, you'll be able to approach your exam after a well-rounded review!

Section Outline

Quantitative Comparison

Multiple-Choice with One Answer

Multiple-Choice with Multiple Answers

Numeric Entry

Data Interpretation

Quantitative Comparison

Perhaps the Quantitative section's most challenging format, Quantitative Comparison quesitons can strike even the most mathematically expert test-takers off-guard and present a significant obstacle to earning scores coeval with your review and your knowledge. The unique difficulty of Quantitative Comparison is generated by the different ways in which quantities can be presented involve different combinations of abstract descriptions and concrete figures, and therefore each one demands a customized approach. In this vein, you have to think not only about how to determine the correct answer, but how *best* to solve the problem at hand; furthermore, you might not be given enough information! On every other Quantitative Reasoning question type, you're given enough data to arrive at the correct answer, but that is not the case here: one of the answer options is "The relationship cannot be determined from the given data." The answer choices are always the same for Quantitative Comparison questions, so you'll never get the reassurance that comes with your calculated answer perfectly matching one of those listed. Furthermore, process-of-elimination tactics don't get you very far with the given choices, since they're all comparative. You won't be able to eliminate all but one answer choice unless you can identify the correct answer anyway; the best scenario you'll be able to get to by eliminating answers is a 50/50 choice between A and B.

By focusing on these unique traits of Quantitative Reasoning questions, we can practice responding to them in ways designed around the details of the format, potentially saving you time, stress, and second-guessing. We can break down these problems into the following categories:

- You have to calculate both Quantity A and Quantity B
- You need to calculate only one quantity because the other is given to you
- You need to put one quantity in terms of the other
- You don't need to calculate either quantity
- You can't calculate either quantity

Let's consider each of these scenarios and practice a method for solving each one.

Solving Problems Where You Need to Calculate Both Quantities

In the most straightforward scenarios, you'll be shown two descriptions of quantities, neither of them numerical nor easily comparable to one another based on the descriptions. You'll need to solve for each quantity individually and compare your results in order to pick out the correct answer. These questions may seem to take the most time, but the calculations they involve are likely shorter and simpler than other Quantitative Reasoning questions, e.g. those where you only need to perform one, perhaps more complex, calculation.

In Sample Question #1, we're given two pairs of points on the coordinate plane and asked to solve for the distance between each pair. Oftentimes when you find yourself having to perform two calculations to solve a Quantitative Comparison problem, you'll find that the quantities mirror one another and that you'll need to perform the same calculation twice over, once for each quantity. In this case, we'll be using the distance formula (shown below) twice

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Just because you need to determine two numeric quantities for a Quantitative Comparison question doesn't mean you can't also save time where possible using your knowledge of the math principles being tested. Take a look at Quantity A: both points have a y-value of 0; therefore, the distance between the points must be the difference in their x-values. $10-6=4$, so the points are four units apart. You just saved yourself some time calculating that answer using the formula! Be on the lookout for opportunities like this, but don't be so eager to save time that you take shortcuts about which you're not sure. You don't want to mess up your calculations and get the problem incorrect just because you tried to save a few seconds of computation!

Quantity B doesn't allow for any such shortcuts, so let's identify its points and plug them into the distance formula. x_1 is 1, x_2 is –2, y_1 is 1, and y_2 is 4. Taking the time to jot down which variable is which can save you a lot of time if you end up needing to or wanting to review your work.

$$d = \sqrt{((-2) - (1))^2 + ((4) - (1))^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18}$$

Uh oh, our answer isn't immediatley obvious. We now need to answer the question, "How does the square root of 18 compare with 4?" If you encounter a similar situation on test day, don't panic: you

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Sample Question #1

Quantity A	Quantity B
The distance between the points $(6,0)$ and $(10,0)$	The distance between the points $(1,1)$ and $(-2,4)$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

have access to a square root key on the simple digital calculator that accompanies the GRE's testing program, and you can also use your estimating skills to determine how the two quantities must relate. Let's solve this problem using estimation for practice. In this case, we can put these two quantities in the same terms in one of two ways: we can consider of what number the square root is four, or we can estimate the square root of 18 by considering the two perfect squares it falls between.

The square root of 16 is 4. The square root of 18 must be larger than the square root of 16. (You can check this rule by considering that the square root of 9 is 3, but the square root of 25 is 5.) Thus, the square root of 16 (Quantity A) must be smaller than the square root of 18 (Quantity B), making B the correct answer. Alternatively, we could consider that the square root of 16 is 4 and the square root of 25 is 5. Since 18 falls between 16 and 25, its square root must fall somewhere between 4 and 5. A number between 4 and 5 noninclusive (Quantity B) must be larger than 4 (Quantity A), so Quantity B must be larger.

Be careful when converting your numeric answer to your multiple-choice response, especially when working with negative numebers. Making a logical misstep anywhere in the process of solving the problem, even as the very last step after you're done calculating, can result in an incorrect response!

Solving Problems in Which You Need to Calculate Only One Quantity

In other Quantitative Comparison problems, you'll only need to perform calculations for one of the two presented quantities. There are various scenarios in which this situation can arise. In one of them, like in Sample Question #2, one of the quantities is a discrete number. Thus, you just need to determine the discrete number value of Quantity A in order to see how it stacks up against Quantity B. This question involves calculating the number of options when selecting 3 students from a group of 8, so we'll need either the combinations formula or the permutations formula. Does order matter in the groups that we're selecting? No—therefore, we need to use the combinations formula.

$$_nC_r = \frac{n!}{(n-r)!r!}$$

Sample Question #2

Out of a group of 8 students, 3 are attending a meeting.

Quantity A | Quantity B

The number of different groups of students from the group of 8 who could attend | 336

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

Out of 8, we're picking 3, so $n = 8$ and $r = 3$.

$$_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5! \times 3!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times 3 \times 2 \times 1} = \frac{336}{6}$$

Wait a second, you might be saying—what's 336 divided by 6? That fraction isn't in reduced form! Here's another trick to solving Quantitative Comparison questions: remember you only need to understand how the terms *compare*—the aim of the game isn't necessariy to determine a numeric value for each quantity. As soon as you can definitively determine which quantity is the larger one, if they're equal, or if you won't have enough information to figure out which is necessarily larger, stop! You've solved the problem. Mark your answer—in this case, B—and move on to the next question!

In addition to those questions that provide you with a numeric quantity, you also only need to perform one calculation for questions in which one of the quantities simply needs to be converted so that both quantities are stated in the same terms. There may be no defined numeric values invovled at all, as in Sample Question #3. In this case, we don't need to do anything to Quantity B—there's no way we can reduce that or further determine its value without additional information. But don't mark D—we can simplify Quantity A.

$$(y^2)(y^4) = y^{2+4} = y^6$$

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Sample Question #3

Quantity A	Quantity B
$(y^2)(y^4)$	y^8

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

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Now that we've put Quantity A and Quantity B into similar terms, we can better compare them. But how does y^6 comapre to y^8? We can substitute different values into these expressions to figure out the correct answer. We could pick any number to substitute, but which ones are most likely to yield relevant insights? If you need to substitute a group of numbers into a pair of expressions to compare them, start with –1, 0, and 1—or at least a negative number, 0, and a positive number. A lot of expressions change their value dramatically when shifting between negative and positive values, and zero can often offer valuable insights as to whether expressions can be equal. Remember, if expressions can be equal but aren't necessarily in the context of a particular problem, you've solved it, and the answer is D, not C!

If $y = 0$, $y^6 = 0^6 = 0$ and $y^8 = 0^8 = 0$, so the two quantities are equal. If $y = 1$, $y^6 = 1^6 = 1$ and $y^8 = 1^8 = 1$—still equal. What if we try $y = 2$? In this case, $y^6 = 2^6 = 64$ and $2^8 = 2^8 = 256$. Do you need to calculate those exact numbers? No! You just need to recognize that whatever 2^6 equals, 2^8 is going to equal that times four, and thus larger than 2^6, since we're working with positive numbers. Thus, based on our substitutions, Quantity A is either equal to or smaller than Quantity B, making D correct.

Solving Problems in Which You Don't Need to Calculate Either Quantity

Sometimes, you don't need to perform *any* calculations to solve a Quantitative Comparison question, though that doesn't mean that determining the correct answer will be easy! This is an extreme version of taking a mathematical shortcut in solving for one quantity—if you can determine that one quantity must be larger or smaller than another without doing any calculations, you've solved the problem, and likely more quickly than you would have if you needed to break out formulae and the calculator.

Take Sample Question #4, for instance. Look at how the two quantities relate. They're exactly the same apart for one important detail: Quantity A includes the phrase "with replacement" and Quantity B

Sample Question #4	
Quantity A	Quantity B

The number of ways to choose 3 cards with replacement from a standard 52-card deck

The number of ways to choose 3 cards without replacement from a standard 52-card deck

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the given data.

the phrase "without replacement." Think about this logically: if you don't replace cards after drawing them, you'll necessarily have fewer choices for your next selections, resulting in fewer ways in which you could choose 3 cards from 52. With no such condition restricting its picks, Quantity B will have more options for each selection and thus there will be more ways to choose 3 cards from the deck. In effffect, the number of options you'd have for Quantity A would be $52 \times 51 \times 50$, whereas for Quantity B, it would be $52 \times 52 \times 52$ —necessarily larger. Thus, we don't actually need to calculate these products, or even determine the exact expressions—we just need to recognize that by involving replacement, Quantity B must involve more ways to pick 3 cards from 52, making B correct.

Solving Problems in Which You Can't Calculate Either Quantity

For some Quantitative Comparison problems, comparison might be entirely out of the question. These questions might rely solely on your understanding of the principles being discussed, specifically how one measurement always affects another measurement in a certain way, or perhaps the direct relationship between an abstract concept and the quantity it is measuring. The latter is the case for Sample Question #5. There are numbers provided in the question stem of this problem, but the information about the mean of each set is extraneous. All you need to do to solve this problem is to realize that the standard deviation is a measure reflecting the dispersion of numbers in a set. The larger the standard deviation of a set is, the larger the dispersion between its elements. Thus, we're told that Set A's standard deviation is 2 and Set B's standard deviation is 20, Set B must necessarily have the larger standard deviation between its elements because it has the larger standard deviation. B is correct. While questions that avoid calcualtions like this one can be very hit-or-miss, the upside to them is that if you're familiar with the concept(s) being tested, they can be solved extremely quickly and you can have great confidence in your responses.

> ### Sample Question #5
>
> Set A has a mean of 4 and a standard deviation of 2. Set B has a mean of 100 and a standard deviation of 20.
>
> | Quantity A | Quantity B |
> | --- | --- |
> | The dispersion of numbers in Set A | The dispersion of numbers in Set B |
>
> **A.** Quantity A is greater.
>
> **B.** Quantity B is greater.
>
> **C.** The two quantities are equal.
>
> **D.** The relationship cannot be determined from the given data.

Multiple-Choice with One Answer

The great majority of questions you'll face on your Quantitative Reasoning sections will seem like standard fare for formal tests: multiple-choice questions with four answer choices each, only one of which is correct. While you might initially expect there to be a dearth of tactics applicable to such a straightforward format, there are a few categories into which you can place any given single-answer multiple-choice problem based on the approach that either can or must be applied to it. By thinking critically about how to solve this type of problem, you can recognize opportunities to apply time-saving strategies and identify the best questions to guess on should you find yourself running out of time. We'll go over these strategies in a loose order of time required for each type of single-answer multiple-choice problem, starting with the fastest.

Process of Elimination Applicable as "Checklist" of Correct Answer Features

Sample Question #1

Which data set would be represented by this box-and-whisker plot?
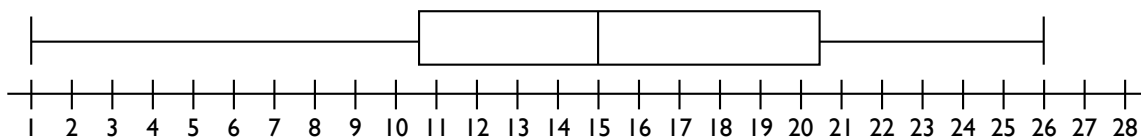
A. {1, 9, 10, 11, 12, 14, 15, 16, 17, 19, 22, 25, 26}
B. {1, 2, 5, 7, 7, 9, 15, 15, 15, 22, 23, 24, 26}
C. {10, 10.5, 11, 11, 12, 13.5, 13.75, 14, 15, 20, 22, 22.5, 26}
D. {1, 11, 12, 13, 13, 14, 15, 20, 21, 21, 22, 23, 26}

The way that certain questions are set up allows clever test-takers to identify features that the correct answer choice must have and immediately eliminate any answer choice that is missing any one of these identified features. Consider these conditions like check boxes: the correct answer will check all of them, whereas incorrect answers will miss at least one, and maybe more. While you need to be very sure of the rules you're using to cull answer choices, this can save a great deal of time than if you analyzed each answer choice fully. Consider Sample Question #1 to see this strategy in action. We're given a box-and-whisker plot and asked to identify the data set used to produce it. There are a number of data points we can gather from a box-and-whisker plot, so a good first step would be to jot down any that we could easily identify in a numeric data set.

Lowest number: 1   Highest number: 26   Median: 15   Q1: 10.5   Q3: 20.5

While we could calculate the interquartile range, it would take quite a bit of math to calculate that metric for each answer choice, and it might not be necessary. Let's see how far we can get toward finding the correct answer with just those more immediate metrics we identified. While it may be necessary to perform more complex calculations on multiple answer choices, if we can narrow down the number of potential correct answers using other information, that will save us time we'd otherwise spend performing extraneous calculations.

Let's start with the basics: do all of our answer choices have a lowest number of 1 and a highest number of 26? Their highest numbers are all 26, but C has a lowest number of 10, not 1, so it cannot be correct. Next, we can check the median of each answer choice. Each of the answer choices is a set composed of 13 elements; recognizing this saves us time because we can just turn our attention to the seventh number in each set, as this will be the median of each. For A and B, that number is 15; for D, it's 16, making D incorrect.

Now we need to turn our attention to quartiles. Q1 is the median of the lower half of the data set, and Q3 is the median of the upper half. Calculating these medians will require finding the average of the third and fourth numbers in a data set (for Q1) and the ninth and tenth numbers (for Q3). For A, Q1 $= \dfrac{10+11}{2} = 10.5$, and for B, Q1 $= \dfrac{5+7}{2} = 6$. Aha! A must be the answer, regardless of how the two answer choices' Q3 values compare, as B does not meet the requirements established by the graph.

Note that the step-by-step nature of applying process of elimination in a "checklist" format eliminates answer choices one by one, making it a useful tactic even if you're forced to guess at the end. Narrowing your options down from four answer choices to two increases the chances you can pick the right answer at random by 25%. Not all questions can be answered using this "checklist" approach, though, so let's take a look at how to apply process-of-elimination tactics while solving another variety of problem.

Process of Elimination Applicable While Solving

If you're running out of time and guessing on questions, this is exactly the type of question you might be happy to see. Certain single-answer multiple-choice questions allow you to eliminate answer choices based on mathematical logic during the process of calculating their answer. While this may not affect your final answer, it can help increase your confidence in it, and if you have to guess, it can help you identify those answers that aren't reasonable responses to the question.

Take Sample Question #2, for example. We're given the mean of a set of three unknown variables—15—and then

Sample Question #2

The arithmetic mean of the positive whole numbers x, y, and z is 15. If $w = 10$, then what is the average of w, x, y and z?

A. 9
B. 12.5
C. 13.75
D. 15

asked how that mean will change upon the addition of a variable with a known value—10—to the set.

We can immediately discount answer choices A and D using mathematical logic. We're told that x, y, and z are positive whole numbers, so some of them must be above 15 and some of them between 1 and 15. If you add in another positive whole number like $w = 10$, you're going to shift the mean, so D isn't going to be correct. Furthermore, the mean isn't going to suddenly jump from 15 to 9 with the addition of 10 to the set: adding a number to the set would not result in a mean lower than that number if we start with a mean larger than that number.

Thus, just glancing at the answer choices and applying some common sense, we can knock out two answer choices. This would be very helpful if we ended up needing to guess at this problem, but we can solve it to figure out whether the new mean is 12.5 or 13.75. We can do this by considering that the following has to be true based on how you calculate mean:

$$\frac{x + y + z}{3} = 15$$
$$x + y + z = 45$$

Now we just rewrite that first equation to include w, which we know to be 10. Let's use a to refer to the mean after the addition of w to the set.

$$\frac{x + y + z + 10}{4} = a$$

That's a lot of variables! Even if we don't know the exact values of $x + y + z$, we know that they have to sum to 45. We can substitute in 45 for $x + y + z$ in our new equation:

$$\frac{45 + 10}{4} = a$$

At this point, all we need to do is find the sum of 45 and 10 and divide it by 4. It turns out that the correct answer is C.

$$\frac{55}{4} = 13.75$$

If you're very lucky, you might come across a question that has been set up to allow you two options: using process of elimination based on a mathematical principle, or performing a long calculation. Sample Question #3 is one such question. While it is not highly likely that the test-writers will include many if any such questions, be on the lookout for ways in which you can use your knowledge of math not only in performing calculations, but also to knock out answer choices and potentially identify the correct response without performing any calculations.

We're asked to identify which of the given equations is a line perpendicular to a line passing through two points. One point is (4,5). Consider where this is: somewhere in Quadrant I. The other point is (8,–1). Consider where this one is: it's in Quadrant IV, and since its x-value is larger than 4, it is farther to the right. This means that we're going to move down and right when moving from (4,5) to (8,–1). Thus, whatever it's linear equation is, it's going to have a negative slope. When finding the equation of a line that is perpendicular to another line, you invert the slope and change its sign. The "change its sign" part is what's relevant in this problem: this means that we have to end up with a positive slope. Only one answer choice is a line that contains a positive slope: B. If you work through the math, it turns out to be the correct one, but you can save a lot of time by applying your knowledge more generally before resorting to the specifics of calculation.

Which of the following is a line perpendicular to the line passing through $(4, 5)$ and $(8, -1)$?

A. $y = -\dfrac{2}{3}x + 13$

B. $y = \dfrac{2}{3}x - 12$

C. $y = -5x - 14$

D. $y = -\dfrac{5}{3}x - 12$

Calculate and Match the Answer to Confirm

Some Quantitative Reasoning questions can be approached linearly—that is, you simply identify the calculation you need to perform, perform it, and see which of the answer choices matches it. Keep in mind that for questions like this, the test-writers may have inserted incorrect answers that match the results you get if you make common math missteps, so if you have time, it is prudent to glance over your scratch work and make sure that your calculations are sound. In Sample Question #3, this occurs with a few answer choices.

Let's solve Sample Question #3 and see how the incorrect answers are purposely designed. We know the perimeter of this triangle as well as algebraic expressions for each of its sides, so we can write the following equation:

$$(x) + (2x - 1) + (x + 9) = 40$$

The perimeter of a right triangle is 40 units. If the lengths of the sides are x, $2x - 1$, and $x+9$ units, then what is the area of the triangle?

A. 60 units^2
B. 68 units^2
C. 127.5 units^2
D. 136 units^2

We can combine the left side's expressions:

$$4x + 8 = 40$$

Now we need to subtract 8 from each side and then divide by four:

$$4x = 32$$

$$x = \frac{32}{4} = 8$$

Knowing that x=8, we can substitute 8 in for x in our original expressions for the lengths of the triangles' sides and find their lengths:

$$x = 8$$
$$2x - 1 = 2(8) - 1 = 16 - 1 = 15$$
$$x + 9 = 8 + 9 = 17$$

Ok, we know the lengths of each of the triangle's sides! Now we need to find its area. We can use the formula $\frac{1}{2}bh$, but to do that, we'll need to identify the base and height. Since this is a right triangle, we just need to identify the triangle's legs, since one will be the base and the other the height. The longest side, 17, must therefore be the triangle's hypotenuse, making its legs 8 and 15.

$$\frac{1}{2}bh = \frac{1}{2} \times 8 \times 15 = 60 \text{ units}^2$$

We've identified the correct answer, A, but consider how you might end up with a result that matches the other answer choices. You get C, 120 units², if you identify the correct legs of the triangle but forget to divide the product of the base and height by two. You get B, 68 units², if you choose the hypotenuse as the base or height along with the 8-unit side, and you get 127.5 units² if you use the hypotenuse and the 15-unit side as the triangle's legs. It's very important to be sure of your work, so keep neat notes to facilitate checking them!

Calculation Required for Each Answer Choice

Occasionally, you might come across a single-answer multiple-choice question that requires you to consider each answer choice individually, often by performing a calculation on each option to identify the correct one. While these questions take a great deal of time, there may not be a faster way to approach them. If this is the case, you can be confident that the test-writers were aware of it as they designed the test to be taken in a certain amount of time. The best thing you can do is to work methodically and keep careful, neat notes.

Let's go through Sample Question #5 to demonstrate this sometimes-necessary approach. In this case, we're given the height and base of a triangle and asked which of the listed options is similar to that triangle. Similar triangles will have the same ratio of side lengths, so we just need to form a fraction from the base and height and compare it to the same fraction of base and height for each answer choice. Let's make a fraction by placing height over base. For the given triangle, this is $\frac{6}{9}$, or, in reduced form, $\frac{2}{3}$. Which of the answer choices, when height is placed over base in a fraction, also reduces to $\frac{2}{3}$?

A: $\dfrac{5}{10} = \dfrac{1}{2}$

B: $\dfrac{7}{12}$

C: $\dfrac{8}{14} = \dfrac{4}{7}$

D: $\dfrac{10}{15} = \dfrac{2}{3}$

> ### Sample Question #5
>
> A right triangle with which one of the following measurements is similar to one with a height of 6 and a base of 9?
>
> **A.** $h = 5$, $b = 10$
>
> **B.** $h = 7$, $b = 12$
>
> **C.** $h = 8$, $b = 14$
>
> **D.** $h = 10$, $b = 15$

The correct answer is D! This problem didn't involve any intense calculations, but if you need to analyze each answer choice individually, it's not too likely that the test-writers will make you perform extremely long and complex calculations on each one. Just remember to consider any alternative approaches that you can use before you resort to tackling the answer choices one by one.

Multiple-Choice with Multiple Answers

Some questions on your GRE Quantitative Reasoning sections present you with five answer choices and the opportunity to select more than one of them as correct. In order to get any credit for the question, you need to select all of the correct options and none of the incorrect options, so these questions can present quite a challenge. As in solving Quantitative Comparison or Numeric Entry problems, process of elimination is not a viable strategy; however, Multiple-Answer questions fall into one of a few categories that you can address specifically.

- Problems with a Range of Correct Answers
- Problems with a Known Number of Correct Answers
- Problems with an Unknown Number of Correct Answers (Individual Analysis Required)

It's worth noting that most of these are still rather time-intensive, as there will not be one clear correct answer if the test is using a Multiple-Answer format, so more consideration will be required of you than on a Single-Answer multiple-choice question. Given this, it may be advisable to skip any particular question that you don't have a clear plan for solving and try to return to give it more consideration later.

Let's tackle a sample problem from each of these categories so that you can familiarize yourself with the ways in which they differ and learn to vary your approach to be most efficient.

Solving a Multiple-Answer Problem with a Range of Correct Answers

Certain Quantitative Multiple-Answer problems explicitly direct you to solve for ranges of possible numbers that fulfill stated conditions and are correct answers if listed. Other questions of this type might allow for this sort of approach in a more subtle manner. After you solve for the range of correct answers for a given problem, you can turn your attention to the answer choices and quickly determine which fall into that range. Since it's a very efficient way of approaching a problem, always take a moment to consider whether you can approach a Multiple-Answer question from a "correct range of answers" perspective instead of considering each answer choice one by one.

Let's see this tactic in action by working through Sample Question #1. Here, we're explicitly asked to identify which of the answer choices fall between two

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Sample Question #1

The mean of a set of data with a standard deviation of 0.32 is 4.45. Which of the following numbers fall between 2 and 3 standard deviations away from the mean?

    A. 3.56
    B. 3.80
    C. 3.84
    D. 5.10
    E. 5.44

and three standard deviations away from the mean. We're told that the standard deviation of the data set is 0.32 and that its mean is 4.45; knowing these two pieces of information, we can solve for the boundaries of one, two, and three standard deviations in either direction from the mean.

Mean - three standard deviations = 4.45 - 3(0.32) = 3.49

Mean - two standard deviations = 4.45 - 2(0.32) = 3.81

Mean + two standard deviations = 4.45 + 2(0.32) = 5.09

Mean + three standard deviations = 4.45 + 3(0.32) = 5.41

Our calculations show that any number between 3.49 and 3.81 is between two to three standard deviations below the mean, and that any number between 5.09 and 5.41 is between two to three standard deviations above the mean. Now that we've identified our range of correct answers, it's time to consider our answer choices. 3.56 and 3.80 fall between 3.49 and 3.81, so A and B are both correct. 3.84 does not fall in either of our identified correct ranges, so C is incorrect. Considering our other identified range, 5.10 falls between 5.09 and 5.41, so D is correct, but 5.44 falls above the range of correct answers, so it is incorrect. Altogether, this problem has three correct answers: A, B, and D. Keep in mind that all five answers could have been correct, or only one of them: we had no information that suggested that there had to be a particular number of correct responses or that the number of correct responses (not the numerals in the answer choices themselves) was limited in any particular way. For some types of Multiple-Answer multiple choice Quantitative Reasoning questions, you may very well be able to determine that a particular number of answer choices could logically be correct. Let's look at that type of question next.

Solving a Multiple-Answer Problem with a Known Number of Answers

When solving certain GRE Quantitative Multiple-Answer problems, you may realize that there is a realistic number of answer choices that could possibly be correct, regardless of the individual numbers or options presented to you. For example, if you're asked to solve for values that could be found in the coordinate pair of point where the graphs of two linear equations intersect, you know you're limited to providing a maximum of two answers: one for the correct x-value, and one for the correct y-value. Because you're only working with the coordinates of a single point, there can't be any other correct answers. Keep in mind that just because you have a maximum of two answers in this situation, it doesn't mean that the problem has two correct responses. The problem might only list one of the coordinates' values amongst its answer choices. In this case, the problem would have only one correct answer, but you could be sure of your answer to the entire problem after you'd found that one answer.

As another example, consider if you are asked to solve a square root in the context of a Multiple-Answer question. Keep in mind that since the product of two negatives is a positive, the square root of a positive number is actually +/- the root, thus creating the opportunity for two answers: a positive

number and a negative number. You could be sure that marking three, four, or five answers would be incorrect, though only one might be listed.

Let's see this sort of scenario play out as we solve Sample Question #2. We can go about solving this problem in a couple of different ways, but before we do, it's important to realize that since we're working with a function that has a highest term of two, its graph will be a parabola, and it will have at most two roots, though it might have only one. Thus, we shouldn't expect to mark three, four, or all five answer choices as correct. This problem will have either one correct answer or two: either the function will have one root, and it will be listed, or it will have two, and either one or both will be listed.

Time to solve the quadratic! We can either use factoring or the quadratic equation to identify this function's roots. We'll go over each method so that whichever you prefer, you can follow along and check your work.

Factoring

Because the coefficient in front of the x^2 is not equal to 1, we need to multiply this coefficient by the constant, which is –4. When we multiply 2 and –4, we get –8. We must now think of two numbers that will multiply to give us –8, but will add to give us –7 (the coefficient in front of the x term). Those two numbers which multiply to give –8 and add to give –7 are –8 and 1. We will now rewrite $-7x$ as $-8x + x$.

$$2x^2 - 7x - 4 = 2x^2 - 8x + x - 4 = 0$$

We will then group the first two terms and the last two terms.

$$(2x^2 - 8x) + (x - 4) = 0$$

We will next factor out a $2x$ from the first two terms.

$$(2x^2 - 8x) + (x - 4) = 2x(x - 4) + 1(x - 4) = (2x + 1)(x - 4) = 0$$

Thus, when factored, the original equation becomes $(2x + 1)(x - 4) = 0$. We now set each factor equal to zero and solve for x. Let's start with $(2x + 1)$.

$$(2x + 1) = 0$$

Subtract 1 from both sides.

$$2x = -1$$

Divide both sides by 2.

$$x = -\frac{1}{2}$$

Now, we set x – 4 equal to 0.

$$x - 4 = 0$$

Add 4 to both sides.

$$x = 4$$

The roots of $f(x)$ occur where $x = -\dfrac{1}{2}$ and where $x = 4$. The correct answers are thus B and E.

Quadratic Formula

Alternatively, we can use the quadratic formula to solve for this equation's roots. First, we need to identify a, b, and c. a is the coefficient of the squared term, b is the coefficient of the variable term, and c is the constant. In this case, $a=2$, $b=-7$, and $c=-4$.

Now we can substitute our variables into the quadratic equation:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-4)}}{2(2)}$$

$$\frac{7 \pm \sqrt{49 - (-32)}}{4}$$

$$\frac{7 \pm \sqrt{49 + 32}}{4}$$

$$\frac{7 \pm \sqrt{81}}{4}$$

$$\frac{7 \pm 9}{4}$$

At this point, we need to calculate two different results of our remaining expression: one that finds the sum of the numerator, and another that finds the difference of the numerator.

$$\frac{7+9}{4} = \frac{16}{4} = 4$$

$$\frac{7-9}{4} = \frac{-2}{4} = -\frac{1}{2}$$

As you can see, no matter which approach we use, we end up with the same answers: this function crosses the x-axis at $x = -\frac{1}{2}$ and $x = 4$. Both of these answers are listed as answer choices B and E, so you'd need to select both of those and no other answer choices to get this question correct.

What if a question doesn't have an apparent relevant limit on the number of correct answer choices and you can't identify a range or ranges into which the correct answer(s) must fall? In this case, you may need to analyze each answer choice individually. While this takes time, if it is the required approach for a given problem, the test writers will be aware of the time required to solve it; however, sometimes shortcuts still become apparent that can help you remove several answer choices from consideration at once. We'll take a look at one problem that demonstrates each of these scenarios in the next section.

Solving a Multiple-Answer Problem Using Individual Answer Analysis

In some problems, going down the list of answer choices and performing certain calculations on each one to determine if it is correct is an unavoidable situation. You can usually spot such questions based on the variety apparent in their answer choices. For instance, in Sample Question #3, you might be tipped of that this problem falls into this category when you see that the question concerns volume and each answer choice involves a distinct three-dimensional shape. Fortunately, for many of this sort of Multiple-Answer questions, you'll need to perform similar calculations on each of the answer choice. Unfortunately, Sample Question #3 doesn't fall into this category completely; while you need to calculate volume for each of the answer choices, each shape requires you to apply a distinct equation.

Keep careful and neat notes when working through this type of Multiple-Answer question. Process-

Sample Question #3

A worker at a recycling factory needs to produce 100 cm³ of aluminum for an order. Which of the following solid aluminum items alone would allow the worker to meet his or her goal? Use 3.14 for the value of π.

   A. A hemisphere with diameter 7.5 cm

   B. A right cylinder with radius 4 cm and a height 2 cm

   C. An cube with side length 4.5 cm

   D. A right rectangular prism with width 5 cm, length 7 cm, and height 3 cm

   E. A sphere with radius 3 cm

of-elimination does nothing for you in this particular context, so it's important to be very confident in your work, and part of that confidence may come from reviewing it should you have extra time. Making your notes easy to glance over can help you check your work efficiently, without confusion, and proceed to reviewing your answers to other questions.

To actually solve Sample Question #3, we'll need to break out our geometry skills. Our notes might look something like this:

A - Hemisphere

$\frac{1}{2} \times V_{Sphere}$

$\frac{1}{2} \times \frac{4}{3}\pi r^3$

$d = 7.5$ cm

$r = 3.25$ cm

$\frac{1}{2} \times \frac{4}{3}\pi(3.25 \text{ cm})^3$

$\frac{1}{2} \times 143.720416667 \text{ cm}^3$

$71.860208334 \text{ cm}^3$

Incorrect

B - Cylinder

$A_{Circle} \times h$

$A_{Circle} = \pi r^2$

$V_{Cylinder} = \pi r^2 \times h$

$r = 4$ cm

$h = 2$ cm

$V_{Cylinder} = \pi(4 \text{ cm})^2 \times 2 \text{ cm} = 100.48 \text{ cm}^3$

Correct

C - Cube

$V_{Cube} = (\text{side length})^3$

Side length = 4.5 cm

$V_{Cube} = (4.5 \text{ cm})^3$

$V_{Cube} = 4.5 \text{ cm} \times 4.5 \text{ cm} \times 4.5 \text{ cm}$

$V_{Cube} = 91.125 \text{ cm}^3$

Incorrect

D - Prism

$V_{Prism} = l \times w \times h$

$l = 7$ cm

$w = 5$ cm

$h = 3$ cm

$V_{Prism} = 7 \text{ cm} \times 5 \text{ cm} \times 3 \text{ cm}$

$V_{Prism} = 105 \text{ cm}^3$

Correct

E - Sphere

$V_{Sphere} = \frac{4}{3}\pi r^3$

$r = 3$ cm

$V_{Sphere} = \frac{4}{3}\pi(3 \text{ cm})^3$

$V_{Sphere} = \frac{4}{3}\pi \times 3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$

$V_{Sphere} = 113.04 \text{ cm}^3$

Correct

In this case, the correct answers are B, D, and E, and we had to analyze each answer choice individually to come to that conclusion. Sometimes, the context of a Multiple-Answer question allows you to come to a certain conclusion halfway through solving the problem that you can then use to sweep aside multiple incorrect answer choices at once. There wasn't an opportunity to do this in Sample Question #3 since each of the answer choices were so independent from one another, but let's see how we can do this in Sample Question #4.

Sample Question #4

Three surveys were given to the same class of students, and the following responses were collected.

	Purple	Green	Red
Favorite Color	17	12	7

	Doughnuts	Pizza	Nachos
Favorite Food	7	8	21

	Action/Adventure	Science Fiction	Comedy
Favorite Movie Genre	12	13	11

If you pick a survey respondent out at random, which of the following categories are you at least 20% likely to choose?

    A. A student whose favorite food is doughnuts

    B. A student whose favorite movies are science fiction films

    C. A student whose favorite food is nachos and whose favorite color is purple

    D. A student whose favorite food is nachos and whose favorite movies are action-adventure films

    E. A student whose favorite food is doughnuts and whose favorite color is green

There are a few ways we can go about solving Sample Question #4; some of them are more efficient than others. It all depends on how we group the answer choices and in which order we approach them. Let's start by analyzing A and B, which each ask about the results of one of the three surveys. Since the surveys were given to the same class, we can add up the total responses for one of them to find the total number of students in the class, a number applicable to both. $17 + 12 + 7 = 36$, as does $7 + 8 + 21$ and $12 + 13 + 11$; it doesn't matter which survey you choose to use to find the total as they all yield the same result. Next, instead of calculating $7 \sqrt{36}$ (the chances of choosing a student whose favorite food is doughnuts) and $13 \sqrt{36}$ (the chances of choosing a student whose favorite movies are science fiction films), you can invert the question and ask "If a response total is above what number will I get a result at or above 20%?" Put a different and more familiar way, 20% of 36 is what? We can format this as an algebraic equation and solve it:

$$0.20 \times 36 = x$$
$$x = 7.2$$

This isn't a whole number like our survey results, but it's still highly informative. The number of

students who answered that doughnuts are their favorite food is 7, and 7 < 7.2, so $\frac{7}{36}$ < 20% and A is incorrect. On the other hand, the number of students who answered that their favorite movies are science fiction films is 13, and 13 > 7, so B is correct. See how approaching the problem a certain way allowed us to save time by reducing calculations? Practice taking an efficient approach like this wherever you can while still maintaining confidence in your work.

We'll need to approach the rest of the answer choices using a different strategy because they involve combinations of survey results. Since these categories are mutually exclusive (e.g. a student liking doughnuts does not influence his or her taste in movies), we can use the multiplication rule to calculate the chances of selecting a student who falls into multiple listed categories.

To find C, the odds of picking one student who likes both purple and nachos, we'll need to multiply $\frac{17}{36}$ by $\frac{21}{36}$. While we could convert these answer choices to decimals and then multiply them together to arrive directly at a decimal answer, we end up getting really long decimals in this instance, so using fractions saves time and potential error in transcribing and manipulating the numbers involved.

$$\frac{17}{36} \times \frac{21}{36} = \frac{357}{1296} = 357 \div 1296 = 0.275462963$$

C is correct! Now for D.

$$\frac{21}{36} \times \frac{12}{36} = \frac{252}{1296} = 252 \div 1296 = 0.194444444$$

D is not correct. There's one more answer choice to analyze: E, the odds of picking out a student who likes both doughnuts and green. But wait—if you take a moment to look at the numbers in the table for these categories, you'll discover that you don't have to calculate anything to realize this answer choice has to be incorrect. The "doughnuts" category had 7 responses, remember, and anything that got fewer than 7.2 responses represents less than 20% of the class opinion. While we need to multiply $\frac{7}{36}$ by another fraction, whenever you multiply a positive fraction by another positive fraction, you get a positive result that is smaller than either factor. So, we know that the result of $\frac{7}{36}$ times any other proper fraction (fraction less than 1) has to be smaller than $\frac{7}{36}$, meaning E is incorrect since $\frac{7}{36}$ < 20% to begin with!

The correct answers to this problem are B and C, but by noting patterns and approaching the problem creatively, we were able to avoid making several unnecessary calculations. Don't try to take too many shortcuts when answering multiple-answer questions, but if you see that mathematical logic clearly prevents an answer choice from being correct, don't waste time calculating it unless you have time to spare!

Numeric Entry

The GRE Quantitative Section's Numeric Entry problems come replete with their own particular set of challenges. It's difficult to miss the main differences between this problem type and others: this one distinguishes itself by not offering any answer choices for your consideration. Instead, it presents a box after the question stem into which you must type the answer you calculate. This creates the opportunity for missteps in rounding, transcription, and in some cases, unit conversion.

When answering Numeric Entry problems, you won't have your calculation somewhat confirmed by its matching a listed answer choice: you have to have confidence in your work. Besides checking your math, make sure that your answer choice is a reasonable response to the question's specific expectations. For example, if you know from the problem's constraints that you need to end up with a negative number and you end up with a positive one, or that you need to end up with a decimal value and end up with a value over 1, you can suspect that something went wrong in your calculations. On a similar note, if you end up with the result that someone reads one million words per minute, you can suspect that your math may be off simply because this is not a realistic human reading speed.

Another effect of having to come up with your answer from scratch is that it's nearly impossible to guess correctly on Numeric Entry problems since you cannot use process-of-elimination to narrow down presented answer choices. Given these constraints, if on test day you find yourself unsure of how to solve a Numeric Entry problem, it might be best to skip it in the hope that you can come back and address it later. This can net you valuable time that you can use to get other questions correct.

On the plus side, there is a definitive numeric answer to each of these problems that doesn't involve complex symbols like π. If a problem requires unit conversion between the data it gives you and the correct answer, you will be explicitly told this and/or the box will likely be labeled with the units the final answer needs.

One thing you may be wondering about is how to present answers with values between 0 and 1. Numeric Entry problems employ different setups for questions that require fractional responses and those that require decimal responses. If you need to submit a fraction as your final answer, a pair of boxes will be provided—one for the numerator, and one for the denominator. A fraction bar will appear between them. If you're only given one box and end up with a fractional answer, you'll need to convert it to a decimal.

Pay particular attention when rounding to make sure you're rounding as instructed by the question stem. If you get the correct answer but round it to the wrong place, it's an incorrect answer, since there's only one correct typed response that receives credit.

Because the Numeric Entry format is so rigid in requiring certain answers to be in particular formats, you can prepare yourself for any format conversions you'll need to perform on test day by

brushing up on how to convert answers between decimals, fractions, improper fractions, and mixed numbers, as well as how to write percentages as decimal multipliers (e.g. 112% as 1.12), how to write probabilities as decimals, fractions, or percentages, and how to write ratios using the colon format and as fractions.

Now that we've got a good handle on how to address certain factors implicit in Numeric Entry problems, let's try walking through a few of them to meet some of these issues head-on.

A prudent first step in solving any Numeric Entry question is to consider what the answer choice needs to look like. Will it have to be a fraction? A decimal? Will it need particular units? In Sample Question #1, we have nothing to indicate that our answer choice will be anything other than a simple whole number— perhaps a decimal. But wait! What are we counting? "Additional bars." You can't purchase half of a candy bar at a store, so those are whole bars. This subtly tells us that we're going to need to round our answer to the nearest whole number. Make a note of that—it could make the difference between getting this problem correct or missing it on the last step!

> ### Sample Question #1
>
> A group of ten candy bars has an average cost of $0.89 per candy bar. How many additional bars must be bought at the cost of $0.72 a piece to bring the average down to $0.80?

To calculate the number of additional bars that would need to be purchased at the new price in order to bring the average cost down to $0.80, we'll need to set up an algebraic equation modeling the situation. Before we even consider the additional bars, let's figure out how much was paid for the initial bars.

$$\frac{x}{10} = \$0.89$$
$$x = \$0.89 \times 10$$
$$x = \$8.90$$

The buyer paid $8.90 for the first ten bars: that's useful information. We can use that to model the "additional bars" situation, using y to represent the number of additional bars needed to get an average cost per bar of $0.80. When calculating an average, you take the sum of the elements as the numerator of a fraction and the number of elements in the set as the denominator of the fraction. We can recast the problem in these terms mathematically: the buyer paid $0.89 for each of the first ten bars, a total of $8.90. This person also then paid $0.72 per bar for an unknown number of additional bars, so that's 0.72y. The sum of those goes in the numerator of a fraction. The total number of bars purchased will be the denominator; that will be the 10 initial bars plus y, however many bars are purchased at $0.72 apiece. All of this needs to equal $0.80, the average cost. Thus, our equation is:

$$\frac{\$8.90 + \$0.72y}{10 + y} = \$0.80$$

We've constructed an equation with a single variable in it, so all we need to do at this point is use algebra to solve for that variable.

$$\$8.90 + \$0.72y = \$0.80(10 + y)$$
$$\$8.90 + \$0.72y = \$8.00 + \$0.80y$$
$$\$8.90 - \$8.00 = \$0.8y - \$0.72y$$
$$\$0.90 = \$0.08y$$
$$y = \frac{\$0.90}{\$0.08} = 11.25 \text{ candy bars}$$

That's a reasonable answer, considering that the buyer purchased ten bars to begin with. But wait: we have to round, remember? Our answer has to be the number of whole bars purchased. If we were rounding out of the problem's context, we'd round 11.25 down to 11, but in this case, we need that quarter of a bar to get the average cost to be $0.80—any less, and we won't get that result. So, the buyer actually needs to purchase 12 bars to get 11.25 bars. The correct answer is 12.

Sample Question #2 asks us to find a percentage, but more than that, it asks us to round to the nearest thousandth. That means that the thousandth place will need to be the last decimal place in our final answer.

To find the percentage of blood in the solution, we need to place the amount of blood over the total amount of solution. Be careful—that's not just the amount of water in the solution, it's the amount of water and blood in it! We can construct our answer a a fraction:

That's a long decimal, but don't round it yet! We need to calculate our answer as a percent, not a decimal. We need to multiply it by 100 and add the percent symbol.

0.000166302 x 100 = 0.0166302%

That's still a large number, but now it's time to round it. First, identify the thousandth's place. (If you forget which place that is, remember that you can divide 1 by 1000 to get a number in which 1 is in the thousandth's place). A 6 is in the thousandth's place, and it is followed to the right by

> ### Sample Question #2
>
> What percentage of a solution is blood if it contains 1.4 ml blood and 8417 ml water? Round to the nearest thousandth.
>
> ☐ %

another 6. 6 > 5, so we round the six in the thousandth's place up to 7. Remember, it doesn't matter what the number in the thousandth's place is when deciding in which direction to round it—it's the number to its immediate right that determines that! Our final answer is 0.017%. Keep in mind that since this question includes the percent symbol as a preset feature of the text box, you don't need to type it. In fact, if you type it, you'll get the question wrong! Pay particular attention to these details as you transcribe your answer.

$$0.0166302\% \rightarrow 0.017\%$$

To answer Sample Question #3, we're going to need to end up with a fraction. For some questions, this will dictate the particular way in which we have to solve it. For other questions, we'll be able to use any of multiple ways to solve it and then simply convert our answer to the required format at the end. While many answer formats are easily interchangeable, e.g. decimals and fractions, sometimes questions may demand an answer with a specific format (e.g. a multiplier of π) that necessarily affects how it must be calculated. It's not a good discovery to find out that you just spent three minutes calculating a decimal when what you needed was a fraction with a specific denominator. Before you start calculating, make sure you know which category your problem falls into, and sketch out a game plan for arriving not only at the correct answer, but the correct answer in the correct format.

> **Sample Question #3**
>
> A solution is made up of 13 parts water, 2 parts orange juice, and 4 parts apple juice. If you wanted the percentage of orange juice to be 25% of the solution, how many parts would you need to add?
>
> $$\frac{\boxed{}}{\boxed{}}$$

To begin, you know that the basic form of the solution has a total of $13 + 2 + 4 = 19$ parts. Now, we know that we are going to have to add an unknown number of parts of orange juice—let's call that x. This means that the new solution will have $2 + x$ parts orange juice and $19 + x$ total parts (since we are adding to the original). Since we want this to be 25% orange juice, we need the following equation to be true:

$$\frac{\text{Parts Orange Juice}}{\text{Total Parts in Solution}} = \frac{2 + x}{19 + x} = 0.25$$

Now that we've got our equation, we need to solve for x. Start by multiplying by $(19 + x)$:

$$\frac{2 + x}{19 + x} = 0.25$$
$$2 + x = 0.25(19 + x)$$

Now we can distribute the 0.25:

$$2 + x = 4.75 + 0.25x$$

We can subtract 0.25x from both sides of the equation to isolate the variable on one side:

$$2 + 0.75x = 4.75$$

At this point, we could divide 2.75 by 0.75, get 3.667, and call the problem done, but remember, we need our answer choice to be a fraction, and an improper fraction at that. Therefore, let's convert 0.75 and 2.75 into fractions. 0.75 converts easily in lowest terms to $\frac{3}{4}$; if we keep a common denominator of 4 for 2.75, what do we get for the numerator? We can set up a quick proportion and cross-multiply to find out:

$$\frac{2.75}{1} = \frac{x}{4}$$
$$x = 2.75 \times 4 = 11$$

2.75 as a fraction with a denominator of 4 is $\frac{11}{4}$. We can now rewrite our equation in fractional terms:

$$\frac{3}{4}x = \frac{11}{4}$$

Now we can solve our problem and get a fractional answer by multiplying $\frac{11}{4}$ by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$:

$$\frac{4}{3}\frac{3}{4}x = \frac{11}{4}\frac{4}{3}$$
$$x = \frac{11}{3}$$

$\frac{11}{3}$ parts orange juice will need to be added for the final solution to be made up of 25% orange juice. To submit this answer, you would type "11" in the top box and "3" in the bottom box, and you would get this question correct!

Data Interpretation

On every GRE, you're sure to encounter a few Data Interpretation questions. These are often grouped together in about three consecutive questions about the same graphs and charts, but you can also be asked independent questions about charts and graphs only provided for and relevant to a single question. No matter whether you're facing a few questions or just one about a graph, chart, or combination of graphics, you can use some of the same strategies to avoid becoming overwhelmed with visual input and wasting time needlessly stressing over these questions. We outline some of these strategies in this lesson.

Keep in mind that Data Interpretation is not itself a question format, but a way of grouping several questions based on one stimulus. The questions themselves in a Data Interpretation trio may be Numeric Entry, Single-Answer Multiple Choice, or Multiple-Answer Multiple Choice, so remember to make use of the strategies we outline for these specific question formats along with the following recommendations.

Consider What The Graphic Can and Can't Tell You

The figure(s) that accompany Data Interpretation questions may be accompanied by a short description and/or a title that provides you with a general context of what's being presented. Initially, take a moment to consider what a graphic of the type facing you can and can't tell you. What is it used to measure? For instance a single pie chart isn't used to show change over time, whereas a line graph isn't often used to show how much support multiple survey options received. Get in the proper mindset for approaching whichever type of graph you're faced with to avoid confusion later and get a jump-start on identifying whichever data it provides are most relevant to the problem at hand.

Read the Graphic Carefully

To approach sets of Data Interpretation questions, begin by briefly scanning the figure to gain a basic understanding of the data. Key points include axes of graphs, legends and labels, magnitude/scale of numbers, and units. Recall that all graphical representations of data are drawn to scale, so use this to your advantage. If the figure involves a topic about which you have prior knowledge (other than common knowledge), be careful not to let this outside knowledge obscure your understanding of the figure, which contains all the information you need to answer the associated questions correctly. Take your time in analyzing the information a graphic presents, especially if multiple graphics are involved in a given prompt.

Jot Down Important Numbers to Avoid Mistakes from Repetitive Reading

One of the most common ways to introduce error when answering Data Interpretation questions is to force yourself to repetitively read the graph for the same data point. Instead of wasting your time and potentially introducing errors accrued through reading complex charts multiple times, slowly and carefully determine a relevant data point on the first read, and then write it down, clearly indicating the exact thing that number stands for. This way, you can refer to the number as a number instead of returning to the graph and performing the action of reading it more times than necessary, which is easy for anyone to mess up, especially in a testing scenario.

Consider What You Know vs. What You Need to Know and Make a Game Plan

At their core, Data Analysis function just like many other Quantitative Reasoning questions. You're given a stimulus—in this case, information presented in the graphs—and asked either to report that information directly or to use it to calculate the answer. The former case represents easier questions that just test your ability to read the graphic at hand accurately. The latter case can easily result in testing overwhelm if you're not sure how to calculate the answer choice and focused only on the question and not the graphs you're given. If you find yourself unsure of how to approach a given Data Interpretation question that requires calculation, start by listing out all of the information you know from the graph. Then, write out exactly what you need to calculate. At this point, you need to use a combination of working forwards from what you know and working backwards from what you need to end up with to figure out how you're going to get from point A to point B. The advantage of taking such notes is that it allows you to focus on just numbers and data without having to stare at the graph in confusion, which can easily just compound any test stress you're already feeling.

Let's now take a look at three sample questions about one graphic premise. This particular one involves two pie charts that are equated in a specific way to show similar statistics for two different industries. Remember, take a moment to familiarize yourself with the charts and figure out what data is being presented before starting to tackle the questions!

The following pie charts apply to Sample Questions #1–3.

Percentage of Time Spent in a Single Role for Employees in Two Industries, 2008

Financial Industry
(12 million)

Construction Industry
(8 million)

5–9 years

18%

50%
10+ years

20%

12%

2–4 years

0–1 years

25%

40%
10+ years

20%

15%
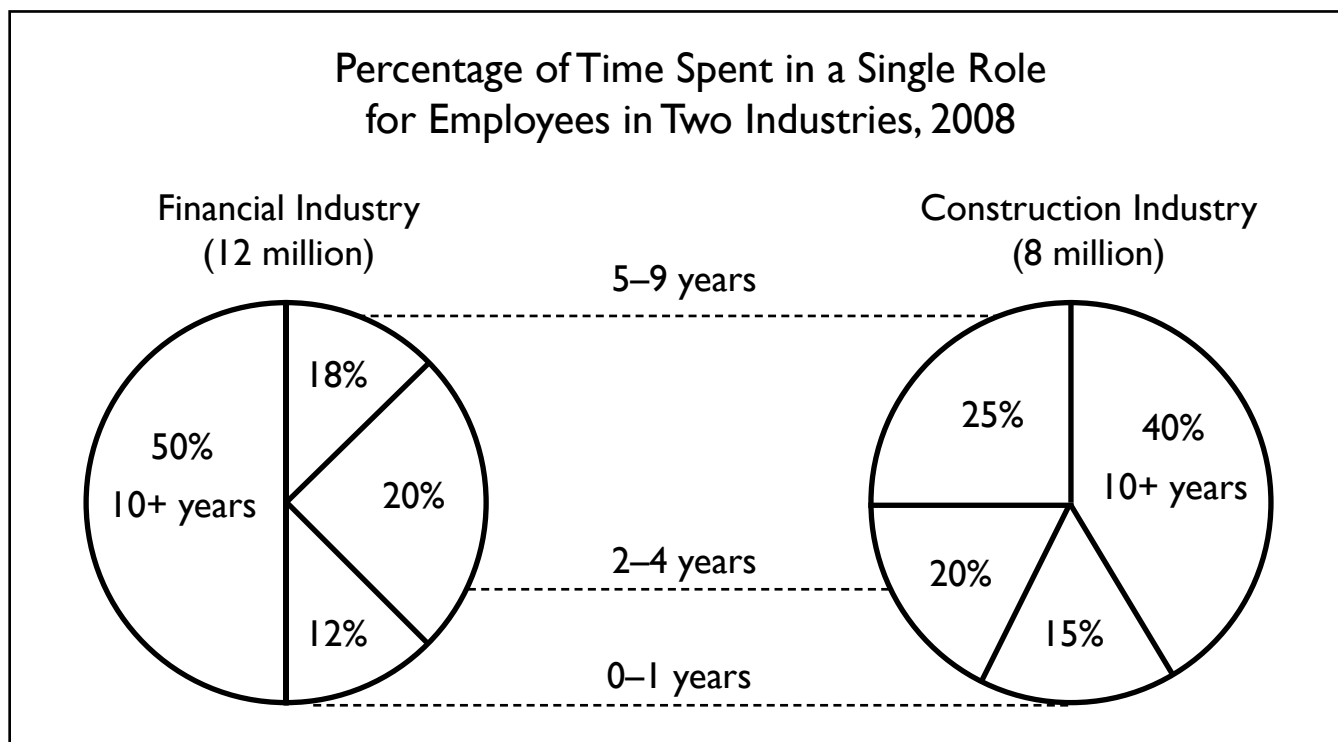
Since the values in the pie charts are presented as percentages (numerators over 100), we must convert them to degrees (numerators over 360°, since within a circle) to solve the question. The "5 to 9 years" portion for the financial industry is 18%, since that is where the "5–9 years" dotted line is pointing; for construction industries, it's 25 percent. We can cross-multiply these values to figure out what percent of their circle graphs each represents:

$$\frac{18°}{100} = \frac{x°}{360}$$

$$100x° = 18° \times 360$$

$$100x° = 6480$$

$$x° = \frac{6480}{100} = 64.8°$$

$$\frac{25°}{100} = \frac{y°}{360}$$

$$100x° = 25° \times 360$$

$$100x° = 9000$$

$$x° = \frac{9000}{100} = 90°$$

Sample Question #1

According the pie chart, the degree measure of the sector representing the number of workers spending 5 to 9 years in the same role is how much greater in the construction industry chart than in the financial industry chart?

A. 7°
B. 18°
C. 22°
D. 25°

Subtract: $90° - 65° = 25°$, so the answer is D. Alternatively, we could first subtract the percentages (25% − 18% = 7%), then convert the 7% to degrees via the same method of cross-multiplication.

The first step is to figure out the percentage of construction employees that have stayed in the same role for 5 years or more—this includes both the "5 to 9 year" and "10+ years" ranges. This is $0.25 + 0.4 = 65\%$ of all construction employees. To convert to the number of employees, we take the percentage of their total, $0.65 \times 8{,}000{,}000 = 5{,}200{,}000$ workers. Since the probability we are attempting to find is of workers between both industries, we must add the 8 million to the 12 million = 20 million workers total. $\frac{5{,}200{,}000}{20{,}000{,}000} = 0.26$, or a 26% chance.

Sample Question #2

If one of the employees across both industries were to be selected at random, what is the probability that the employee will be a construction industry worker who stayed in the same role for 5 years or more?

A. 10%

B. 17%

C. 20%

D. 26%

Sample Question #3

The ratio of the number of financial employees who remained in the same role for 2 to 9 years to the number of construction employees who remained in the same role for 0 to 4 years is closest to which of the following?

A. $\dfrac{8}{5}$

B. $\dfrac{3}{2}$

C. $\dfrac{5}{3}$

D. $\dfrac{12}{7}$

For this problem, we need to find the number of employees who fall into the categories described, keeping in mind that multiple portions of the pie chart must be accommodated for. Then, we can fit them into a ratio. For the "2 to 9 years" portion of the financial industry, include

$$(0.2 + 0.18)(12{,}000{,}000 \text{ workers}) = 4{,}560{,}000 \text{ workers}$$

For the "0 to 4 years" portion of the construction industry, include

$$(0.15 + 0.2)(8{,}000{,}000 \text{ workers}) = 2{,}800{,}000 \text{ workers}$$

Now divide and simplify to find the ratio:

$$\frac{4{,}560{,}000}{2{,}800{,}000} = \frac{8}{5}$$

Analytical Writing

Many standardized tests involve a written essay response; the GRE involves two. It may be easy to assume that both will somehow align with writing tasks you've encountered on previous exams, but this is only partially true, if at all. In the Analyze an Issue task, you're provided with a statement and asked to argue cogently for or against it. The prose you produce should be persuasive, and you're expected to use the general knowledge you bring to the exam to supplement your argument with evidence and examples. In contrast, the Analyze an Argument task provides you with a short written stimulus. It's your job to take it apart, pointing out all of the logical flaws and missteps that are present.

You can prepare for these similar but distinct tasks by practicing skills applicable to both of them and each one individually. We've structured our Analytical Writing section with this two-pronged approach in mind. The first section, "Components of Strong Writing," reviews various characteristics of cogent and clear prose including thesis statements, evidence, and thoughtful organization and style. You'll need to demonstrate your mastery of these general requirements on both your Issue and your Argument essays to achieve good scores.

Each writing task also presents its own unique challenges for which you can review specifically; with this in mind, we spend a section looking over each one's idiosyncrasies. In our "Issue Analysis" section, we focus on how to decide whether to argue for or against the prompt's statement and select appropriate evidence from your knowledge and experience in response to it. We also consider how to embed counterarguments within your essay to address any potential holes in your argument while also strengthening your essay as a whole. In our "Argument Analysis" section, we focus on the logical features and problems that will form the nuts and bolts of your response. We'll go over how to identify the argument being made in the sample passage as well as how to analyze the effectiveness and validity of its evidence, inferences, and critiques. With each of our task-focused sections, we've include a full sample response to a mock prompt so that you can see how our recommended essay features and strategies can come together in whole-essay form.

It can be easy to bypass reviewing Analytical Writing section topics because so much of the section rests on you and the writing that you produce on test day. There are no formula or vocabulary terms to memorize—the section is measuring your writing skills, which rely on more abstract features of your prose. After reviewing this section, you should understand exactly what each of the GRE's writing tasks asks you to do, and you'll have practiced many of the foundational techniques that can serve as stepping-stones to great work on test day.

Components of Strong Writing

No matter the particular task you're given or the particular prompt you need to respond to, the GRE's Analytical Writing section scores part of your essay based on how well it adheres to the principles of cogent writing—the topics we cover in the following section. While it's important to familiarize yourself with what differentiates the "Analyze an Issue" task from the "Analyze an Argument" task, the essays you compose will need to state a point clearly and then provide evidence to back up your claim if you hope to achieve good Analytical Writing scores. We begin this section by considering how to come up with a purposeful thesis statement. Next, we consider how to expand upon your initial claim by providing relevant and strong supporting evidence. What you say is important, as is how you say it, so in our final two lessons of the section, we take the time to consider how to organize your essay for optimum clarity and how to make the most of word choice and sentence structure to lend interest without distracting from the task at hand. Whatever that task is, you can apply the principles we review in this section to craft essays well-grounded in the fundamentals of effective persuasive prose.

Section Outline

Establishing Main Idea, Thesis, and Purpose

Development of a Position

Organization, Structure, and Focus

Vocabulary and Syntax

Establishing Main Idea, Thesis, and Purpose

The easiest way to prepare for the analytical writing section of the GRE is to remember that it replaced an older *logic* section of the exam. Although the writing section is not, strictly speaking, a logic exam, it *does* test your ability to express yourself in a coherent, logical manner. Students often benefit from remembering a helpful image: your writing is like an outlined argument covered with the "flesh" of words that express that argument in a persuasive manner to your reader. Therefore, you should think of two major architectural points in your essay: the basic "skeleton" of your logic and the "flesh" that includes the rhetorical aspects that encourage your reader to continue through the brief passage that you are writing. In this lesson, we are going to consider the basic skeleton that you create with your thesis statement (though this skeleton *does* have important implications for the rhetorical efficacy of your passage as well).

Students have varied reactions to the analytical writing section. This section requires you to take a position on a random topic. Some students have more experience than others formulating an opinion "on the fly." Therefore, you should begin your preparation by making sure that you can generate an opinion on a topic with relative celerity. Using some of the many resources available online and in publication, you can find a variety of sample topics for writing.

These prompts should be thought of as being "pro / con" questions. You need to take a position either for or against what is being presented by the text. Choose a number of these sample prompts and ask yourself, "Which side seems more arguable?" Sometimes, it is obvious that you must agree with one side of a given prompt. Consider the following examples:

- Given the dangers to the environment, every septic system should be removed from properties in the United States. (You clearly cannot agree with this, given that there are many, many rural areas in the US, not having sewage systems available. The word "every" makes it too strong of a claim.)

- An effective way to improve one's vocabulary is through extensive reading and writing. (This is so simple that you just can't help agreeing with it!)

However, in other cases, you might be faced with a prompt that is more ambiguous. Indeed, this will likely happen more frequently than not. In such cases, the question will become uniquely personal. You will need to ask yourself, "Which can *I* argue well?" Notice the emphasis on "*I*." You will need to generate a main idea and supporting examples quickly. It will be easier for you to do so if you have some personal reason or experience with the given answer that you have provided. Your own knowledge of the subject will make your claim more easily arguable and realistic. This will prevent you from defending a wild claim! Consider the following example prompt:

- Because of its numerous benefits, music education should be required of all students who are capable of learning to sing or to play an instrument.

There are numerous perspectives from which this could be answered. Merely consider the following three:

- The perspective of someone who has extensive experience teaching students to play music:

Presuming that these experiences have been positive, such a person could write about the benefits that are accrued even by those who do not have greatly developed skills. Of course, examples would depend upon the writer's own experience.

- The perspective of an artist of another genre: This sort of writer might disagree, insisting that there are benefits in other arts and skills. Based on his or her particular knowledge and experience, he or she would then provide examples of how those skills are also enriching.

- A philosophy major who has read Aristotle's remarks on music in the curriculum of ancient Greece: This odd sort of writer may have little to no experience in musical education in contemporary society; however, he could use his or her knowledge of Aristotle's discussion of music as a vehicle for teaching leisurely and contemplative activity as a way for justifying such musical education. On the other hand—*and this is important*—he or she may also argue that music is an aristocratic kind of knowledge, really only to be learned by those who are capable of leisurely living. *Notice how one's knowledge can enable one to take both sides of an issue. Do what feels most natural and defensible—though always make sure that you are not just expressing a simplistic bias.*

Once you have a general idea of the argument that you plan to make, it is time to write your thesis statement. This should be a direct and obvious statement of your opinion on the matter, clearly expressing this to your reader. Remember, clarity and directness are important. It should not be a mere direct "yes" or "no" to the prompt. Therefore, you should not write something like, "Music should indeed be required of all students." That is rather uninformative. Even if you agree with the prompt, you need to craft your thesis statement such that it expresses your own unique reason for supporting the assertion expressed in the prompt. Thus, given the final prompt considered above, you could write, "Music education should be broadly available and required as a vehicle for teaching students discipline and, more importantly, the importance of non-utilitarian goods."

Okay, at this point you have a basic statement of where you are going to go with your essay. Although every student has a unique writing style, it is helpful to come back to our opening analogy, as it will provide you with a basic plan for approaching the task at hand. Recall: think of your essay as the fleshed-out skeleton. The basic skeleton will have the following form:

- Opening paragraph
- Main point 1
- Main point 2
- Main point 3
- Closing paragraph

Forming your thesis statement will require you to think through your reasons for defending a position. Continuing with the thesis statement written above, we can generate the following two argument points pretty quickly: (1) Music teaches discipline; (2) Music teaches students to present non-utilitarian goods. Often, the progression in your essay should include some small concession to the other side of the question. You can make such a concession in the paragraph containing your third main point. You shouldn't overturn your main assertion, of course. Therefore, carefully craft the third main point such that you can close by saying, "But nonetheless, the overall point remains clear…

(i.e. that the main assertion of this essay is correct)."

So, from this general consideration of your thesis statement, you will be able to generate the complete skeleton of your essay! It can be arranged so that the argument flows very readily from one point to the next:

- Opening paragraph: Introduce the basic question and state your thesis / overall argument.

- Point 1: Music teaches discipline to students, and this is an important aspect of education. Then, defend with concrete examples (a point to be discussed in detail in other lessons). You can transition to the next point by hinting that even though music is useful in this manner, there are even more important non-utilitarian reasons for learning music.

- Point 2: Build on the closing remark in the previous paragraph and introduce the topic of teaching non-utilitarian goods. You can explain how education always involves teaching values to students. It is important to teach that there are some things and activities that are just good for themselves. Music helps to accomplish this.

- Point 3: Here, you can build in a concession. You can note that there are other activities that fit these criteria. However, you may even here buttress your claim by noting the social benefits of playing music in a group and how this is positive in a way that does not hold for some activities. (Your examples will be unique to your own experiences. Remember, you do not need a "full proof" and magisterial argument—only a *logical* and generally cogent argument.)

- Conclusion: Round out your considerations and restate your position in light of the discussion above.

Now, you will need to work on the details of writing body paragraphs in other lessons. It is important for you get a general idea of all that you can generate from your thesis statement. You will know that you have an acceptable statement when you are able to craft an outline argument like this, providing you with a strong "skeletal framework" on which you will be able to hang the further rhetoric of your passage.

Development of a Position

In the analytical writing section of the GRE, you will be required to develop a coherent argument that progresses through several main points in a thoughtful manner. The key here is to develop your thesis without simply repeating it. Instead, you should think of your thesis as a seed that contains in its initial potentiality all that will develop from it. It is a starting point that will give rise to the various points that you will defend in the course of your argument. A seed does not develop by becoming a great deal of other seeds, it grows into a plant, and that plant grows leaves and fruit and branches. The plant can always be traced back to the seed, but the plant looks and feels much different than the seed and is inherently more complex because it has grown. The same should be true of your essay; it should be unrecognizably more complex, beautiful, and developed than your reader could have imagined having just seen the wee seedling of your initial thesis!

The two types of analytical writing questions in the GRE *both* require you to write with a sturdy overall developmental structure in place. Of course, the nature of your content will be a bit different in each case. In the issue essay, your main points will contain arguments that support your position, each building on the previous one to further your argument toward its conclusion. In the argument-analysis essay, your main points will indicate the various ways in which you are critiquing the given argument. As a general rule, you should try to have three main points that support your thesis, each one expressing a unique aspect of the issue you are discussing or the argument that you are critiquing.

In both of these essay types, the use of evidence, references, and examples will be very important and will, indeed, be the primary means of expanding and developing your position. You should constantly remind yourself while gathering your thoughts: "Be specific!" This helps you to express yourself with complete clarity, ensuring that the person scoring your essay will see and understand its overall argumentative structure and development. By having specific, thoughtful pieces of evidence, you will not only be supporting your argument, but also expanding and complicating the scope of your argument. Good evidence does not simply support an idea, it raises ideas of its own!

Let's take an issue essay as an example first. Consider a prompt that states, "Democracy is the best form of government and should be instituted everywhere in the world." This prompt is luckily rather simplistic, so we can formulate an essay that disagrees with it. From the very beginning, you will need to formulate clear and unique points that support your position. Thus, you may write a thesis such as, "While democracy has many beneficial aspects, it is not appropriate for all types of people depending upon their unique social and historical circumstances." Now, this general thesis helps you to generate two general main points that you should discuss: (1) How democracy is not feasible because of certain social circumstances; (2) how democracy is not feasible due to certain historical circumstances. Likely, you can write a third body paragraph that acknowledges some of the benefits of democracy, while simultaneously noting that these can be integrated into other forms of governance as well.

It is helpful to consider a contrast between two manners of approaching these points. Let's take as our example the first proposed point above. It would be *very* ineffective for you to make some

general claims such as, "Many societies are not ready for democratic processes. Indeed, before societies become ready for democracy, they need the correct amount of social development to provide structures of interaction and public deliberation. These kinds of deliberation take time and require societal development, something often lacking in many nations that are not currently democratic."

We must be quite clear on this point: such a paragraph is useless. It is one long string of repeated points. Although it states many additional words about the topic, it does not actually express much in the way of new content or thoughtful discourse. When you are adding words to your essay, you need to make sure you are adding content. This is to say that any new point should be unique or add a layer of complexity to a previously stated point. It is never appropriate, or useful, to simply repeat or assert a point without support in your Analytical Writing Essay.

Examples and references help to structure the new content that you should have in your body paragraphs. Of course, such examples should not be merely anecdotal statements. It is not very helpful for you to say, "My hometown, for example, is full of many people who have no clue about public life and what needs to be done in the city. Such people really should nt be given democratic rights." Instead, you should use more broadly applicable examples from history and contemporary life. You do not need to give all the details of a full reference. You are not writing a research paper, but you *do* need to specify some context for your reference. Thus, instead of giving the anecdote noted above, you could say, "Democracy requires that voters have some awareness of the projects being undertaken by their community. While this may be possible on a small scale in most locales, it is not possible for a nation that has not reached a sufficient level of economic and educational development. For example, poorer nations with economies in need of development require time and expansion to guarantee the educational and technological resources needed for truly democratic communication."

Of course, this is still a general statement. It does not cite a particular economy or nation, but this can be fine in some circumstances. The example is clear enough for your reader, so it helps you to develop your argument. After giving this example, perhaps you can cite something that you know from a smaller scale (e.g. your own local community or state) without making it sound anecdotal. This would help to show just what you think regarding the limitations that must be placed on the hopes of making a given nation or group of people democratic, taking into account social conditions. The important rule: be specific but not anecdotal. This basic rule will provide must, if not all, of the guidance needed for your writing.

In the argument essay, you will need to craft your examples to match your critiques of the passage. This is a little different from the issue essay, which requires you to defend one side of an argument. Consider a passage that asserts the benefits of being vegetarian, arguing that a certain kind of bean is a great source of protein. Even if you personally think that everyone should be a vegetarian, it will still be necessary for you to show why this particular argument is weak.

For this example, it will be helpful for you to draw on some basic notions that you hopefully recall from a biology class. You do not need to know and recount all of the details of bean protein content and the human metabolism of those proteins. You can, however, note that the author is assuming

that these beans contain all of the proteins needed for human life. By observing that there are various kinds of amino acids needed for humans to produce the proteins used in day-to-day life, you can state that the given argument needs to complicate or expand its treatment of the nature of the proteins found in the one kind of bean.

Likewise, you can note the fact that the author is only writing about one kind of bean; in other words, you can note a deficiency in the scope of the given argument. By noting that nothing is said about the rarity of the bean, the environment in which it is grown, and other such facts, you can call into question the validity of the overall argument. Depending on these details, the argument might be very weak, for perhaps the beans are not widely available and hence would be very expensive. Even if you cannot come up with a real-world example of a given fruit or vegetable, it is acceptable (and quite helpful) for you to *create* evidence that is *based* on reality. For example, you could note that the author has said nothing about the current growth of the bean in question. Even if it provides adequate protein nutrition for humans, perhaps it is only grown in some remote areas of the world. You could contrast this to the growth of coffee beans, which have a large industry developed around them. This industry developed only over the course of many centuries. While modern technology might make this expansion quicker today, it still takes time to develop a full industry in all of its aspects—growing, packing, shipping, etc. By using this latter example of the coffee industry, you can help to ground your semi-fabricated example on reality, thus lending it validity.

The point here is that general knowledge is helpful, but never strictly required for your analytic writing essay. There will always be a way for you to critique the logic of a given passage, and a way for you to generate evidence and examples without specific knowledge of the field of study discussed. If you happen to have this specific knowledge, so much the better for you!

In both kinds of essays, it is important to use the kinds of specific but globally applicable examples we've outlined to develop your argument. In addition to keeping your argument clear and concrete, examples can help in crafting the transitions between the main structural points of your essay. This will guarantee that your argument reads like a developing narrative instead of a brute repetition of the same assertion multiple times. With this kind of specificity and development, your essay will have increased clarity and strength—both very good things when trying to communicate a logical and persuasive argument! The seed that is sown in your thesis statement should ultimately blossom forth in a fully formed, coherent narrative!

Organization, Structure, and Focus

In this lesson, we will consider the basic structural elements that are needed for crafting a persuasive and cogently argued essay. Every student has a different style of writing. While you should not shackle yourself to a particular methodology, it is important that you teach yourself some basic organizational skills for the GRE essays. Since these essays are not a venue for showing all of your linguistic and authorial skills, it is better to err on the side of a conservative style as opposed to your own unique voice. Therefore, even if you have very strong writing skills, you should try to retain a general overall structure akin to what we will discuss in this lesson.

In either kind of GRE essay, you are basically arguing a point. In the "Analyze an Issue" essay, you are arguing on behalf of a particular position. In the "Analyze an Argument" essay, you are arguing that the proposed argument is actually weak in some respects; therefore, you can structure these essays in very similar ways, even though their content will be significantly different, given the difference in tasks presented to you in each.

We should begin with a rule of thumb—one that should be repeated to yourself over and over: "You are *fleshing out* a *logical skeleton*." In the human body, the skeleton prevents you from collapsing into a blob on the ground. So too, in your essays, the logical steps of your argument need to be the hard skeleton to which all of your rhetoric attaches. You are primarily presenting an argument, and that argument is like the hard skeleton of your essay. Make sure that your reader can see that you have crafted this skeleton.

The basic pieces of your argument are twofold: the statement of the thesis and your defense of that thesis. In general, you should plan to defend your thesis in a given essay by marshaling three particular main points. Perhaps you will get stuck when you are preparing your essay, thus limiting you to two main points; however, the optimal situation will find you writing three main points in your essay. Aim for this. In addition to these two main components, you should "round out" your discussions with a concluding paragraph that helps to summarize your argument.

When you are brainstorming at the beginning of your essay writing, you will list out the main points of your argument. In your opening paragraph, you should make these main points very clear. In the introduction, you need to introduce the problem being discussed. Do this in a brief manner, as this initial discussion should clearly lead to the explicit statement of your thesis. Once you state your thesis, it can be best to spell out the basic reasons that you will adduce in defense of your thesis. This means that your introductory paragraph will have the following basic format:

- Introduction to the problem
- Thesis—explicitly stated
- Statement of argument 1
- Statement of argument 2
- Statement of argument 3

Alternatively, you might rearrange the pieces of this introduction a bit to lead up to the thesis with your argument statements:

- Introduction to the problem
- Statement of argument 1
- Statement of argument 2
- Statement of argument 3
- Thesis—explicitly stated

These pieces of the argument have been spelled out for you here in order to stress the importance of being explicit regarding your argument. While you should avoid writing in a very tedious and dull way, it is better to be *explicit* than to be *unclear* regarding your argument. If your first paragraph clearly states the parts of your essay, your reader will clearly see the skeletal structure of your argument. This clarity will show the reader that you have indeed put together a coherent argument. He or she will not need to guess at your reasoning if it is stated with clarity.

Your three argument paragraphs should have obvious connections from one to the next. Consider an issue essay that states, "College students should be taught logic early in their studies." To this prompt, you may put together the following set of arguments to support the claim:

- An understanding of logic is necessary for validity and cogency in all human thought.
- It can be taught in flexible manners depending on the curriculum in question—i.e. in writing classes, philosophy courses, and even in discrete mathematics.
- Granted, the *detailed* philosophical discussion of logic is not needed for every college student.

Notice how this essay builds on each point. It opens by making a universal claim in support of the thesis on behalf of logic education early in the curriculum. When writing this paragraph, you can then transition into the second point by noting how this universal applicability makes it easy to teach the basic points of logic in several different courses, depending on a particular student's major. Notice an important word in the previous sentence—"basic." So long as you emphasize this point in the second paragraph, you then can transition into the third paragraph, which notes that more advanced topics do not need to be taught to all students.

Notice how the last paragraph is a kind of concession. It helps to explain in what sense logic should *not* be taught early in the college curriculum. If you are careful in the way that you craft your argument, this kind of planned concession can add a great deal of weight to your overall argument. It helps you to focus on the exact sense of your claim. In this particular example, it helps you to focus on the fact that *basic* logic is what is needed for early college studies. By means of careful connections between each main idea, you can help to ensure that this point is present in your thesis as well as in your first and second paragraphs; therefore, when you come to this qualification in the third paragraph, you will actually be strengthening your argument instead of undermining it!

Such final "qualifications" help to show the importance of a concluding paragraph after the main arguments are presented. The conclusion can be thought of as saying either, "But nevertheless, we can see why my main argument holds in the face of this qualification," or (where such a qualification is not present), "And therefore we can see why my thesis is cogent." Of course, you should avoid writing with such obvious self-assurance.

Remember: the most important thing is the *skeleton*. You must show your reader that you have a strong logical argument in your essay. Although it is not a clean-cut formal syllogism, your essay *is* making a logical claim on behalf of your argument. In your opening paragraph, be very clear on what you are going to argue (i.e. your thesis) and how you are going to argue it (i.e. the content of the body paragraphs of your essay). In each of your body paragraphs, connect your points closely together so that by the conclusion of the text your argument is clearly articulated and obvious to your reader. Make the *skeleton* very visible—otherwise, the actual "flesh" of your essay will be nothing other than a jumble of disconnected thoughts that support no claim whatsoever!

Vocabulary and Syntax

The essays for the GRE's Analytical Writing section are primarily concerned with content—indeed, with coherent and focused content. Above all, this means that the most important aspect of your essay is its ability to communicate an argument cogently and clearly. This must be kept in mind as you write your Issue and Argument responses. Syntax and vocabulary are important aspects of your writing, but remember: you are tasked with writing a clear and focused essay; therefore, your deployment of the English language does not need to show that you are the greatest stylist since Shakespeare or the most knowledgeable lexicographer since Samuel Johnson. Instead, your use of the English language must make your ideas shimmer, allowing your content to shine forth with a clear structure and a convincing narrative. Of course, this is all a bit pretentious in its tone! When you write your essays, you should try to avoid such verbal exploits.

Vocabulary

It is gratifying to use an extensive vocabulary, but that does not mean that such use is also stylistically clear. With your vocabulary, you should try to choose words that fit the situation well. For example, while metaphors can be deployed, you should be very contained in your usage of them. Indeed, even the use of "shimmer" at the end of the last paragraph may not be appropriate for a GRE essay. While its meaning is clear, it still has a pretentious ring to it. Thus, it would be better to write, "Instead, your use of the English language should express structural clarity and rhetorical force." Notice how direct that vocabulary is: "should express structural clarity and rhetorical force." This direct style expresses just what is needed, paring down the distractions caused by extra verbiage.

Also, be careful to use vocabulary that you know. If you are in doubt with regard to the meaning of a word, use a synonym or a synonymous expression. While you do have a little room for honest mistakes, the misuse of vocabulary generally will be counted against your score. Be sure to study up on the uses of basic contractions and the meanings of frequently misused words (such as "affected" and "effected" and so forth). To this purpose, we follow this lesson with a guide sheet pointing out some of the most commonly confused English words.

On the whole, it is important for you to show that you have a strong vocabulary; however, use words without drawing attention to their use. Read the following two sentences out loud:

> The adequacy of his scholastic verbiage was interrogated by the supervisory directorate holding the perpetual sovereignty concerning pedagogical inquiries.

> The supervisory board, which had sole authority over pedagogical matters, questioned the professor's use of arcane terminology.

Both of these sentences utilize vocabulary well; however, the first sentence should stop you on many occasions, hopefully exasperating you. Its obvious verbal haughtiness is matched only by its ability to obfuscate its own meaning! Use a noble vocabulary, but make sure it has a noble simplicity.

Syntax

We can think of syntax as the way the parts of the sentence fit together. Since there are different parts involved, there are different levels of syntax. The first level of syntax that you should consider is the basic grammatical syntax of a sentence. A major concern at this level is the choice of active or passive voice. Master the use of the passive voice and use it only when it is truly necessary. Likewise, be sure to know basic issues of grammatical objects, such as when you should use "whom" as opposed to "who." You may need to review other basics of grammatical syntax, depending upon your experience with writing college-level essays. Be sure to practice these skills using example prompts.

The next "level" to consider is that of sentence-to-sentence relationships. Each sentence has a structure as a particular kind of whole. While you do not have to be groundbreaking in your sentence styles, you should vary the structure of your sentences. Most importantly, do not unleash a litany of sentences that all have the same overall appearance. For example, consider the following three sentences.

> It is obvious that philosophy is an important subject for societal progress. It is one of the most influential of all subjects, even if people do not realize it. It is quite important, therefore, that it be taught so that people are aware of its influence.

This set of sentences is not the strongest of arguments. It also has a very boring structure. Notice how every sentence begins with the same basic subject and verb pairing: "It is . . ." Avoid all such repetitions like the plague! (Also, recall: avoid metaphors like a mild plague!)

On the whole, avoid writing labyrinthine sentences. It is not necessary that every sentence be direct and brief. Nevertheless, you should always err on the side of clarity and brevity. Thus, avoid winding, snake-like sentences like, "The building, towering high above the towers that were in its vicinity, was given to undergoing a swaying motion in response to the winds that pounded it in the morning time." This could be rephrased in much more direct language such as, "The building, which towered above the neighboring buildings, swayed in the morning winds." Remember, you are making an argument, not trying to express the full poetry of the English language!

You should also consider another level of syntax, namely that of the complete paragraph. Each paragraph has an overall structure. Granted, you will likely use a standard format consisting of an introduction, several body paragraphs, and then a conclusion. Still, you should strive to avoid two kinds of paragraph-level syntactic errors. First, avoid repeating yourself in your conclusion. It is very easy for students to write a conclusion that is almost an exact copy of the introduction. This does little to help your overall argument. Instead, make sure that the conclusion integrates the details that you have discussed throughout your essay. Often, the transition from the preceding paragraph into the conclusion will help you to do this effectively. Second, avoid writing body paragraphs that all look the same. While you will be presenting facts and justifications for each of your main points in these paragraphs, it is best if you do this without writing a series of paragraphs that appear to be copies of each other. Just as sentence variety is important, basic paragraph-level variety is also important for showing your ability to craft a cogent and effective argument.

While English syntax and vocabulary are not the only aspects of writing assessed in the GRE analytical writing section, they are of pivotal importance. Think of these aspects as being the "building blocks" of your overall argument. You would not build a house out of loose, sandy blocks; so too should you avoid writing with a loose and slipshod deployment of syntax and vocabulary.

Commonly Mixed-Up Words

"Its" = possessive
"It's" = contraction of "it is"

It's a good thing you gave the dog **its** chew toy before it chewed on our shoes!

"They're" = contraction of "they are"
"Their" = possessive form of the pronoun "they"
"There" = demonstrative adverb conveying location at a distance

They're really lucky that **their** luggage was at the hotel when they got **there**.

"You're" = contraction of "you are"
"Your" = possessive form of the pronoun "you"

You're probably wondering where all the food in **your** fridge went.

"Who's" = contraction of "who is"
"Whose" = possessive form of the pronoun "who"

Whose idea was it to toss out the paper that said **who's** sitting by whom?

"We're" = contraction of "we are"
"Were" = past-tense indicative or subjunctive verb, depending on context

We **were** all just discussing this, and **we're** all in agreement that **were** we to explain where all your food went, you might get a bit frustrated with us.

"Effect," noun = the result of an action or cause
"Effect," verb = to cause a particular specified change to take place
"Affect," noun = a false mannerism, often one used with the intention of impressing others
"Affect," verb = to alter or cause a change in a specified subject

Rule of Thumb: you effect (*cause*) a change, which might affect (*alter*) others.

People who put on **affects** often do so because they are trying to **affect** the way that others view them, but often, others see through the charade, so it doesn't have its intended **effect**. As a result, **affects** are not an advisable way of **effecting** positive changes in the way others perceive you.

"Elude" = to avoid or remain out of reach
"Allude" = to refer in a work of literature to another, famous work of literature

By naming the main character of the novel "Ishmael," the writer **alludes** to *Moby Dick*; unfortunately, this literary connection **eluded** most of the students taking the exam, who did not notice it.

Issue Analysis

Likely the more familiar of the GRE's writing tasks, the "Analyze an Issue" task presents you with a statement of opinion on a generalized topic and asks you to pick a side and argue for it in cogent, convincing prose. While this task may have various characteristics in common with essay-writing portions of other standardized tests, the GRE's Issue prompts are notably much more general than those found on common lower-level exams. No data is provided to you, and no specific instances of the topic are mentioned. This formulation of the prompt requires you to shoulder much of the intellectual work of coming up with relevant evidence to support your assertions. At the end of the day, there is no "right answer"—you can argue for either side and still receive the best score. The section is testing your ability to argue well, not the side you've picked.

In this section of the book, we walk you through specific skills you'll need to demonstrate as you write in order to receive a great score. We begin by considering how to approach the prompt and structure the bare bones of your argument, and then spend a lesson considering how to brainstorm relevant and convincing examples of your claims as evidence for them. Finally, we provide a sample response to an "Analyze an Issue" prompt so that you can see how all of the recommendations and strategies we've outlined can come together at the essay level.

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Section Outline

Constructing an Argument

Selecting Evidence

Counterarguments

Sample Issue Analysis Essay

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Constructing an Argument

Issue Analysis Prompt

"The prevalence of social media creates an overload of information in which fact becomes indistinguishable from fiction, resulting in a decline in media reliability."

Write a response in which you discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider ways in which the statement might or might not hold true and explain how these considerations shape your position.

Pre-Writing: Pros and Cons

When you begin an Issue essay like this, you should write out some initial thoughts regarding both sides of a given issue. This will help you to gather your thoughts. Do not write lengthy notes; your time limits necessitate speed. Nonetheless, some initial "pro and con" arguments should be listed. This step can be difficult for some students, especially those who are not accustomed to partaking in disagreements and discussions on random topics; therefore, in order to prepare for the exam, you should practice responding to a number of prompts to ensure that you are prepared to marshal an argument in a timely manner on test day. Also, if you find that you have significant difficulty forming opinions, it might be helpful for you to begin reading some newspapers or magazines that contain opinion articles. This will help you to acclimate yourself to the general mental environment involved in writing these sorts of essays.

With those preliminary remarks in place, let's consider both sides of this argument. You may well generate lists that differ from these, but the following brief lists should help you understand the task at hand. (Also, depending on your style, these lists will likely be briefer in some ways. As long as you're clear and can read your own work, feel free to abbreviate claims on test day.)

Arguments for the prompt's claim

- Does create an overload of trivial information—brief snippets of information
- Most of this information is based on opinions, feelings, or things like reactions to dinner food (i.e. pictures of personal life events)
- People have an interest in presenting themselves in a good light

Arguments against prompt's claim

- The 24-hour news cycle has created quite a media overload of information
- While social media has had a negative influence, this general trend of media does just as much to create a decline in media responsibility
- Existing media has the appearance of veracity; deception actually easier
- Social media is an open market of ideas; can prove and disprove each other

While it does appear that either claim could be made, the second claim (i.e. the arguments against the prompt) is perhaps a bit stronger. Notice how each element in the list builds off the one before it in some way. There is a tight cohesion in the information provided by those observations; therefore, this argument against the prompt will help to yield an essay that is stronger on the whole. As always, remember that the goal for these essays is to show a strong and logical argument. Every other matter of rhetoric and style is secondary to that goal.

Thesis Statement

Next, you should formulate your thesis. This statement will guide the format of the remainder of the essay. Given the information that we gathered above, you could argue the following thesis:

Although the prevalence of social media causes many new pressures in the media world, it is unclear that it is a cause of significant decline in media reliability.

Now, this claim is rather general, so you are going to need to be quite explicit about what you are going to say. Therefore, in your opening paragraph, you need to follow this statement up with some clear points that will guide the body of your essay. Thus, the remainder of your introduction should continue something like this:

While social media has caused an increase in information channels, the contemporary 24-hour news cycle has already created a situation of information overload. The addition of social media is merely one more channel in this situation. Indeed, without taking into account social media's influence on general media responsibility, the media can be charged with mixing fact and fiction. Social media merely feeds into this already existing trend. It does not create it. Most importantly, existing media has appearances of veracity and authority that are not found in social media. Since these appearances encourage the mixing of fact and fiction, the existing media situation creates a situation that is actually less reliable by its very nature, prior

to any considerations of social media information vectors. In what follows, I will discuss each of these points in greater detail.

Body Paragraphs

In the body of your essay, you will need to clarify each of these points in greater detail. It is always helpful to use examples or concrete cases to explain the points that you make. While these should not be anecdotal, they should provide clear images to help your reader understand the point you are making. In this lesson, we will merely consider some points to help you develop your skills in generating the full body of your essay.

In the first body paragraph, you should stress the fact that social media is not the cause of further media unreliability. Instead, it is just one more aspect of the contemporary information overload found in the media. In this paragraph, you will introduce the general fact that the 24-hour news cycle (along with the prevalence of cable plans with hundreds of channels) has allowed for there to be many vectors for information. This situation has already caused many of the problems of unreliability mentioned in the prompt. In the second body paragraph, you can buttress this claim by showing how social media information vectors are just one more aspect of this situation. Here, examples from your own experience will help. (See below for some further recommendations on how you could structure these first two paragraphs.)

The final body paragraph will be critical, as you will need to show this situation still does not impute guilt to social media for making a bad situation worse. For this point, you will likely want to combine the last two points mentioned in the list above. You can stress that the "pre-social-media media" already has an appearance of truthfulness and authority. You can discuss the general idea of the "news industry" and the cultural expectations of truthfulness. The preexisting mixture of fact and fiction in this world is much worse than the mixture found in the world of social media. Social media doesn't make the same claims to authoritative truthfulness and editing as the standard media. You will need to make clear to your readers that it is more irresponsible for traditional media to abuse the boundaries of fact and fiction than for social media outlets to do so. The traditional media makes claims to authority not found in social media. This actually blurs the boundaries between fact and fiction.

Depending on your mood and mindset while writing, it might be necessary for you to alter your argument a bit. The fourth point above actually might help. Notice above how the first two body paragraphs are quite closely related? You may want to combine them. As you are writing, you may notice that the final point actually can be made into a separate paragraph that supplements what we have called above the "final body paragraph." Instead, after talking about the authority attributed to traditional media, you could continue by discussing the way that social media is self-critiquing. Give some examples of how it is a "free marketplace of ideas" in a way that is not found in traditional media. You will need to explain how this helps to sort out the difference between fact and fiction.

This would actually serve to increase the reliability of the media (and provide vehicles for critiquing deceptions offered by other forms of media).

Of course, each case is unique. Indeed, many prompts can be answered either affirmatively or negatively. Perhaps, given your own experience, you would have written in defense of the opposite claim for this essay. That is fine. Nonetheless, you will need to follow a procedure akin to what we have discussed in this lesson. The overall structural and argumentative clarity of your essay are of utmost importance. Thus, no matter what might be the essay content, be sure to clarify your main points and use them to craft an organized essay.

Selecting Evidence

Issue Analysis Prompt

"A teacher's most important role is the academic education of his or her students in the content covered in core scholastic subjects."

Write a response in which you discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider ways in which the statement might or might not hold true and explain how these considerations shape your position.

When writing the body paragraphs of your issue essay for the GRE, you will need to bring forth evidence to strengthen the claims that you make in the opening paragraph in which your thesis is stated. Not all evidence is equal, of course. In your haste, you must avoid providing weak evidence for your claim. As has been mentioned in other lessons, concrete examples are helpful; however, this doesn't mean that anecdotes are always the best kind of evidence. For example, given the prompt above, it would be wholly unconvincing if you were to write, "My best teacher in high school was my calculus teacher. Thanks to his focus on the academic subject matter, I have never forgotten the concepts we learned." This point makes little in the way of a general argument. Indeed, by itself, it doesn't even focus on the general claim that such focus is the most important role for a teacher. Strong evidence should have a strong logical structure as well as a clear connection to the general claim being made in the essay that you are writing.

This can be illustrated by considering in greater detail the prompt given above. This is a very general claim, almost certainly arguable from both sides of the issue; however, for the sake of brevity, let's only consider one side of the argument. The following are some basic general claims that could be made in support of the thesis presented above. They are not the only arguments that you could make, but we will use them as a general context for discussing weak and strong evidence. It will be easiest to contrast weak and strong evidence for the second and third of the following claims:

- Teachers are primarily employed to teach subjects
- Core subjects are most influential on the development of student's future lives
- The moral formation of students is too difficult for teachers to do in detail

At first sight, the second claim can seem quite problematic. While academic subjects are very important for the future development of students' lives, is it really the case that they are most influential? You can certainly think of examples of weak evidence that could be used to support this claim. For example, in sloppy haste, you might write, "Students benefit most from learning the subjects needed for their future careers." This claim is so open that it allows for many critiques. It presupposes that education is primarily undertaken for the sake of guaranteeing careers for the

students. Likewise, it presumes that the core subjects in a curriculum will provide this content for such career training. You will need to discuss these matters in greater detail if you wish to make the claim expressed in the second claim.

Therefore, you might try to strengthen the claim by making the logic of your argument more specific. For example, you could write, "In general, core academic subjects are applicable in all areas of life. Therefore, it is most important that teachers focus on this content in order to prepare them for life." While this argument is indeed slightly more explicit, it still makes a very debatable point. Your reader could quickly ask himself or herself, "Are all core subjects applicable? Are they applicable in the same way? What do you mean by applicable for life—for further education, for practical living at home, for vocational training?" The argument is just too vague and thus requires greater detail to increase its cogency.

To give definite boundaries to your argument, you could reformulate the point as follows:

Within the curriculum, core scholastic courses are most important for other subjects. Theses subjects—such as mathematics, reading, writing, and the basic sciences— provide a foundation for the particular life-paths taken by all students. Whether a student is planning to follow a particular labor track, to continue for many years in collegiate studies, or to prepare to head a household, these core subject contain content that is critical for all such undertakings. Several examples help to show this point. For instance, someone owning an automotive repair business will require various numerical and communications skills that must be learned in the core subjects of school. Without these skills, as well as basic mastery of scientific concepts, he or she will be unable to run a business and unable to work through the day-to-day technical tasks involved in his or her trade. Likewise, someone pursuing a long track of academic studies will require a firm foundation in the core subjects of study so that more detailed studies can be undertaken with ease. Numerous other examples could be deduced, but the point remains the same: it is critical that teachers focus above all on presenting the academic content in core scholastic subjects. This is their primary task in these important classes.

Of course, more detailed arguments would have to be made if this essay were a public policy paper; however, for the GRE, this level of detail suffices. Importantly, it helps to delineate the claims being made. The paragraph focuses on core scholastic subjects (as does the prompt). Likewise, it considers several examples that help to prevent you from implying that education is only useful for vocational training. The several examples at least function to indicate the importance of learning the academic content of core academic subjects.

This does leave some questions open, especially regarding the moral formation of students. Someone may object that this is an important part of ensuring the success of students. How could teaching effectively achieve its goals without addressing this?

Therefore, you can strengthen your claim by providing a further paragraph that brings forth more evidence regarding this latter point. Let's briefly consider how this could be done. A weak argument would follow by stating, "Teachers must focus on academic subjects because they do not have the authority to form the morals of their students." This argument is based on an assumption that can be questioned. Surely, the very fact that parents send their children to school indicates that they transfer some authority to the teachers. Let's consider how you could strengthen this last point.

Sometimes, it is useful to concede a bit of the argument, though always with an eye to strengthening your overall claim. For this final point, you could note that parents spend far more time with their children than does any one teacher. Each individual teacher only has so much time with the pupils, meaning that he or she will have relatively little impact on the character of the students in his or her class. The most important thing is for parents to form the characters of their children. The school can encourage this, but it would be preposterous to think that any given teacher should make this his or her goal in teaching. Instead, you could argue that the school as a whole should work to inculcate certain values in students—especially ones that are most relevant to the academic work being done (e.g. studiousness, industriousness, honesty, friendliness, etc). You can grant that these are important aspects to individual success; however, you can "turn" the argument to your advantage, for you can stress that such non-academic aspects are merely supported by the teachers. They are not their primary focus in the classroom. Each teacher's primary (and, hence, most important) role will be the education that he or she offers in the core subjects.

Notice how this last set of evidence can be integrated into the first point mentioned in the list at the beginning of our discussion. This helps to provide a strong integration within all the evidence being presented. The overall structure of your essay will benefit from the strong evidence thus provided. Thus, you can think of strong evidence as having two functions. On the one hand, each individual "bit" of evidence will be more convincing the stronger it is; however, these strengthened claims will better integrate together, given their improved thoughtfulness. Therefore, your overall argument will have a coherence that would be lost in a scatter-shot essay filled with weak evidence.

At the beginning of this lesson, it was mentioned that these skills are applicable to the issue essay. Do note that they will also be useful when you write your argument essay. Although it is a different genre of writing, it will require you to bring forth strong evidence to support your critiques of the argument presented to you.

Counterarguments

When writing an argument, it is sometimes helpful to argue against yourself! To express the point with a bit more propriety, it is sometimes helpful to acknowledge an apparent counter-argument to your claims. This kind of "about-face" should not weaken your overall argument, though. You are making an argument, so continue to make it. Counter-arguments are therefore a carefully crafted mechanism for considering another side of an issue. It must be clear to your reader that this counter-argument does not defeat the defense of your thesis. A counter-argument can occur briefly in the midst of one of the main points of your essay, or an entire body paragraph can be presented as a particular counter-argument to be considered. For the sake of clarity, we will use the prompt above to explain various counter-argument strategies. For each of these examples, let us presume that you wish to disagree with the prompt's assertion.

One such strategy uses a counter-argument to add precision to your overall argument. You can think of this as stating implicitly, "It may seem that X is the case, but really Y is. They are related, however, so let me explain . . . " So, for example, you could have a first body paragraph that argues that many individualistically rash acts of bravado would indeed be a threat to society. In this paragraph, you would make clear, however, that rashness is not the same as heroism, even though it might look the same. This would allow you to differentiate between acceptable and unacceptable behavior. Thus, you would concede to the reader that there is a kind of individual boldness that can be a danger to society. You can explain what it would look like and, briefly perhaps, how it could be handled. Finally, you would show that this really doesn't affect your argument. It helps to make very precise what you mean by "individual heroism." By placing this at the beginning of your essay, you would help to add precision. You do this by making a careful distinction for your reader. Hence, you are following the helpful maxim: "Never deny, rarely affirm, always distinguish."

Another way of expressing a counter argument is to use your final body paragraph to concede some point carefully. For example, you may discuss the fact that much of society relies upon obedience. If an excessive number of people desire to be heroes in a society, the social fabric will hold together less tightly. After granting all of this, you can then turn the point to your advantage: you can then

discuss how none of this has to do with individual heroism *per se*. Instead, it is just something that is a risk, not a certain occurrence. Indeed, you could go on to explain how the society itself relies upon heroism in many important moments—at its founding, in the midst of great moral crises, in the midst of war, etc. Without this, the society also could crumble. The important thing, you can say, will be that society is able to "absorb" the strong personalities who often want to be heroes. Thus, it is not so much a problem as it is an invitation for the society to adapt and use the virtues of its citizens. Notice how this carefully turns the argument in your advantage. In a way, it also makes a distinction—indeed, as the old Aristotelian philosophers would note, it distinguishes between the essential and the accidental (or, the *per se* and the *per accidens*).

Often, a final counter-argument will arise in the very midst of your writing of the essay. Consider the previous example. Perhaps you choose to write a second body point on the many ways that strong characters are needed in a society. Perhaps you discuss at length the role of courage in the social fabric—whether in day to day life or in large-scale events; however, as you are writing, you begin to notice that you do seem to be waxing poetically about rather dangerous characters. Perhaps your examples are of soldiers, entrepreneurs, and others who have a kind of heroic bent. You should give into your sense that this is perhaps a bit "too much to swallow." That will lead you to express a counter-example akin to what was discussed in the previous paragraph. This will show your reader that you are capable of seeing where your argument "could go" if it is misunderstood.

Of course, the simplest form of counter example is that which takes a form like, "You might argue X; however, that is wrong for the following reasons . . ." Really, this is just a variant on the earlier forms. In this case, you present quite directly an argument against your thesis. For example, you could say, "It could be argued that insofar as individuals enter society, they give up their individuality, making heroism unnecessary." You could even add to this claim by noting that something like this could be derived from political philosophers like Hobbes and Rousseau. This would seem to give some weight to your claim—and thus, also, give weight to your argument insofar as you can show that this seemingly strong claim really isn't sound. (Of course, always be careful when citing—always leave room for other interpretations that might be favored by the person evaluating your essay!)

After stating this argument, you can then present a brief set of arguments explaining why the proposed remark is wrong. We do not need to go into the details of social theory, of course, but you can give examples of how individuals remain individuals even in society. You can explain how it is good for society that some individually heroic things even have nothing to do with society per se. (For example, it is good that a person is heroically willing to rescue his or her neighbor from a building or to undergo great want in order to provide for his or her family.) You could argue about how there are various levels to society, further showing that social life does not totally destroy individuals' interests and actions. Each essay will be unique, and some topics will easily allow you to formulate a clear counter-argument in this style, readily dispatching it by showing how it is wrong. Generally, it will probably be easier to write one of the earlier kinds of counter-examples, using it as a chance to clarify by means of a distinction; however, you will sometimes be graced with a very clear case in which you wholly dispatch an opposing view!

Sometimes, you can use a small counter-argument in the midst of one of your main points. In this case, you would not dedicate an entire body paragraph to the counterexample or counterpoint. Instead, you would mention a competing possibility and then explain why it does not weaken your argument. Then, you would just continue on with the remainder of the paragraph.

Thus, as a general rule, you can think of counter-arguments as taking a form like this:

- "It seems that what I have been saying is not the case." OR "Well, this point X shows that this position actually can't hold." Or some similar form . . .

- Then, you respond in a manner that addresses the issue. Sometimes, you will completely disprove the point. Other times, you will show that the point may be true, but that it does not apply to this case. This is a particularly powerful strategy, as it concedes a point while also helping to clarify just what you are talking about. Recall again the maxim, "Never deny, rarely affirm, always distinguish."

- Finally, you make clear that your main point is not overturned. The tone will express the idea, "Thus, the objection really does not hold."

It is important that you show your reader that you can consider multiple sides of a given issue. Each prompt will require you to do this in a unique way. What is key, in any case, is that you use that ability to clarify your own argument and show your reader that you have indeed thought through the exact sense of what you are asserting. Thus, you can turn a possible objection into a chance for advancing your argument.

Sample Issue Analysis Essay

Issue Analysis Prompt

"Ignorance has become glamorized in Western culture, as knowledge has become a less desirable financial and intellectual burden."

Write a response in which you discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider ways in which the statement might or might not hold true and explain how these considerations shape your position.

Every age decries its own time as one of decline. One should thus hesitate to assert too boldly the negative drift of one's cultural situation; however, if one surveys the current state of Western culture, particularly in nations under the sway of the customs and cultural pressures exerted by the United States of America, it is fair to say that a glamorization of ignorance has developed in Western culture. To avoid rash generalization, it is best to consider one nation, so the United States of America will be taken as a paradigm case of this cultural drift. I will argue that the desire for economic advancement and media stardom have replaced the centrality of pure knowledge in U.S. culture. This is not equivalent to claiming that utter barbarism and idiocy have overtaken the West; however, it does mean that a true concern for knowledge has become significantly relativized, seeming to be justified only insofar as it serves as a means to some other end.

It is arguable that the primary cause of this drift in culture has been caused by the instrumentalization of knowledge. Although it is not universally true throughout U.S. culture, knowledge is rarely seen as an end in itself. Instead, knowledge is generally seen as a means to various other ends. The well-known maxim "knowledge is power" illustrates the idea, as does the early Modern call to become "masters and possessors of nature." The overall outlook is understandable, of course: unprecedented development has arisen from the application of human knowledge to various technical, social, and economic goals. The standards of living of many millions throughout the world have risen thanks to this very societal pressure existing in the West. While this is certainly a gain in many ways, it does have the detrimental effect of inverting the order of priorities in human life. Accustomed to the idea that knowledge is best used when it is applied, it is quite difficult for a contemporary U.S. citizen to believe in the primacy of knowledge for its own sake. It seems impossible to justify pure knowledge—even though pure knowledge of the world is a necessary antecedent to any thoughtful human activity whatsoever!

This trend is well exemplified in the current educational mania for the STEM subjects—that is, for the subdisciplines of science, technology, engineering, and mathematics. Well-meaning politicians and educators in the U.S. stress the advantages of extending the amount of education in

these subjects. Above all, it is said that this kind of education best fits the coming economy of the twenty-first century. While it certainly is necessary for education to match the needs of society and employment, an overemphasis on a set of applied technical subjects can make it quite difficult to see the importance of knowledge for its own sake. In such a cultural climate, the primary criterion will be whether or not some activity produces sufficient results, especially for economic advancement and employment. Indeed, even within the fourfold STEM categorization, the subjects falling into technology and engineering implicitly take precedence to pure science and mathematics. Since the desire for employment and application of knowledge is paramount from such an outlook, the "pure" subjects will be seen as instruments for other undertakings.

While these sorts of economic and technical concerns exert an important role on the U.S. idea of knowledge, we can also see similar pressures at play in the U.S. media. Once again, it is necessary to note the instrumentalization that is involved in these trends. Knowledge is here sought for the sake of something else, whether it be quick and pleasant experience for the viewer, "getting the first scoop," or merely for the fame that accrues from publishing the information as quickly as possible. The slow process of carefully learning and retaining knowledge is thus given a second-rate status. Indeed, careful consideration shows that even in this domain, the influence of economic concerns is strong, once more making knowledge an instrument for other ends, namely the desires of media companies for financial gain. While one should not overstate this point in a conspiratorial manner, it is quite important to note that, in general, the same force is at play, namely that of instrumentalization.

As these considerations come to a close, we should note, of course, that this situation does not indicate that the West has succumbed to complete ignorance. Instead, it is better to think of the state of affairs as indicating a particular kind of ignorance. Certainly, there is much technical and social knowledge being utilized in today's societies. Indeed, if we sufficiently broaden the definition of "practical disciplines" to include subjects like medical studies, various kinds of engineering, and technology, we can rightly say that practical knowledge has an immense breadth and import in contemporary U.S. (and, by extension, Western) society. Indeed, even in the so-called humanities and "soft sciences," the great number of technical studies undertaken today attest to a great devotion to achieving new knowledge; however, even in these seemingly impractical disciplines, much of the knowledge learned is judged in terms of its usefulness—whether that usefulness apply to social betterment, increased health, technological advancement, or merely to success in an academic career.

Thus, in conclusion, the general point remains. The major ill of the West's culture is its instrumentalization of knowledge. Due to the various economic, social, and media trends briefly discussed above, the pursuit of knowledge for its own sake does not naturally garner many supporters in such a cultural climate. This supports the general claim that ignorance—though only of a very particular kind—has become glamorized in the West. Certainly, knowledge remains a desirable burden for society and individuals to bear; however, such "knowledge" retains a tinge of the practical, social, and economic expectations of the West today. In so doing, knowledge itself is imperiled, for in such a scenario, it is difficult to see how knowledge itself is something good, regardless of the uses to which it might be put.

Argument Analysis

The GRE Analytical Writing section's "Analyze an Argument" task likely catches many test-takers off-guard with its demand for logical focus. If you've never taken a class in formal logic and have not reviewed with this particular task's challenges in mind, it can be all too easy to find yourself flustered and grasping at straws when composing your response. As the Analytical Writing section is the first section you face when taking the GRE, such stress can start you off on the wrong foot for the rest of your testing period, besides having consequences for your Analytical Writing score. It's thus very important to understand exactly what the "Analyze an Argument" task will ask of you when you take your exam. It varies dramatically from other essay sections on other standardized tests you may have taken. You're not asked to compose a persuasive essay based on your opinion and examples—instead, the topic of your essay will be the flaws in the reasoning of a short provided response.

In this section, we walk you through the idiosyncrasies of the style of prompt you'll be given, focusing on helping you to practice skills that can help you jump into a logical-analytical mindset that can be very beneficial when facing this part of the GRE. First, we consider how to identify the argument that the prompt passage is making. Tenuous reasoning can often make this a challenge in and of itself, but identifying the argument being made is a first step toward critiquing it. Next, we practice taking a critical look at the evidence that the prompt passage presents and consider various ways in which evidence might not be as logically sound as it initially seems. After that, we consider how unsupported assumptions and inferences can weaken the passage's argument. We also devote a lesson to investigating how you can effectively critique a passage's logic by bringing up points and examples that emphasize the fallacies in its reasoning. Finally, we provide a full-length essay answer to a sample argument analysis prompt so that you can see how the topics addressed in the section's lessons can be implemented in a single response. The argument analysis portion of the Analytical Writing section may initially seem imposing, but with practice and preparation, you can face it with confidence!

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Section Outline

Identifying Argument

Effectiveness of Evidence

Assumptions and Inferences

Critique and Counterpoint

Sample Argument Analysis Essay

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Identifying Argument

Argument Analysis Prompt

Over the past several years, a town in Alaska has seen its residents attacked by bears at an increasing rate. 10 years ago, there were 4 recorded bear attacks, while 5 years ago there were 7 recorded attacks. Last year there were 9 bear attacks on residents. Over the same period, the town's population has nearly tripled, and it has become a popular tourist destination. The town council has decided that something must be done about the growing number of bear attacks, so they've suggested increasing the length of the deer hunting season from 3 months to 4 months. If there are fewer deer around town, there will be fewer bears, and since bears eat deer, there will be fewer bear attacks. Furthermore, the town council has recommended that the length of the bear hunting season from 2 months to 3 months to further reduce the number of bears in the area, and they support the requirement that all local residents must use bear-proof trash containers to dispose of waste.

Write a response in which you discuss what questions would need to be answered in order to decide whether the recommendations and the argument on which it is based are reasonable. Be sure to explain how the answers to these questions would help to evaluate the recommendations.

The GRE argument essay needs to be carefully distinguished from the issue essay that you will also write at the beginning of the exam. Some students are tempted to approach both essays in the same way. Since you are arguing a point in both essays, it can be tempting to approach the argument essay as though you were to take a side regarding the argument presented therein; however, this is the way to approach the issue essay, not the argument essay. In the argument essay, you will be presented with a complete argument on behalf of some conclusion. It is your job to critique the argument. In this lesson, we are going to discuss some basic skills that you will need to apply for a successful completion of this essay section of the GRE. In particular, we will focus on the importance of identifying the claims being made by the author.

Obviously, ascertaining these main claims will be pivotal to crafting an adequate response. You cannot critique what you haven't tried to understand! Generally speaking, the main claims will be rather obvious and will often stand at the very beginning of the passage. Based on this main claim, there will often be some recommendation made by the author—as we will see in our passage; however, you need to begin by framing your reading in terms of the opening assertions made in the argument. Let's consider the passage given as an example for this lesson.

At the beginning of this passage, you can tell immediately what will be the overall claim. The author wishes to argue that a particular town has had an increasing rate of bear attacks on residents.

Likely, he or she will make a recommendation to ameliorate this situation. (Indeed, this will be the case, as we will see).

As you note the opening claim, you will need to be very clear regarding just what the author is asserting. The author is speaking about a particular town. That is, the claim is limited in its geographic scope. Likewise, he or she is writing only about bear attacks. This is an obvious point, but you need to note such small details. In some prompts, you will read an argument that slides from one topic to another, thus making a weak argument. (For example, a passage might go on to defend this claim by giving some extraneous details regarding coyote attacks, having nothing to do with bears at all). Notice too that the claim is not merely that the attacks have been increasing in number. The claim also is being made that the attacks have been happening at an increasing rate. This detail may or may not be important as you go on to critique the argument. Finally, you should note the other small details as well—such as the claim's limitation to the past several years and to residents.

Now, the argument does not end there. Instead, as you read on, you will see that the town council is making a decision regarding the situation. In the middle of the passage, their recommendations are noted, namely the alteration of the deer-hunting and bear-hunting seasons' lengths as well as a requirement regarding trash containers. Notice as well that some small justifications are made in the course of explaining these recommendations. As you craft your response, you will need to address the weaknesses inherent in these justifications. For now, merely note that the claims are being made. This will delimit the subject matter of your critique.

Thus, you will summarize the general claims being made in a manner as follows. It might be helpful on test day that you write a shorthand version of this on your scratch paper so that you can organize your critiques quickly and in an orderly manner:

There has been a recent increase in the rate of bear attacks in this town in recent years.

The council wants to reduce these attacks.

Recommendation 1: Lengthen deer-hunting season; this will drive out the bears, which eat deer.

Recommendation 2: Extend bear-hunting season; this will reduce population of bears.

Recommendation 3: New trash containers that are "bear-proof;" the meaning of "bear-proof" is not explained in detail

This initial investigation is very important. It will give you the appropriate "lens" for viewing the remainder of the passage. You are not reading this passage as you would read one of the reading comprehension passages in a later verbal section. From the very beginning, you have one outlook: "How can I critique this argument?" By gathering the various claims being made by the author, you will be ready to look for inadequacies in the details provided in the remainder of the passage.

As has been stated, your overall attitude is one of critique. You should be asking, "What is wrong with this argument? How is it weak?" Now, you shouldn't approach the passage as though it is making a completely foolish argument. Think of your role as being like someone helping a friend to overcome some weakness. This point is very important, so we should emphasize it quite strongly: **Your attitude should be that of recommending what needs to be supplied to make the argument stronger. Continually think, "Well, that might be true, but you need to add X, Y, etc. to make this argument overcome the following objection or objections."** If you approach the passage in this manner, you will quickly see the weak points that need to be addressed.

Some weaknesses may require the argument to be changed a bit. Recall the remark we made above about the word "rate." It was claimed that the attacks are happening at an increasing rate. Well, the passage actually notes that the town's population has increased quite a bit. This actually means that the rate lower in the final year than it was ten years earlier. (Based on the data, the same rate 10 years ago would now number at 12 attacks, given that the population has tripled). This means that the argument itself should be modified. It should not speak of "rate" but only of absolute amounts of attacks.

Recall too that we paid attention to some other small details. The argument discusses the "past several years;" however, the data is actually spread over a ten-year period—and only in a limited fashion, giving us merely three data points from that period! The data doesn't match the general assertion. You could devise counter-examples to show that such limited data could be read in a way that is detrimental to the argument being proposed. We aren't concerned with those details in this lesson, however.

Sometimes, the data itself introduces problems. The argument is focused upon resident-related attacks; however, the details of the passage include remarks about the increased tourism to the town. This opens up whole new vistas that are not addressed. Just to give one example: Even though the data is limited to residents, perhaps there are no attacks on the visitors. In an indirect way, this would show that the bear problem is perhaps even less bad, making council action undesirable.

We have thus far discussed only the initial argument; however, like many of the argument essay questions on the GRE, this passage makes a recommendation as well. Thus, we had the council's various recommendations. For each of these points, you should consider situations that might cause the plan to "backfire." In this lesson, we won't go into details. We are more concerned with your assessing of the general purpose of the passage. What is important to learn for now is the central importance attached to expressing this general intent. By noting that the passage not only is making a claim about bear attacks but is equally making recommendations regarding the actions to be taken, you will have an organized template for crafting your critical response to the author's overall claims. This will set you on your way to having a strong essay that deserves an high score.

Effectiveness of Evidence

Argument Analysis Prompt

Alan's Coffee Shop should start serving sandwiches like Brittany's Coffee Shop did last year. Brittany's Coffee Shop was only making a 2% profit at the start of last year, but by the end of the year, they were making a 14% profit. There are only three businesses that serve lunch in Alan's Coffee Shop's town versus seven that serve lunch in Brittany's Coffee Shop's town, so sandwiches would be even more popular at Alan's Coffee Shop than at Brittany's Coffee Shop. In addition, Alan's Coffee Shop has more regular customers than Brittany's Coffee Shop and is located near a cluster of local workplaces, so it is sure to see an even greater rise in profits if it starts serving sandwiches.

Write a response in which you discuss what specific evidence is needed to evaluate the argument and explain how the evidence would weaken or strengthen the argument.

In this lesson, we will focus on building skills at assessing the strength of evidence provided in the prompt for an argument essay. Of course, the strength or weakness of evidence depends in part upon the claim that is being made. We will be asking, "How well does this evidence support the claim being made?" In order to answer this question, we will at least need to identify the general claim. Luckily this argument makes a very simple claim, namely that Alan's Coffee Shop should start serving sandwiches. The evidence for this claim will all draw on facts gathered from Brittany's Coffee Shop. For more information on the importance of discerning the central claims of your argument, reread the lesson that discusses this important skill. If you aren't clear about the claim, you will never be clear about the details that defend that claim! You must know your topic in order to talk about it!

When you begin to scan the passage for evidence, make an inventory of the general data being presented for consideration. In the example passage, note the following data being used by the author:

• The profit data for Brittany's Coffee Shop, focusing on data gathered after the changes regarding serving sandwiches

• The lunch-serving-restaurant statistics for each shop's town

• The regular customer data for each shop

When you are assessing data like this, you will need to ask yourself two questions. The first is, "Why does the author believe that this supports the overall claim?" The second question is evaluative, "Does it actually support the overall claim?" Let's use each of the data-points from the example argument to guide our general discussion about making such evaluations.

The first argument is based upon a change in profit. Clearly, the argument wants you to infer that

Brittany's Coffee Shop reaped the total increase in profits due to the addition of sandwiches; however, this inference is liable to the famous fallacy—*post hoc, sed non propter hoc*. Literally, this means "after this, but not on account of this." That is, just because Y happens *after* X does not mean that X was the *cause* of Y's happening. Thus, just because Brittany's Coffee Shop added sandwiches last year, we cannot immediately say that the change in profit was due to the change in menu. The change in profit could have arisen from any number of unstated factors—changes in the economy, changes in customer patterns, alterations to staff, other items added to the menu, and many other possible factors.

Indeed, notice that the data itself is ambiguous. It speaks of the *early year* profit and the *end-of-year* profit. This isn't even giving you an average yearly profit. It could well just be indicating that a lot of people come to the shop at the end of the year, hence increasing profits greatly! In any case, the point is quite obvious: based on the meager data, you have no justification for believing that the addition of sandwiches to the menu was the cause of the increased profit; therefore, by extension, you have no reason to think that it will help Alan's Coffee Shop either! You must demand that your author defend the causation that he or she implicitly claims! That is, you must demand that the argument show that it was *propter hoc* and not merely *post hoc*.

The second example argument appeals to the number of lunch-serving businesses in each town. The author wants you to infer that since there are so many more potential competitors in the town containing Brittany's Coffee Shop, there were fewer potential customers who would want to come to her shop. This argument requires you to infer vast amounts of unfounded information! It wants you to think that both situations are exactly the same—that the towns are of equivalent size, that the desire for lunch is the same, that the shops draw the same sorts of clientele at the same times of the day, and many other such details. When you are reading an argument, always be on the lookout for this kind of temptation. You need to ask yourself, "How could the the situations be different?" Thus, you might ask whether the market is already saturated in Alan's town, whether his customers even come for food right now (before changing the menu), whether he has the skill for making sandwiches that people will want to buy, and so forth. Never assume any of these data for the argument.

In your essay, state that the author must provide more such details, and **be clear by listing examples**. As you recall from the issue essays, you must make your logic clear to your reader. Do not make general claims without showing exactly what you mean. In the argument essay, you do this by clearly expressing examples of what is missing and what should be added. This will help show your reader that you can articulate the manifold assumptions that must be made in order to strengthen the argument. This helps to show that you are aware that the data provided may be totally irrelevant, at least based upon the way that it is formulated in the argument as now stated. Explicit examples make your own thought explicit. Given that you want your thought to be "seen" by the reader, this is quite important!

The final data point is similar to the second one. Once again, the argument requires a great deal of extrapolation. Notice that some claims can actually make several claims at one time. This sentence is a good example of this. It makes claims about the following: (1) What regular customers want to buy; (2) the total number of customers based upon data restricted only to regular customers; (3) what will be bought by people at workplaces in a given area. You can find weaknesses in all three of these claims. If you can articulate these manifold problems, you will greatly impress the person evaluating your essay! To help you build your skills, let's briefly consider how you would address these issues.

First of all, it is arguing that regular customers will want to buy sandwiches. Since Alan's Coffee Shop has more regular customers, there will be more sandwich sales—or at least that is what the writer wants you to think! You can point out that you know *nothing* about the buying patterns of regular customers, however. Just to give one example: perhaps they are all early-morning customers who only want coffee.

As regards the second point noted above, notice that this claim draws an equivalence between a greater number of *regular* customers and a greater *overall* number of customers. Based on the author's words, there is no justification for drawing such an equivalence. Always beware of these kind of implied equivalences that may not be true. "More regular customers" does not mean the same thing as "more customers." Perhaps Brittany's shop gets a lot of tourist traffic! That would immediately undermine the argument. An argument needs clear terms, and you can register that fact when critiquing the argument.

Finally, this argument implies that the workplaces will yield customers who want sandwiches. Once again, you can craft numerous counter-examples that would weaken this claim; however, we are not concerned to provide every possible critique of the argument provided.

As you can see, the data and arguments offered to support a given claim can often be subject to significant degrees of imprecision, equivocation, and bad logic. From passage to passage, you will need to identify how these factors weaken the argument presented for your evaluation. Directly indicate any formal or informal logical fallacies, noting that these must be ameliorated before the argument can stand at all. Likewise, be clear about any implied assumptions that the argument presents, noting that these details cannot be left unsaid but, instead, must be explicitly stated if the argument is to be cogent and firm. Finally, do not allow your writer to use terms in a loose manner, whether by equivocating or by implying that several different terms are the same when in fact they are not. Demand precision from the argument, for an imprecise argument should be recognized for what it is—a bad argument!

Assumptions and Inferences

When you are preparing to write a critique for the GRE argument essay, it is important to pay attention to assumptions that are made in the course of the argument presented to you. In almost every case, the author of your argument will include assumptions that are not warranted by the facts presented. If these assumptions are not justified, the argument is significantly weakened. It is your task to point out such assumptions, incorporating them into your critique of the argument.

There are two main classes of assumptions for which you should evaluate the argument presented to you. On the one hand, there are global assumptions. These are assumptions that underlie the entire argument. As we will see in the passage presented to us here, there are two global assumptions that immediately undermines the overall premises on which the argument is founded. Sometimes there will be no global assumption, sometimes only one, and (obviously) sometimes multiple such assumptions. Keep your eye open for these cases, as they indicate significant weaknesses in the argument. On the other hand, there can be various assumptions that we can call "local" insofar as they only pertain to particular portions of the argument presented to you. Often, various assumptions—whether global or local—will interact with each other depending on how you approach the argument in your critique. Even though there is this kind of interaction, it is helpful to consider these two "levels" separately, for such a division between global and local assumptions will help you to organize your critical thoughts.

Before we consider the details of the argument presented here, an additional procedural remark will be helpful. As you are working through your critiques of various assumptions, you can craft a clear narrative by proceeding in two stages for each assumption. First, note and critique the assumption in question. Be clear with the reasons why the assumption is problematic. Explain what *could be* the case, given that the assumption does not provide clear enough details. As you complete your critique, you then can use a second step to help transition to your next remark about other assumptions. You can indicate that even if one grants the assumption that you just critiqued, there are still further

issues in the essay (namely the remaining assumptions you will then discuss). While you should try to avoid being stiff in the wording that you use to exercise this two-stage structure, such an approach shows your reader that you are able to isolate each problem, noting that each particular assumption is a problem on its own. That is, even if every other assumption were granted, this or that particular assumption would still be an issue.

Turning now to the sample argument provided, there is at least one glaring global issue. Perhaps we can even say there are two, as you will see. Above all, however, the argument works on the assumption that statistics for online streaming services are going to apply just as well to a given local movie theater. Think of all the assumptions with which this argument is laden! It presumes that theaters experience the same viewership dynamics as online streaming at home. Even if we grant that popcorn has a role to play at all, we cannot presume that people rate movies in the theater more highly because of this same "popcorn dynamic." People may generally have good memories of eating popcorn *at home* or eating their favorite popcorn brands, or some other influence on their recalled liking for a given movie; however, these same dynamics do not necessarily apply to theaters. We need to have a good bit of proof to know that they do. Likewise, even if they did apply to theaters in general, notice that this argument is referring to *the* local theater. Perhaps in this *one* theater does not experience the same dynamic. For example, perhaps it is in a community of older people who are worried about the role of salt in their diets, or perhaps it is in a community of people for whom popcorn is not a pleasant experience (for some cultural reason).

As we progress, a quick aside will draw attention to something that we are doing. Notice that with every critique should come **examples**. In all of your GRE essays, concrete examples are helpful. Notice as we continue to discuss various assumptions how these help to show our reasoning clearly. This is certainly something you want your readers to see as they are grading your exam!

Now, it was mentioned above that there is a second global issue. This second global issue is the general use of popcorn eating as the determining factor for the higher ratings. Perhaps it is always true that the eating of popcorn is followed by higher ratings; however, this could well be a case of *post hoc, sed non propter hoc*. Just because something (i.e. higher ratings) comes temporally after something else (i.e. eating popcorn while watching the movies), that does not mean that the latter was the cause of the former! Indeed, perhaps it is not the popcorn that makes the people happier about the movie but, instead, it is the sweet soda that they drink along with the salty popcorn that causes them to be thirsty. You could come up with many possible scenarios such as this to show that the general ascription of causality to the popcorn is also problematic. (**Note again the importance of concrete examples!**).

Now, let's grant a good deal of the assumptions that we have critiqued. Presume that the popcorn *does* have a positive effect—both at this particular theater and at home for the streaming service. Even in this case, several local assumptions remain at play. Notice that it is being recommended that the theater take take two steps: (1) the reduction of charges; (2) the offering of larger bucket sizes for popcorn. Both of these recommendations are based on shaky assumptions.

First, you have no guarantee that the reduction of cost is going to drive up popcorn sales. You would need to know the general desire of patrons in this regard before you make any decision on that matter. Perhaps people do not buy popcorn because of health concerns; therefore, no matter how low the price is dropped, such people will not add the buttery and salty product to their movie experience. When you are writing your essay, you can supplement this with additional examples of how this assumption might play out.

Likewise, the argument assumes two points regarding the offering of larger bucket sizes. First of all, it assumes that people are going to buy those bucket sizes (likely at the reduced prices). Once again, there is just not enough information provided to justify this assumption. Perhaps people who buy popcorn are happy with the amount that they are already buying. This may not change a thing! Even at the lower prices, such people will just gladly buy their normal size—now at a lower cost! In addition, this assumes that when people do buy large buckets of popcorn that they will eat them. Perhaps such people will not actually consume all of the extra content. Yes, popcorn is very tasty! Perhaps it is the case, however, that people can only stomach so much of it in one sitting! Thus, we need to have some data that shows that offering larger buckets will both tempt people to buy those new larger sizes as well as to eat the contents.

As an additional concern, you can note that you do not have comparisons regarding the amounts of popcorn at home and at the theaters. Perhaps at home people eat a modest amount of popcorn. If the theater starts to tempt people to stuff themselves full of popcorn, this might leave them in a bad mood when they leave the theater—being filled with salty snacks that leave them thirsty! This could have detrimental effects for the ratings. Once again, you need to have more information—here, regarding role of popcorn volume on the ratings outcome.

Of course, when you are writing your essay in response to an argument, you will find your own, unique critical remarks to make about the assumptions. No matter what, it is important that you presume that such assumptions have been made. Generally speaking, the GRE questions have been crafted by explicitly including such unstated assumptions. The authors of the questions are well versed in rhetoric and logic. Remember well a folksy adage regarding Aristotle's view concerning one's mastery of logic: "A good logician *knows* well how to make a bad mistake!" Look for the mistakes and assumptions that are "planted" in the argument! As you read each sentence, continually ask yourself, "What is this trying to get me to believe?" Also ask, "Does the passage actually justify me in holding that position?" This kind of "assumption hunting" is incredibly important for the GRE's argument essays. Honing your abilities at this skill will greatly improve your chances of success on test day!

Critique and Counterpoint

When you are looking to critique an argument, its most relevant aspect is its overall logical structure. When someone makes an argument, he or she does not merely present a flat assertion. An argument states a conclusion and explains the reasons for that conclusion. The strength of an argument rests upon the strength of its premises. In other lessons, we have discussed factual and structural issues that can weaken an argument. In this lesson, we will focus on some basic logical problems that can undermine an argument. As you craft your critical essay, you should pay attention to these logical issues. When you can articulate the weakness in an argument's logic, you will provide strong evidence that the argument needs to be reformulated.

We cannot here provide a complete lesson in logic. Nonetheless, we should consider some basic points so that you can have some skills in detecting fallacies—that is, bad reasoning. We will consider two main classes of fallacies, namely formal fallacies and informal fallacies. You will most likely find informal fallacies in the arguments that you will be critiquing; however, this category of bad reasoning is a bit nebulous. Therefore, it will help if we say a few things about formal fallacies first.

When we call a fallacy "formal," we loosely mean that it represents reasoning that invalidly uses the relationships that hold in logical forms. Let's take an obvious example. Consider a mother who says to her child, "You can have ice cream if you eat your peas and are nice to your sister." If the child then says, "I ate my peas, so I get ice cream," we know that he is not reasoning well. Both parts of the conjunction are needed—eating peas *and* being nice to his sister. We need proof of the latter if we are going to assume that the he is going to get ice cream.

There are many types of valid and invalid forms of arguments. Sometimes, these categories can differ from logic teacher to logic teacher. However, you should be familiar with several forms of reasoning that deal with if-then relationships. Let's take as our general example, the statement, "If you eat ice cream, then you will be happy tonight." The if portion is called the "antecedent," and the then portion is called the "consequent." For the sake of brevity, we will list several valid and invalid forms:

Two valid forms:

- "I ate ice cream; therefore, I will be happy tonight." This is called modus ponens. In less formal terminology, you can think of it as the rule that says conditional ("if-then") statements hold. If you "place" the antecedent (the cause associated with the "if" portion—here, having eaten ice cream), you can also "place" the consequent (the effect associated with the "then" portion—here: being happy later).

- "I am not happy tonight; therefore, I did not eat ice cream." This is called modus tollens. If you deny the consequent, you can also deny the antecedent. Given the initial statement, you know that if you did eat ice cream, you would have been happy.

Two invalid (i.e. fallacious) forms:

- "I didn't eat ice cream; therefore, I will not be happy tonight." This is called "denying the antecedent." All you know is that if you eat the ice cream, you will be happy. If you don't eat the ice cream, perhaps something else will make you happy.

- "I am happy tonight; therefore, I ate ice cream." This is called "affirming the consequent." This is similar to denying the antecedent. Since an if-then relationship does not assert a one-to-one correspondence between the antecedent and consequent, you cannot guarantee that your happiness tonight was caused by the ice cream. It could have had another cause.

Now, these kinds of formal fallacies occur merely because the form of the argument; however, sometimes there are fallacies that are caused by bad content or other less strictly "formal" issues. These get lumped together into the category of "informal fallacies." For example, the *ad hominem* fallacy is committed when you attack someone for some personal characteristic and use that as an argument against his or her logic. For example, you could say, "Peter is a clown. Therefore, this argument is weak." Well, that is not a lock-tight counterargument at all. The fact that Peter is a clown has nothing to do with the validity of his argument.

There are a number of informal fallacies that you may find in passages. For example, you may find a hasty generalization, where only several items are considered in a group, though these items are taken to be representative of the whole group. This is sometimes also called the "fallacy of composition." In contrast to this, the fallacy of division attributes something to an individual in a group based upon the properties of the group. An argument can avoid these issues by being more precise about the relationship between parts and wholes in a group.

The most frequent issue with arguments is the relationship of cause to effect. The fallacy known as *post hoc, sed non propter hoc* frequently occurs in GRE passages. People often presume that just because one thing temporally comes after another that it was caused by what came before it. In GRE passages, you may find an author stating something like, "The amount of rain increased over the past five years, and during this time, people stopped walking outside. Therefore, the rain must have been the cause of the decrease in walking." While this seems like an obvious conclusion, you should not follow the author's assumptions. The mere temporal ordering cannot guarantee causation. You will likely find yourself noting this kind of informal fallacy many times. In general, carefully watch causal reasoning. Many informal reasoning errors are committed in this way in GRE passages.

There are numerous other types of logical fallacies, but we will now take a look at a sample prompt to consider some examples of bad reasoning.

Argument Analysis Prompt

Bicycle sales across the city fell between 1992 and 1993, suggesting that fewer people must have been using bicycles to commute and travel around the city. Local car sales did not increase, the average price of gas was lower in October 1993 than during the same month of the previous year, and there were no notable increases in applications for parking permits between 1992 and 1993, so more city residents must have been walking as a primary mode of transportation. In June of 1993, the majority of the dozen commuters interviewed for a local newspaper story at various centrally located businesses reported walking to work whenever the weather permitted. Also, when one hundred randomly selected residents were asked in a poll about what transportation method they would prefer to use to commute, most replied that they would prefer to walk to work. Neighboring cities experienced increases in the number of pedestrians frequenting their city centers since between 1992 and 1993. The increase of walking commuters between 1992 and 1993 must be the clear cause of these aggregate trends.

Write a response in which you discuss one or more alternative explanations that could rival the proposed explanation and explain how your explanation(s) can plausibly account for the facts presented in the argument.

The very first argument of this passage commits an informal logical fallacy. It presumes a causal relationship that factually is not necessary. Strictly speaking, this is not a logical problem, but it does represent a poor use of cause-and-effect relationships in constructing the argument. It is taking the decrease in bicycle sales to be a sign of decreased desire to commute to the city via bicycles. It could well be the case that people are just not buying new bicycles. That does not mean that they will then stop riding.

We can also note a formal issue in the first sentence. When you first read it, it seems to say, "If sales fall, then fewer people will use bicycles for travel." The sales are taken as an effect, which means that the cause is the actual desire to use the bicycles. You could formulate the argument as, "If fewer people desire to use bicycles to commute and travel to the city, then bicycle sales will fall." Hence, we more correctly state, "If less desire, then less buying." Now, the argument states that the sales fell, but recall from above that you cannot affirm the consequent (the then portion—here, bicycle buying) and presume that the antecedent (the if portion—here, the decrease in the desire to use bicycles) is true. Thus, even if we presumed that there were not an informal fallacy present, we still would be faced with a formal fallacy of the type "affirming the consequent."

Regarding the survey from June 1993, there are several cases of hasty generalization. First of all, notice that only a dozen commuters were interviewed about their habits. It is quite imprudent to infer from such a small sample size anything that would apply to all commuters to the city. In addition to the numerically small size of its sample set, it was directed only to workers at centrally

located businesses. This is a small group of workers, many of whom could well live downtown! Their situation and opinions cannot be extrapolated to apply to the whole commuting population. Finally, they were only asked questions about being able to walk when weather permits. Once again, you cannot extrapolate even about their own general opinions about walking from such a limited criterion—let alone extrapolate the data to infer the opinions of everyone! Perhaps the area is very rainy; or, perhaps it is sunny. We do not know and cannot extrapolate based on such limited assertions and data.

The other survey is problematic in this same way. It argues from a group of residents and infers that this survey applies to all residents and commuters. Indeed, the latter point makes the extrapolation potentially very weak. In addition, it argues from preferences to what people would actually do. People may prefer to walk, but they could live in an inconvenient location or perhaps need to use their cars after work for family functions. As you can see, the links of causation are weak in the argument as presented. A mere preference cannot guarantee what will actually be the case.

There are several other issues in this argument, but these examples should help you to see some of the ways that attention to formal and informal fallacies can help you in fashioning your critique.

Sample Argument Analysis Essay

Argument Analysis Prompt

The following appeared in a citizen appeal to a local legislator.

The intersection of High St. and Main St. is too dangerous and the city needs to install a traffic light. Over the past three years, the city has opened one elementary school and one high school within walking distance from this intersection. Increased automotive and pedestrian traffic creates a situation that puts too many people at risk. Last year, there were a total of seventeen calls that resulted in police response to the intersection and eight calls that resulted in ambulance deployment. Installing a traffic light will employ a system to ensure right-of-way for pedestrians, cyclists, and cars, reducing the number of incidents at the intersection.

Write a response in which you discuss what questions would need to be answered in order to decide whether the recommendation is likely to have the predicted result. Be sure to explain how the answers to these questions would help to evaluate the recommendation.

Although it is understandable that the city believes that a light should be installed at the intersection of High St. and Main St., it is necessary that a stronger argument be fashioned before this decision is ratified by those in charge of these matters. There are several weaknesses in the argument on behalf of such a change at the intersection. First of all, the argument insufficiently connects the opening of the new schools to supposed alterations in traffic patterns at this intersection. Second, the argument does not present enough historical data to justify its claims that there has been an increase in dangerous situations at this intersection. Third, the data presented are quite vague, not providing enough information regarding the details of the supposedly increased accidents. Finally, it is not guaranteed that the proposed solution will provide a safer flow of people through the intersection in question. In what follows, we will consider each of these points in greater detail.

At the beginning of the argument, the opening of two new schools is mentioned. It is quite understandable that the argument implies that these schools were causes of increased traffic. Indeed, as presented, the argument implies that there has been increased pedestrian traffic through the intersection of Main St. and High St., given the close proximity of the schools to the intersection; however, one can only hastily assume that the schools would cause increased traffic through this particular intersection—whether that be increased walking traffic or increased vehicular traffic due to the transport of students. Before making this sort of claim, the authors of this argument need to show that this intersection is one that would be traveled by pedestrian students and school-related vehicles. Depending on the layout of the city, it is possible that all school traffic is directed away from this intersection. Lacking details regarding the traffic flow to and from the schools, it is imprudent to presume that the schools are causing increased traffic at this intersection.

As regards the data provided, the argument does not provide an adequate historical basis for its claims. Arguing from one year's statistics, we know absolutely nothing about the changes that have occurred in recent years. While any human injuries are lamentable, we cannot presume that the situation is worsening at an intersection merely because there were accidents last year. Instead, the argument needs to be strengthened by adding explicit details showing that accidents have, in fact, been increasing over the course of a number of years. In addition, it should be shown that such accidents were indeed caused by alterations in traffic flow and not merely several accidents that were unfortunate cases of personal fault on the part of drivers. The argument needs to justify that the intersection itself is a danger and that the situation is, in fact, worsening.

Indeed, quite explicit data should be provided regarding the character of the accidents that have occurred at this intersection. Those calling for a new light should provide information such as times of day, types of vehicles, weather conditions, and so forth. Likewise, this data should include information about the roles of pedestrians and bikes in such accidents. It is quite possible that such data will show that the issue in question is not related to traffic patterns. Instead, the issues being experienced could well be caused by other environmental conditions that would not be addressed by the addition of a new stoplight.

Finally, the argument functions on the assumption that the introduction of new traffic patterns will not be the potential cause of new dangers. As has already been mentioned in part above, the argument does not provide significant details regarding the pedestrian and driving patterns at this intersection. Before altering the way that the intersection functions, it is necessary to understand the types of users who cross through the intersection. For example, it may well be the case that cyclists do not use the intersection at this time. Based on the argument, it is not clear how the cyclists will be accommodated to ensure that they have a right-of-way through the intersection. If the installation of the lights include the designation of cyclist lanes, this could increase cyclist traffic through the intersection, given that bikers would perhaps now feel safer. This increase in cyclists could well introduce new dangers for drivers, who may not be accustomed to driving on roads that are shared with bikers. In addition, the cyclists may not follow traffic laws strictly, causing increased hazards through the intersection as vehicles and pedestrians presume that they are, in fact, going to follow the general transportation laws. Likewise, this arrangement may change traffic patterns in the city, leading cyclists to use new routes and thus causing potential new hazards in the town. Although the argument is concerned with one intersection, such collateral effect should be taken into account before any changes are made.

These points indicate some of the weaknesses in the argument presented on behalf of the new light at the intersection of High St. and Main St. It is understandable that the city would like to address any dangers that might be leading to the accidents that are occurring at this intersection in the town; however, before making any decision to install a light and alter the traffic patterns at the intersection, it is necessary to address the various issues discussed above. In light of such expansions, the argument on behalf of this change may well be justified, but it is necessary that such discussions be undertaken and clarifications provided before proceeding forward with the proposed changes.

Full-Length GRE Practice Test

Section	Tasks	Time Allotted
Analytical Writing	2	30 minutes each
Verbal Reasoning #1	20	30 minutes
Verbal Reasoning #2	20	30 minutes
Quantitative Reasoning #1	20	35 minutes
Quantitative Reasoning #2	20	35 minutes

Instructions

The test that follows is similar to the one that you will see on test day when you take your Revised General GRE. During your exam, you are allowed to skip questions and return to them later as long as you have time to do so; however, this opportunity is only available for questions in the section on which you are currently working. For example, if you finish up a Verbal section early, you cannot use the extra time to edit either of your essay responses or change answers on a Math section you completed earlier in the exam.

We recommend using this full-length practice exam to simulate the time constraints of the actual exam in order to gauge your time-management skills and stress levels. With that in mind, each section of the test includes an indication of how many questions are present on that section and how long you have to complete them.

Keep in mind that when taking the Revised General GRE, you will have scratch paper available, as well as a simple digital calculator to use on Quantitative sections.

Analyze an Issue - 30 Minutes

Write a response in which you discuss which of the following views more closely aligns with your own position and explain your reasoning for the position you take. In developing and supporting your position, you should address both of the views presented.

> "Some people believe that the most effective means of advancing the development of a society is by focusing on the contributions resulting from the goals of each individual. Others believe that goals pertaining to the whole society provide a necessary unifying direction, without which individual contributions are meaningless."

Analyze an Argument - 30 Minutes

Write a response in which you discuss what specific evidence is needed to evaluate the argument and explain how the evidence would weaken or strengthen the argument.

> Dr. Benson is a renowned scientist known for his work in biochemistry and cardiology to establish the link between high blood pressure and heart disease. In a paper published ten years ago, he postulates that consuming salty foods increases the salt content of the blood and puts those individuals at greater risk of heart disease due to high blood pressure. A recent study published by Dr. Benson shows that consuming some salty foods is healthy, but warns that consuming sugary foods can lead to heart disease. At a recent convention, he gave a presentation concluding that high blood pressure is definitely linked to heart disease.

For each question, indicate the best answer using the directions given. If a question has answer choices with ovals, then the correct answer consists of a single choice. If a question has answer choices with square boxes, then the correct answer consists of one or more answer choices. Read the directions for each question carefully.

Text Completion: For each blank, select one entry from the corresponding column of choices. Fill all blanks in the way that best completes the text.

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1 Although I felt confident that my mathematical abilities would give me an edge in Vegas, having tried my hand at gambling, I now realize I am a bit of a _____.

malcontent
tyro
bourgeois
teetotaler
masochist

---

2 His _____ approach to the sport was copied by competitors and quickly became the standard method.

dangerous
lax
rehashed
novel
ineffective

---

3 The museum's collection is vast and (i) _____, covering thousands of years of worldwide art history; thus, to (ii) _____ it, it would take several days at least.

Blank (i)	Blank (ii)
lofty	scrutinize
painstaking	undertake
comprehensive	appreciate

4

The famous story that H.G. Wells' *War of the Worlds* caused panic in the streets is (i) _____; instead of running in fear, many listeners were simply (ii) _____ by the storytelling, staying inside in order to not miss a single moment of it.

Blank (i)	Blank (ii)
apocryphal	harrowed
salubrious	perused
arcane	riveted

5

The (i) _____ cat refused to walk around tamely on a leash, but the (ii) _____ dog not only heeled, but obeyed each of its master's commands obediently and never needed to be (iii) _____ for not listening.

Blank (i)	Blank (ii)	Blank (iii)
pernicious	prodigal	countenanced
recalcitrant	complaisant	impeded
credulous	insipid	admonished

6

George decided to (i) _____ eating red meat to reduce his high cholesterol, as he had been told by his doctor that his current diet was (ii) _____ to his overall health; it was a difficult lifestyle change, as he had a(n) (iii) _____ for hamburgers.

Blank (i)	Blank (ii)	Blank (iii)
impugn	deleterious	anachronism
cease	volatile	predilection
bolster	disparate	abeyance

Questions 7–9 are based on the following passage.

[1] During photosynthesis, plants convert carbon dioxide and water into oxygen and glucose, a sugar used for energy storage. [2] Each carbon dioxide molecule is broken into oxygen gas and a carbon atom, which, with other carbon atoms, is used to generate glucose in a process called "carbon fixation." [3] There are a few different carbon-fixation pathways plants can take. [4] The C3 pathway, named for a three-carbon molecule produced during the process, is most common. [5] Certain plants have evolved and can use a C4 pathway instead of a C3 pathway when it is more efficient to do so. [6] The C4 pathway results in the saturation of a particular enzyme, RuBisCo, with carbon dioxide. [7] RuBisCo's job is to fix carbon, and when saturated with raw materials, it can perform more efficiently.

[8] CAM plants have evolved a different method of making photosynthesis more efficient. [9] These plants do not alter the saturation of carbon dioxide around RuBisCo. [10] Instead, they change the way in which they collect the raw ingredients of photosynthesis from the environment. [11] Plant leaves contain "stomata," openings through which carbon dioxide is collected, but through which water can also be lost. [12] CAM plants close openings in their stomata during the day and open them at night. [13] This prevents water from evaporating out of their stomata during the day and is a useful adaptation in hot, arid environments.

[14] You may be wondering: don't CAM plants need carbon dioxide and sunlight at the same time to photosynthesize? [15] They do, and this has a dramatic effect on when they photosynthesize. [16] The reactions involved in photosynthesis fall into two categories: the "light" reactions require light, and the "light-independent" or "dark" reactions do not. [17] At night, CAM plants do not completely photosynthesize anything; they collect carbon dioxide, perform the light-independent part of the process, and wait. [18] In the daytime, they perform the light-dependent reactions and use the available sunlight and the carbon dioxide they collected at night to generate glucose and oxygen.

7   Which of the following is implied by the underlined sentence?

A. Any plant that can use a C4 pathway can use a C3 pathway.

B. Any plant that can use a C3 pathway can use a C4 pathway.

C. Only C4 plants contain RuBisCo.

D. Only C3 plants contain RuBisCo.

8   C4 plants are similar to CAM plants in that _____, but different in that _____.

A. they have both evolved to increase photosynthetic efficiency by saturating RuBisCo with carbon dioxide . . . they gather that carbon dioxide at different times of day

B. they both close their stomata during the day . . . CAM plants perform only the light-dependent photosynthetic reactions during the day whereas C4 plants perform only the light-independent reactions during the day

C. they are both well suited to arid environments . . . C4 plants use RuBisCo, but CAM plants lack RuBisCo

D. they both use evolutionary adaptations to perform photosynthesis more efficiently . . . they use different mechanisms to do so

9 You're talking to a friend about what you learned in the passage when your friend says, "But isn't photosynthesis just one cycle? Why would it function differently in the day than at night?" Which sentence most directly answers your friend's questions?

> Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

Sentence Equivalence: Select the <u>two</u> answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole <u>and</u> produce completed sentences that are alike in meaning.

---

10 The musician's attempts to brighten the mood continually failed, as all his songs sounded like _____.

- [ ] encomiums
- [ ] dirges
- [ ] waltzes
- [ ] jigs
- [ ] laments
- [ ] threnodies

---

11 With a handlebar mustache and a loping walk, the reenactor _____ the look of a nineteenth-century baseball player; he seemed to have stepped right out of a previous era.

- [ ] subverted
- [ ] epitomized
- [ ] underscored
- [ ] mocked
- [ ] exemplified
- [ ] satirized

Questions 12–14 are based on the following passage.

[1] As the logical positivism rose to ascendancy, poetic language was increasingly seen as merely emotive. [2] Wittgenstein's influential *Tractatus* argued that only language corresponding to observable states of affairs in the world was meaningful, thus ruling out the value of imaginative language in saying anything about the world. [3] Poetry's contribution was rather that it showed what could not be said, a layer of reality which Wittgenstein called the "mystical." [4] Despite Wittgenstein's interest in the mystical value of poetry, his successors abandoned the mystical as a meaningful category, exiling poetry in a sort of no man's land where its only power to move came through the empathy of shared feeling.

[5] Yet some thinkers, like Martin Heidegger, reacted strongly to the pretensions of an instrumental theory of knowledge to make sense of the world. [6] Heidegger, Hans-Georg Gadamer, and Paul Ricoeur all gave central value to poetry in their philosophical method, signifying a growing sense among continental thinkers that poetic knowing was an important key to recovering some vital way of talking about and experiencing the world that had been lost.

Passage adapted from "Poetry and Philosophy" by Justin Bailey (2013)

12 The author is primarily concerned with _____.

A. exploring the contribution of philosophy to discussions of poetic method and appreciation

B. enumerating the reasons why Wittgenstein and his successors were misguided in their philosophical approach

C. arguing that given the current trajectory of philosophy, poetry will soon no longer be studied in mainstream society

D. describing the mainstream marginalization of poetry among philosophers of a certain period before noting significant exceptions

13 Select the sentence in the passage in which the author contrasts the position of a group of philosophers against those who followed Wittgenstein.

Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

14 **Select all answers that apply.**

Which statements can be inferred from the passage?

A. Some of Wittgenstein's successors used his work to exclude something that was important to him.

B. Philosophers agree that instrumental theories of knowledge are sufficient in understanding the world.

C. Most positivists followed Wittgenstein in arguing for poetic knowledge as a meaningful category in philosophy.

15 Having finally fixed the engine, the mechanic took a moment to listen to it
_____.

- [ ] purr
- [ ] hum
- [ ] rattle
- [ ] clank
- [ ] whine
- [ ] drone

---

16 Frequently, beginning pilots find themselves unable to trust their instruments,
_____ them in the air and leaving them upside down.

- [ ] stranding
- [ ] bewildering
- [ ] informing
- [ ] reassuring
- [ ] mystifying
- [ ] misinforming

Questions 17–20 are based on the following passage.

The future of poetry is immense because in poetry, where it is worthy of its high destinies, humanity, as time goes on, will find an ever surer and surer stay. There is not a creed which is not shaken, not an accredited dogma which is not shown to be questionable, not a received tradition which does not threaten to dissolve. Our religion has materialized itself in the fact, in the supposed fact; it has attached its emotion to the fact, and now the fact is failing it. But for poetry the idea is everything; the rest is a world of illusion, of divine illusion. Poetry attaches its emotion to the idea; the idea is the fact. The strongest part of our religion today is its unconscious poetry.

Passage adapted from "The Study of Poetry" in *Essays in Criticism: Second Series* by Matthew Arnold (1888)

17 With which of the following assertions would the author most likely agree?

A. The appeal of art focused on abstract emotion will outlast that of art focused on representing historical events.

B. A religion that involved no emotion could nevertheless be popular in the right cultural milieu.

C. Poetry should only be used to describe fanciful, unrealistic events.

D. While certain dogmas may fade as time progresses, we're likely to see others gain a stronger cultural hold.

18 The author contrasts poetry most strongly against which of the following?

A. Ideas

B. Facts

C. Religion

D. Emotion

19 Which of the following is the best paraphrase of the underlined sentence "Poetry attaches its emotion to the idea; the idea is the fact?"

A. Poetry derives its effects from abstract concepts, not physical realities.

B. The emotions felt most strongly by readers of poetry are those inspired by unrealistic situations.

C. When reading poetry, readers are expected to suspend their disbelief and pretend that even outrageous scenarios could potentially occur.

D. Poetry places more importance on the novelty of the concepts discussed than on their objective truth value.

20 The author's use of the underlined phrase "the supposed fact" accomplishes which of the following?

A. It suggests that religion and poetry are not mutually exclusive.

B. It reprises the author's point that peceived facts are subject to revision as more is learned.

C. It introduces the idea that a reliance on facts might be somehow subpar to a reliance on emotions.

D. It suggests that some people perceive as facts ideas to which they have attached strong emotions.

End of Section   **STOP**   **End of Section**

	Answer	Question Type / Concept Tested
1	tyro	*Single-Blank Text Completion – Cause and Effect*
2	novel	*Single-Blank Text Completion – Comparisons*
3	(i) comprehensive (ii) scrutinize	*Single-Blank Text Completion – Positives and Negatives*
4	(i) apocryphal (ii) riveted	*Text Selection – Inferences*
5	(i) recalcitrant (ii) complaisant (iii) admonished	*Three-Blank Text Completion – Positives and Negatives*
6	(i) cease (ii) deleterious (iii) predilection	*Three-Blank Text Completion – Positives and Negatives*
7	C	*Mutliple-Choice with One Answer – Argument*
8	D	*Mutliple-Choice with One Answer – Purpose*
9	Sentence 16	*Mutliple-Choice with One Answer – Summarizing*
10	dirges, threnodies	*Sentence Equivalence*
11	epitomized, exemplified	*Sentence Equivalence*
12	D	*Mutliple-Choice with One Answer – Main Idea*
13	Sentence 5	*Text Selection – Argument*
14	A	*Mutliple-Choice with Multiple Answers – Inference*
15	purr, hum	*Sentence Equivalence*
16	bewildering, mystifying	*Sentence Equivalence*
17	A	*Mutliple-Choice with One Answer – Inference*
18	C	*Mutliple-Choice with One Answer – Main Idea*
19	A	*Mutliple-Choice with One Answer – Paraphrasing*
20	B	*Mutliple-Choice with One Answer – Purpose*

1 Although I felt confident that my mathematical abilities would give me an edge in Vegas, having tried my hand at gambling, I now realize I am a bit of a _____.

malcontent
tyro
bourgeois
teetotaler
masochist

The leading word "although" lets you know there will follow a contradiction or logical conflict in this sentence. In this case, it is between the speaker's imagined skill at gambling, as evidenced by the phrase "I felt confident that my mathematical abilities would give me a leading edge in Vegas," and the word you aim to find. You should be on the lookout, then, for a word that inverts "having the leading edge," keeping in mind that the speaker specifies this is their first time gambling. "Tyro" means *novice* or *extremely inexperienced person*, so it is the best fit here.

---

2 His _____ approach to the sport was copied by competitors and quickly became the standard method.

dangerous
lax
rehashed
novel
ineffective

The approach to the sport "became the standard method" only after being "copied by competitors." This implies it was not the standard method at first, and the correct answer will reflect this. "Novel," meaning *new and innovative*, is the best choice.

---

3 The museum's collection is vast and (i) _____, covering thousands of years of worldwide art history; thus, to (ii) _____ it, it would take several days at least.

Blank (i)	Blank (ii)
lofty	**scrutinize**
painstaking	undertake
comprehensive	appreciate

The sentence notes that the collection "covers thousands of years of art history," which speaks of breadth, not complete depth. Thus, "comprehensive" (*far-ranging, impressively complete*) is the best answer. In regard to the second blank, the second clause of the sentence speaks of it taking a long time to do something to the collection. "Undertake" (*begin to do something*) does not work well by its definition. One could potentially "appreciate" the collection after only seeing a small part of it, so this isn't the best answer choice. "Scrutinize" (*examine carefully*) would, indeed, be something that would take several days to do in a large collection. Thus, the correct answer is "expansive . . . scrutinize."

4

The famous story that H.G. Wells' *War of the Worlds* caused panic in the streets is (i) _____; instead of running in fear, many listeners were simply (ii) _____ by the storytelling, staying inside in order to not miss a single moment of it.

Blank (i)	Blank (ii)
apocryphal	harrowed
salubrious	perused
arcane	**riveted**

You can use context clues in the sentence to figure out which word to use. Here, you have the "famous" story about panic, where, instead, people did something else; therefore, while the story may be a lie, it is thought of as a fact that needs to have a counterpoint ("instead . . ."). For the first blank, neither "salubrious" (*lewd and lasviscious*) nor "arcane" (*known to only a few*) makes sense in context; "apocryphal" (*doubtful, though believed to be true*) is the best answer.

In regard to the second blank, the storytelling made the listeners want to stay inside, so you can conclude that the storytelling was interesting to the listeners. "Perused" doesn't fit grammatically (a person cannot be "perused by" a story), nor does it fit as well as "riveted" (*engrossed*). "Harrowed" (*terrified*) doesn't work well with "simply" and misses the main point of the sentence: that the listeners were interested in the story. Thus, "riveted" both fits and is the correct answer. This makes the correct answer "apocryphal . . . riveted."

5

The (i) _____ cat refused to walk around tamely on a leash, but the (ii) _____ dog not only heeled, but obeyed each of its master's commands obediently and never needed to be (iii) _____ for not listening.

Blank (i)	Blank (ii)
pernicious	prodigal
recalcitrant	**complaisant**
credulous	insipid

Blank (iii)
countenanced
impeded
admonished

Since the cat refuses to be leashed, we need an adjective for the first blank that means something like "stubborn" or "resistant." "Recalcitrant" (*obstinately uncooperative toward authority*) is the best answer, as "pernicious" (*mean-spirited*) and "credulous" (*gullible*) don't make sense in context. For the second blank, we need an adjective that conveys the dog's obedience— "complaisant" (*willing to please others; obliging*) is the best answer. "Prodigal" (*extravagant in spending*) and "insipid" (*dull, bland*) don't work in context. For the third blank, we need a verb that means something like "scolded"; "admonished" (*warned or reprimanded firmly*) is the best choice, so the answer is "recalcitrant . . . tractable . . . admonished."

6

George decided to (i) _____ eating red meat to reduce his high cholesterol, as he had been told by his doctor that his current diet was (ii) _____ to his overall health; it was a difficult lifestyle change, as he had a(n) (iii) _____ for hamburgers.

Blank (i)
impugn
cease
bolster

Blank (ii)
deleterious
volatile
disparate

Blank (iii)
anachronism
predilection
abeyance

For the first blank, we need a verb that means something like "stop," so "cease" is the best choice. Neither "impugn" (*dispute*) nor "bolster" (*provide support for*) make sense. For the second blank, we're looking for an adjective that means something like "causing harm"; "deleterious" (*damaging*) is the best answer. While something "volatile" can be damaging, in this context it means *changeable* or *unsteady*, which isn't the meaning we need. "Disparate," meaning *distinct*, doesn't make sense. For the last blank, we need a noun that means something like "fondness"; "predilection" (*a preference for something*) is the best answer. "Anachronism" (*mistake in portraying something in the wrong time period*) and "abeyance" (*state of not being used, e.g. of a building*) don't work. The answer is "abstain from . . . deleterious . . . predilection."

7

Which of the following is implied by the underlined sentence?

A. Any plant that can use a C3 pathway can use a C4 pathway.

B. Any plant that can use a C4 pathway can use a C3 pathway.

C. Only C4 plants contain RuBisCo.

D. Only C3 plants contain RuBisCo.

Sentences 5 and 6 are underlined:

> [5] Certain plants have evolved and can use a C4 pathway instead of a C3 pathway when it is more efficient to do so. [6] The C4 pathway results in the saturation of a particular enzyme, RuBisCo, with carbon dioxide.

The conclusions listed in the answer choices fall into two categories: A and B relate C3 and C4 pathways, and C and D relate specific pathways to containing RuBisCo. Nothing is stated in the selection that suggests that only a specific subset of plants contain RuBisCo; the difference between the C3 and C4 pathways here is only whether RuBisCo is saturated or not. So, neither C nor D can be correct. The first underlined sentence—specifically the phrase "instead of" and "when it is more efficient to do so"—tells us that only part of the set of plants that can use a C3 pathway can use a C4 pathway, making B the correct answer.

8 C4 plants are similar to CAM plants in that _____, but different in that _____.

A. they have both evolved to increase photosynthetic efficiency by saturating RuBisCo with carbon dioxide . . . they gather that carbon dioxide at different times of day

B. they both close their stomata during the day . . . CAM plants perform only the light-dependent photosynthetic reactions during the day whereas C4 plants perform only the light-independent reactions during the day

C. they are both well suited to arid environments . . . C4 plants use RuBisCo, but CAM plants lack RuBisCo

D. they both use evolutionary adaptations to perform photosynthesis more efficiently . . . they use different mechanisms to do so

The passage discusses C4 plants as one of the "few different carbon-fixation pathways plants can take," and it then introduces CAM plants as yet another pathway: "CAM plants have evolved a different method of making photosynthesis more efficient." We're not told that either type of plant lacks RuBisCo, though we're told that CAM plants saturate the enzyme and C4 plants don't. This knocks out answer choices A and C. Only CAM plants open and close their stomata at different points in the day, so B isn't correct either. D is correct: both C4 and CAM plants have adapted to perform photosyntheiss more efficiently, but they use different mechanisms (pathways) to do so.

----

9 You're talking to a friend about what you learned in the passage when your friend says, "But isn't photosynthesis just one cycle? Why would it function differently in the day than at night?" Which sentence most directly answers your friend's questions?

Sentence 16: "The reactions involved in photosynthesis fall into two categories: the 'light' reactions require light, and the 'light-independent' or 'dark' reactions do not."

To answer your friend's question, you need to provide information about why photosynthesis would function differently during the day and during the night. The correct sentence is the one that explains that photosynthesis is composed of reactions that can be divided into light-dependent and light-independent categories. If some of the reactions involved in photosynthesis rely on the presence of light, this would explain why photosynthesis would necessarily function differently in the day and in the night.

10 The musician's attempts to brighten the mood continually failed, as all his songs sounded like _____.

- ☐ encomiums
- ■ dirges
- ☐ waltzes
- ☐ jigs
- ☐ laments
- ■ threnodies

"Waltzes" and "jigs" are both types of dances or songs, but both terms have positive or at least neutral connotations, and we need terms with sad connotations. "Laments" and "encomiums" capture this sad connotation, but are not specifically types of songs. "Dirges" and "threnodies" hit both required points: they both have sad connotations and are specifically types of songs, making them the best answer choice pair.

---

11 With a handlebar mustache and a loping walk, the reenactor _____ the look of a nineteenth-century baseball player; he seemed to have stepped right out of a previous era.

- ☐ subverted
- ■ epitomized
- ☐ underscored
- ☐ mocked
- ■ exemplified
- ☐ satirized

The characteristics described at the start of the sentence and the clause following the semicolon tell us that the baseball player is an accurate reenactor. We're not given any clues that suggest that he is making fun of the appearance of nineteenth-century baseball players, so even though "mocked" and "satirized" are close synonyms, they are not the correct answers, and we can ignore "subverted" for the same reason. "Underscored" simply means *emphasized*, so it doesn't make much sense in the context of the sentence. This leaves us with the correct answers, "epitomized" and "exemplified." These terms both mean *acted as a great example of*, which fits the sentence's context perfectly.

---

12 The author is primarily concerned with
_____.

A. exploring the contribution of philosophy to discussions of poetic method and appreciation

B. enumerating the reasons why Wittgenstein and his successors were misguided in their philosophical approach

C. arguing that given the current trajectory of philosophy, poetry will soon no longer be studied in mainstream society

D. describing the mainstream marginalization of poetry among philosophers of a certain period before noting significant exceptions

The first paragraph states the main argument: that as positivism grew more popular, poetry had less of a place in the study of philosophy. This can be gleaned from the first and last sentence of the first paragraph. The second paragraph introduces a contrast with the word "yet" and then proceeds to enumerate three examples of philosophers who made poetry a part of their philosophical method. Thus, the author is concerned with this general movement and countermovement centered on whether poetry is significant to philosophy. A gets the "direction" of this interest wrong, suggesting that the author is interested in how philosophy applies to poetry, not how poetry applies to philosophy. B suggests that the passage's main point is to critique Wittgenstein, when the author presents Wittegenstein, his successors, and the counterclaims put forward by Heidegger et. al. in the same tenor as he focuses on presenting an unbiased view of the historical progression of philosophy on a certain topic (poetry). So, B isn't correct either. C argues that poetry will soon no longer be studied at all, when the author's entire second paragraph focuses on philosophers who see poetry as a relevant part of their work. As A, B, and C are incorrect, D must be the correct answer, and it is: in the passage, the author describes the marginalization of poetry and significant exceptions. D captures the point-and-counterpoint structure of the passage well.

13 Select the sentence in the passage in which the author contrasts the position of a group of philosophers against those who followed Wittgenstein.

> Sentence 5, "Yet some thinkers, like Martin Heidegger, reacted strongly to the pretensions of an instrumental theory of knowledge to make sense of the world."

The author first mentions Wittgenstein's successors in Sentence 4, so our sentence must necessarily follow this one, as it introduces the group being contrasted against. Sentence 5, the transition into the second paragraph, introduces "some thinkers, like Martin Heidegger" in contrast to the position held by Wittgenstein and his supporters. Heidegger and thinkers like him think poetry is still relevant, we find, whereas Wittgenstein's successors do not see a place for poetry in philosophy. Sentence 5 introduces this contrast, so it is the correct sentence.

14 **Select all answers that apply.**

Which statements can be inferred from the passage?

A. Some of Wittgenstein's successors used his work to exclude something that was important to him.

B. Philosophers agree that instrumental theories of knowledge are sufficient in understanding the world.

C. Most positivists followed Wittgenstein in arguing for poetic knowledge as a meaningful category in philosophy.

In the first paragraph, we're told that Wittgenstein wanted to make room for poetry in his category of the mystical, but his successors simply abandoned it, citing his work. This supports answer choice A. B is disproven by the first sentence of the second paragraph, in which the author talks about how Heidegger and thinkers like him "reacted strongly to the pretensions of an instrumental theory of knowledge to make sense of the world." The "Pretensions" suggests that these thinkers see such theories of knowledge as insufficient. C is disproven by the passage's opening sentence: "As the logical positivism rose to ascendancy, poetic language was increasingly seen as merely emotive." Thus, positivism is introduced as being at odds with a view of poetry as significant, as is conveyed by the word "merely." Thus, even though you could select multiple answers, only A is correct.

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15 Having finally fixed the engine, the mechanic took a moment to listen to it _____.

■ purr   ☐ clank

■ hum   ☐ whine

☐ rattle   ☐ drone

We need words with positive connotations to complete this sentence. All of the answer choices describe different types of sounds that an engine could literally (e.g. "clank," "hum") or figuratively ("purr," "whine") make, but only "purr" and "hum" have notably positive meanings that pair well with the mechanic's long-worked-for success in repairing the engine conveyed by "finally."

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16 Frequently, beginning pilots find themselves unable to trust their instruments, _____ them in the air and leaving them upside down.

☐ stranding   ☐ reassuring

■ bewildering   ■ mystifying

☐ informing   ☐ misinforming

The best answers for this question are those words that convey a sense of confusion that could "leave [pilots] upside down" "in the air" after they don't trust their instruments. While "stranding" and "misinforming" each make sense, they don't pair with any of the other words to create sentences that have the same meaning. The terms "bewildering" and "mystifying" are close synonyms that also fit the tenor of the sentence, so these are the best answer choices.

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17 With which of the following assertions would the author most likely agree?

A. The appeal of art focused on abstract emotion will outlast that of art focused on representing historical events.

B. A religion that involved no emotion could nevertheless be popular in the right cultural milieu.

C. Poetry should only be used to describe fanciful, unrealistic events.

D. While certain dogmas may fade as time progresses, we're likely to see others gain a stronger cultural hold.

To infer with which position an author might agree, we have to understand his or her argument in the passage and how it relates to each of the suggested topics. Religion is discussed in the context of what it does with its emotion: according to the author, it "has attached its emotion to the fact, and now the fact is failing it." It seems that the author sees emotion as a part of religion; nowhere in the passage does he suggest that a religion without emotion could be popular (B). He aligns poetry with "the idea" instead of "the fact," but doesn't suggest that certain topics it should or should not address (C). His discussion of "the idea" merely gets at the abstract concepts discussed in poetry, and these aren't declared to be necessarily fantastic and fictional.

Poetry's "ever finding a surer and surer stay" is contrasted against "There is not . . . an accredited dogma which is not shown to be questionable." Nowhere does the author suggest that some dogmas will last whereas others won't; he lumps them all together in suggesting they won't last. The author does claim that poetry will last because it is baed on abstract emotion, whereas other traditions based on "the supposed fact" will fade away. (A) transfers this same argument to art, so it is the best answer.

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18 The author contrasts poetry most strongly against which of the following?

A. Ideas

B. Facts

C. Religion

D. Emotion

This question can be tricky because multiple concepts are contrasted with one another and alligned with one another in the passage, despite its brevity. "Ideas" and "facts" may jump out at you because they are involved in contrasts, but they are not contasted against poetry—they're contrasted against one another. Poetry is associated with "ideas" and religion is associated with "facts," and "emotion" (D) is seen as a component part of both poetry and religion both. Look at the contrast that the idea-fact contrast brings into focus, though: poetry and ideas are contrasted against religion and facts. Poetry is thus contrasted most strongly against religion, making (C) the correct answer.

19 Which of the following is the best paraphrase of the underlined sentence "Poetry attaches its emotion to the idea; the idea is the fact?"

A. Poetry derives its effects from abstract concepts, not physical realities.

B. The emotions felt most strongly by readers of poetry are those inspired by unrealistic situations.

C. When reading poetry, readers are expected to suspend their disbelief and pretend that even outrageous scenarios could potentially occur.

D. Poetry places more importance on the novelty of the concepts discussed than on their objective truth value.

The author contrasts "the idea" against "the fact" in the parts of the passage leading up to this statement. Both are generalities. Earlier in the passage, the writer states, "Our religion has materialized itself in the fact, in the supposed fact; it has attached its emotion to the fact, and now the fact is failing it." This sentence contrasts poetry against religion; it suggests that poetry, unlike religion, depends upon abstract ideas for its emotion, not on reality. Thus, A is the best answer.

---

20 The author's use of the underlined phrase "the supposed fact" accomplishes which of the following?

A. It suggests that religion and poetry are not mutually exclusive.

B. It reprises the author's point that peceived facts are subject to revision as more is learned.

C. It introduces the idea that a reliance on facts might be somehow subpar to a reliance on emotions.

D. It suggests that some people perceive as facts ideas to which they have attached strong emotions.

The author states, "Our religion has materialized itself in the fact, in the supposed fact; it has attached its emotion to the fact, and now the fact is failing it." This contrasts religion against poetry, which the author says "will find an ever surer and surer stay," so (A) can't be correct. Similarly, it does not conflate the ideas of emotions and facts (D); these present a consistent, strong divide throughout the passage. To answer this question, we need to consider the sentence that precedes the indicated one: "There is not a creed which is not shaken, not an accredited dogma which is not shown to be questionable, not a received tradition which does not threaten to dissolve." We can't correctly claim that the phrase "the supposed fact" introduces the idea that a reliance on fact might be worse than a reliance on emotions (C). This work has already been done by the preceding sentence. Instead, it "reprises" the author's point that perceived facts are subject to revision over time (B). This captures the gist of the preceding sentence.

For each question, indicate the best answer using the directions given. If a question has answer choices with ovals, then the correct answer consists of a single choice. If a question has answer choices with square boxes, then the correct answer consists of one or more answer choices. Read the directions for each question carefully.

Text Completion: For each blank, select one entry from the corresponding column of choices. Fill all blanks in the way that best completes the text.

1 His demeanor was considered _____ due to his aloof manner and biting comments.

gentlemanly
haughty
froward
exemplary
peevish

2 Critics dismissed the new TV show as _____, calling it "a retread of every show of its kind we've seen in the last few years."

putative
dull
fresh
invigorating
banal

3 Although the wine aficionado _____ Tori for her love of rosé, he did appreciate her for her _____ the opinions of a fellow wine critic, with whom he always disagreed.

Blank (i)	Blank (ii)
adored	support of
rebuked	concern for
excused	lambasting of

4 The electrician was finally _____ for his work, though the client had originally tried to _____ him.

Blank (i)	Blank (ii)
recognized	bilk
billed	extort
remunerated	reimburse

5 With his black eye, ripped jacket, and gruff demeanor, the grimacing bodyguard had a _____ look about him that convinced Jaime that this was not a person to _____ lightly.

Blank (i)	Blank (ii)
stoic	gainsay
minatory	disabuse
intransigent	encourage

6 While Ellen's friend-group tended to be studious and hardworking, she was not as _____ when it came to her schoolwork and often _____ her academics to pursue her private hobbies. Although her friends tried to appear _____ when she informed them that she had been accepted to a top-tier university, it was difficult for them to conceal their shock.

Blank (i)	Blank (ii)	Blank (iii)
sedulous	underpinned	discomfited
perfunctory	shirked	nonplussed
hidebound	validated	unperturbed

7 Often a person who is a mere _____ can appear to have a vast _____ of knowledge when he or she has a merely superficial grasp of _____ topics.

Blank (i)	Blank (ii)	Blank (iii)
pundit	assimilation	elementary
dilettante	repertoire	recurrent
greenhorn	focus	sundry

Questions 8–12 are based on the following passage.

1 The word "blue," say certain philosophers, means the sensation of color that the human eye receives in looking at the sky. **2** Now, say they further, as this sensation can only be felt when the eye is turned to the object, and as, therefore, no such sensation is produced by the object when nobody looks at it, therefore the thing, when it is not looked at, is not blue; and thus (say they) there are many qualities of things which depend as much on something else as on themselves. **3** The qualities of things that depend upon our perception of them, and upon our human nature as affected by them, metaphysicians call "subjective"; and the qualities of things which they always have, irrespective of any other nature, as roundness or squareness, they call "objective."

4 Now, the word "blue" does not mean the sensation caused by a <u>gentian</u> on the human eye, but it means the power of producing that sensation; and this power is always there, in the thing, whether we are there to experience it or not. **5** Precisely in the same way, gunpowder has a power of exploding. **6** It will not explode if you put no match to it. **7** <u>But it has always the power of so exploding, and is therefore called an explosive compound, which it very assuredly is, whatever philosophy may say to the contrary.</u>

8 Hence I would say to these philosophers: if, instead of using the sonorous phrase, "It is objectively so," you will use the plain old phrase "It is so," and if instead of "It is subjectively so," you will say, in plain old English, "It does so" or "It seems so to me," you will be more intelligible to your fellow-creatures; and besides, if you find that a thing which generally "does so" to other people does not so to you, on any particular occasion, you will not fall into the impertinence of saying that the thing is not so, or did not so, but you will say simply that something is the matter with you. **9** If you find that you cannot explode the gunpowder, you will not declare that all gunpowder is subjective, and all explosion imaginary, but you will simply suspect and declare yourself to be an ill-made match.

8 Based on the way the word is used in Sentence 4, what is a "gentian"?

A. A feature of an object that the object has regardless of human interaction

B. The ability to produce a subjective sensation in a viewer

C. Something blue

D. A type of thought experiment often employed in philosophical debates

9 **Select all answers that apply.**

Sentence 7 does which of the following?

A. It draws a sharp distinction between two ways of defining physical qualities.

B. It introduces the author's opposing argument.

C. It uses a familiar example to make metaphysicians look foolish.

10 **Select all answers that apply.**

Which of the following criticisms does the author raise against metaphysicians?

A. They incorrectly assume that their perspective is is universally applicable.

B. They couch their claims in language that is hard to understand.

C. They refuse to recognize that they have no evidence to support their claims.

11 The author's use of "say they" in a parenthetical in Sentence 2 _____.

    A. helps clarify to which of two groups of philosophers the argument at hand is attributed

    B. reminds the reader that metaphysicians have no evidence for the claims being presented

    C. emphasizes that the argument he's laying out is not his own

    D. helps the author contrast what a certain group says and what it does

12 Which of the following questions most closely parallels the passage's debate?

    A. If you keep dividing a thing in half, at what point can it be observed to no longer have the qualities of the original thing?

    B. How can we be sure that a source causes the same sensory experience in one person as in another?

    C. Can you ever absolutely know something to be true?

    D. If a tree falls in the woods and no one is around to hear it, does it make a sound?

Sentence Equivalence: Select the two answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole and produce completed sentences that are alike in meaning.

---

13 The teacher's lectures tended to _____ instead of delving into the grand ideas of history.

    ☐ minutiae

    ☐ ontologies

    ☐ esoterica

    ☐ hermeneutics

    ☐ hypotheses

    ☐ abstractions

---

14 The report put the _____ of the problem on the city's police and minimized the fault of other groups.

- [ ] onus
- [ ] solving
- [ ] focus
- [ ] burden
- [ ] resolution
- [ ] exegesis

15 Caught off guard, Alfred ventured a guess, which to his relief, his professor _____.

- [ ] validated
- [ ] derided
- [ ] corroborated
- [ ] ignored
- [ ] dismissed
- [ ] queried

16 The talk show host drew _____ from critics for his harshly confrontational interviews that were often painful to watch.

- [ ] ire
- [ ] acclimation
- [ ] suspicion
- [ ] ardor
- [ ] flak
- [ ] praise

17 Her answers rarely _____ the facts of the case, increasing the police's suspicion.

- [ ] jibed with
- [ ] proved
- [ ] led to
- [ ] dovetailed with
- [ ] opposed
- [ ] differed from

Questions 18–20 are based on the following passage.

^{1} Peculiarities in the material I have used to elucidate the interpretation of dreams have rendered this publication difficult. ^{2} The work itself will demonstrate why all dreams related in scientific literature or collected by others had to remain useless for my purpose. ^{3} In choosing my examples, I had to limit myself to considering my own dreams and those of my patients who were under psychoanalytic treatment. ^{4} I was restrained from utilizing material derived from my patients' dreams by the fact that during their treatment, the dream processes were subjected to an undesirable complication—the intermixture of neurotic characters. ^{5} On the other hand, in discussing my own dreams, I was obliged to expose more of the intimacies of my psychic life than I should like, more so than generally falls to the task of an author who is not a poet but an investigator of nature. ^{6} This was painful, but unavoidable; I had to put up with the inevitable in order to demonstrate the truth of my psychological results at all. ^{7} To be sure, I disguised some of my indiscretions through omissions and substitutions, though I feel that these detract from the value of the examples in which they appear. ^{8} I can only express the hope that the reader of this work, putting himself in my difficult position, will show patience, and also that anyone inclined to take offense at any of the reported dreams will concede freedom of thought at least to the dream life.

Passage adapted from "Introductory Remarks" in *The Interpretation of Dreams* by Sigmund Freud (trans. 1913)

18 What evidence does the author offer to justify his choice to omit dreams described in scientific publications?

A. He doesn't give direct evidence, but he suggests that his text will explain this decision.

B. Those dreams came from patients diagnosed with various psychological conditions, so they introduced additional uncontrolled variables.

C. Scientific publications edit the dreams they publish and may omit parts that would greatly change the author's interpretation of said dreams.

D. The author had to interview people about their dreams to make sure that the questions people were prompted with were unbiased.

19 Select the sentence in which the author most directly elicits the reader's sympathy.

Sentences are not numbered on the GRE; you click them. Here, each sentence is preceded by a superscript number to aid you in recording and checking your answers to select-in-passage questions.

20 **Select all answers that apply.**

The passage supports which of these inferences?

A. In the work that follows, the author discusses embarassing things that occurred in his dreams and leaves nothing out.

B. The author believes himself to have no "intermixture of neurotic characters."

C. Some readers might be offended by the content of some of the dreams the author discusses.

Answer Key: Verbal Reasoning Section #2

	Answer	Section / Concept Tested
1	haughty	*Single-Blank Text Completion — Positives and Negatives*
2	banal	*Single-Blank Text Completion — Agreement*
3	(i) rebuked (ii) lambasting of	*Two-Blank Text Completion — Comparisons*
4	(i) remunerated (ii) bilk	*Two-Blank Text Completion — Cause and Effect*
5	(i) minatory (ii) gainsay	*Two-Blank Text Completion — Agreement*
6	(i) dilettante (ii) repertoire (iii) sundry	*Three-Blank Text Completion — Agreement*
7	(i) sedulous (ii) shirked (iii) nonplussed	*Three-Blank Text Completion — Comparisons*
8	C	*Mutliple-Choice with One Answer — Word Usage in Context*
9	A and C	*Mutliple-Choice with Multiple Answers — Text Structure*
10	A and B	*Mutliple-Choice with Multiple Answers — Main Idea, Meaning, Purpose*
11	C	*Mutliple-Choice with One Answer — Tone and Style*
12	D	*Mutliple-Choice with One Answer — Inferences, Conclusions, Applications*
13	minutiae, esoterica	*Sentence Equivalence*
14	onus, burden	*Sentence Equivalence*
15	corroborated, validated	*Sentence Equivalence*
16	censure, flak	*Sentence Equivalence*
17	jibed with, dovetailed with	*Sentence Equivalence*
18	A	*Mutliple-Choice with One Answer — Strength of Support and Evidentiary Choices*
19	Sentence 8	*Text Selection — Perspective and Bias*
20	B and C	*Mutliple-Choice with Multiple Answers — Inferences, Conclusions, Applications*

1 His demeanor was considered _____ due to his aloof manner and biting comments.

gentlemanly
haughty
froward
exemplary
peevish

For this sentence, we're looking for a negative term that could characterize someone with an "aloof manner" and who offers "biting comments," neither a good thing. "Gentlemanly" and "exemplary" are too positively-connoted to be correct, whereas "froward" (*stubbornly oppositional*) and "peevish" (*short-tempered*) are each negative, but don't relate to the described characteristics. The best answer is "haughty," which means *arrogantly condescending*. This word best pairs with the described characteristics.

---

2 Critics dismissed the new TV show as _____, calling it "a re-tread of every show of its kind we've seen in the last few years."

putative
dull
fresh
invigorating
banal

We can tell from the word "dismissed" and the description provided for the new TV show that we're looking for a negative word to go in the blank. "Fresh" and "invigorating" are positive terms, and besides, it wouldn't make much sense to call a show being dubbed a "re-treat" either "fresh" or "invigorating." "Dull" might work, but the term is a bit general; let's consider the other answer options. "Putative," a synonym of "supposed," doesn't make sense in the sentence's context, but "banal" (*clichéd, overdone*) fits the sentence well, incorporating not only a negative connotation but also a specific meaning that supports the TV show's specific description, which focuses on its unoriginal elements.

---

3 Although the wine aficionado _____ Tori for her love of rosé, he did appreciate her for her _____ the opinions of a fellow wine critic, with whom he always disagreed.

Blank (i)
adored
rebuked
excused

Blank (ii)
support of
vexation of
lambasting of

In this sentence, "although" the wine aficionado performed the first blank's action towards Tori, he then "appreciated" her for other actions. In other words, he "appreciated" her in spite of whatever action he took towards her in the first blank. Therefore, "adored" doesn't quite fit, as "adored" and "appreciated" are not opposites. "Excused" has a similarly positive connotation, so it doesn't work well in contrasting against "appreciated." "Rebuked," though, means *reproved sharply*; therefore, the later appreciation/approval that Tori receives makes "rebuked" fit.

Considering the second blank, the wine aficionado "appreciated" Tori for her doing something in regard to the opinions of a rival, so we know that Tori does not agree with that particular critic's opinions. Both "lambasting" and "vexation" are negative, but "lambasting" (*thoroughly criticizing*) requires direct action by Tori that can act on the opinions of the wine critic, whereas "vexation" means that Tori directly vexes (*annoys/irritates*) the wine critic's opinions, which doesn't make much sense grammatically; therefore, "rebuked . . . lambasting of" is the correct answer.

---

4 The electrician was finally _____ for his work, though the client had originally tried to _____ him.

Blank (i)
recognized
billed
remunerated

Blank (ii)
bilk
extort
reimburse

"Though" and "originally" are working together in this sentence to create a logical hinge in which the first blank happened despite the fact that the second blank was attempted. The sentence concerns an electrician and his client, so "recognized" doesn't make much sense in this context; in addition, none of the options for the second blank contrast against it in the way that we're looking for. "Billed" and "remunerated" both have to do with money, but it would be the client who would be "billed," not the electrician,

who in this case is the service provider who would be collecting the money. "Remunerated" (*paid for serviecs rendered*) works best for the first blank. All of the options for the second blank have to do with money, so we'll have to look at the specifics of each word's meaning. It wouldn't make sense to say that the electrician was finally paid for his work though the client had tried to reimburse him, as "reimburse" means *give money back for something*. This leaves us with "bilk" and "extort." They're similar in meaning, but "bilk" is the better option; "extort" means *to leverage power to gain money or something valuable*, whereas "bilk" means *get money from someone or refuse to give money to someone—to defraud*. Since our specific circumstance is that the client withheld money, "bilk" better fits the sentence's context.

5

With his black eye, ripped jacket, and gruff demeanor, the grimacing bodyguard had a _____ look about him that convinced Jaime that this was not a person to _____ lightly.

Blank (i)	Blank (ii)
stoic	**gainsay**
minatory	disabuse
intransigent	encourage

What sort of word might go in the first blank? We need something that means something like "threatening" or "imposing." "Stoic" means *emotionally calm* and "intransigent" means *stubborn*, but "minatory" means *threatening*, so it's the best choice for the first blank. What might you not want to do to a threatening bodyguard? "Encourage" doesn't fit the sentence's context. "Disabuse" might look potentially correct because of how negative the word looks, composed of the negative prefix "dis-" and "abuse," but the term actualy means *to convince someone that something they believe is not correct*. Grammatically, you disabuse people of ideas or notions, but you don't just "disabuse people"; that doesn't make sense. The best answer for the second blank is "gainsay," which means *contradict, challenge,* or *oppose*. It makes sense to conclude that you would not want to contradict a menacing-looking bodyguard unless you absolutely had to.

6

Often a person who is a mere _____ can appear to have a vast _____ of knowledge when he or she has a merely superficial grasp of _____ topics.

Blank (i)	Blank (ii)
pundit	assimilation
dilettante	**repertoire**
greenhorn	focus

Blank (iii)

elementary
recurrent
sundry

To find the best terms for this sentence, we need to unravel its complex chain of logic. At its core is a contrast: a person can appear one way, but that gives a false picture of his or her abilities. The first blank's options offer us two reasonable choices: while "pundit" doesn't make sense, "dilettante" (*one who dabbles in many different areas without becoming an expert*) and "greenhorn" (*newbie*) each seem reasonable. Let's see which meshes better with the rest of the sentence. The second blank is preceded by the adjective "vast." It doesn't make sense to have a "vast assimilation" or a "vast focus," but it does make sense to have a vast "repertoire" (*collection of acquired skills or knowlege*). Now we're getting somewhere! A person might appear to have a vast repertoire of knowledge when he or she actually only has grasped what kind of topics? To appear to have a "vast repertoire," we'll need a word that involves breadth of knowledge. This makes "sundry" (*varied*) the best answer. Now we can go back to the first blank and pick out the best answer: "dilettante" is more closely related to the concept of breadth of knowledge than is "greenhorn."

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7   While Ellen's friend-group tended to be studious and hardworking, she was not as _____ when it came to her schoolwork and often _____ her academics to pursue her private hobbies. Although her friends tried to appear _____ when she informed them that she had been accepted to a top-tier university, it was difficult for them to conceal their shock.

Blank (i)	Blank (ii)
sedulous	underpinned
perfunctory	**shirked**
hidebound	validated

Blank (iii)

nonplussed
unflustered
unperturbed

These sentences present two contrasts to sort through. The first sentence contrasts Ellen's study habits against those of her friend-group, and the second sentence is an "although" sentence telling us that her friends tried to appear a certain way but that it was hard for them "to conceal their shock." Let's start with the last blank, since there are two blanks in the first sentence and only one in the second sentence. Ellen's friends want to "conceal their shock" but have a hard time doing so, so which word describes how they want to appear? We need a word that means something like "calm." "Nonplussed" means *shocked*, so that's the opposite of the term we're looking for. While "unperturbed" works, it suggests that her friends were trying to look like they weren't annoyed (perturbed), not like they weren't shocked. The best answer is "unflustered," which directly relates to emotional instability and shock ("flustered").

Now let's turn our attention to the first sentence's blanks. Why might Ellen's friends be shocked that she got into a top university? Ellen's study habits are contrasted against her friends' "studious and hardworking" habits. Ellen's are "not as" something, so we need a word that means something like "studious and hardworking." "Sedulous" (*dedicated, hardworking*) is the best option. The second blank tells us that Ellen did something to her academics in favor of focusing on her hobbies. She probably ignored her academics in some way. The only word that makes sense in this context is "shirked."

---

8   Based on the way the word is used in Sentence 4, what is a "gentian"?

   A. A feature of an object that the object has regardless of human interaction

   B. The ability to produce a subjective sensation in a viewer

   C. Something blue

   D. A type of thought experiment often employed in philosophical debates

The word "gentian" appears in the first sentence of the second paragraph: "Now, the word 'blue' does not mean the sensation caused by a <u>gentian</u> on the human eye, but it means the power of producing that sensation." We have a lot of other concepts to sort through here, but we can isolate "gentian" in the phrase "Now, the word 'blue' does not mean the sensation caused by a <u>gentian</u> on the human eye, but it means . . ." The logic here gets tricky, but the author is using a gentian as an example of something blue to get at the difference between the sensation and the ability to produce it. C is the best answer.

9 **Select all answers that apply.**

Sentence 7 does which of the following?

A. It draws a sharp distinction between two ways of defining physical qualities.

B. It introduces the author's opposing argument.

C. It uses a familiar example to make metaphysicians look foolish.

Consider the underlined sentence in context:

> Precisely in the same way, gunpowder has a power of exploding. It will not explode if you put no match to it. But it has always the power of so exploding, and is therefore called an explosive compound, which it very assuredly is, whatever philosophy may say to the contrary.

This selection appears in the second paragraph, where the author is drawing a distinction between sensations (e.g. something looking blue) and the ability to cause those sensations in onlookers. He introduces the gunpower example after an example about a "gentian" (a type of flower). This second example works to help the author distinguish between a sensation and the ability to cause it; (A) correctly summarizes this argumentative situation. (B) isn't correct; if we wanted to discuss an "opposing argument," it would be the ideas discussed in the first paragraph, so the underlined sentence wouldn't be "introducing" it. Considering (C), look at the end of the sentence, specifically at the phrase "whatever philosophy may say to the contrary." Here, the author makes it seem as if philosophy would argue that gunpowder doesn't explode—a rather silly conclusion that everyone can recognize as silly because the example is so obvious and relatable. (C) is thus also correct.

---

10 **Select all answers that apply.**

Which of the following criticisms does the author raise against metaphysicians?

A. They incorrectly assume that their perspective is is universally applicable.

B. They couch their claims in language that is hard to understand.

C. They refuse to recognize that they have no evidence to support their claims.

At the start of the last paragraph, the author recommends that metaphysicians use simpler language, supporting (B). He goes on to suggest that "if you find that a thing which generally 'does so' to other people does not so to you, . . . you will not fall into the impertinence of saying that the thing is not so, or did not so, but you will say simply that something is the matter with you." This in turn supports (A). (C) is perhaps most tricky, because the author's claims such as those about gunpowder in the second paragraph might appear to support this answer choice; however, nowhere does the author make his conflict with the metaphysicians about evidence specifically. They're just characterizing sensations in a certain way—a way with which the author vehemently disagrees. He may think their ideas are silly, but their definitions are silly because they're unrealistic, in the author's perspective, not because they lack evidence.

11 The author's use of "say they" in a parenthetical in Sentence 2 _____.

A. helps clarify to which of two groups of philosophers the argument at hand is attributed

B. reminds the reader that metaphysicians have no evidence for the claims being presented

C. emphasizes that the argument he's laying out is not his own

D. helps the author contrast what a certain group says and what it does

The parenthetical to which this question is referring is underlined below.

² Now, say they further, as this sensation can only be felt when the eye is turned to the object, and as, therefore, no such sensation is produced by the object when nobody looks at it, therefore the thing, when it is not looked at, is not blue; and thus (say they) there are many qualities of things which depend as much on something else as on themselves.

Note how the author begins the sentence with "say they further." In the structure of the passage overall, he is here relaying the claims of philosophers with whom he does not agree.

He only discusses one group of philosophers—metaphysicians—so A cannot be correct. He's also not underscoring a difference between metaphysicians' statements and acts, so D isn't correct either. This leaves B and C. The author has not claimed that metaphysicians have no evidence for their claims (B); he's just arguing that they are wrong. He discusses a few counterclaims in paragraph two, but his earlier use of "say they" emphasizes that while the author is describing a line of argument in detail, he does not adhere to it himself. C is correct.

---

12 Which of the following questions most closely parallels the passage's debate?

A. If you keep dividing a thing in half, at what point can it be observed to no longer have the qualities of the original thing?

B. How can we be sure that a source causes the same sensory experience in one person as in another?

C. Can you ever absolutely know something to be true?

D. If a tree falls in the woods and no one is around to hear it, does it make a sound?

This sentence requires you to distill the overall debate the passage treats and select a question that parallels that debate. This can be tricky because you can't point to one specific point in the passage that gives you the answer; you have to summarize the entire thing. The author discusses one way of looking at sensation and the ability to cause sensation before claiming that suggests that a human observer is necessary for "subjective" characteristics and not for "objective" ones. The author then argues that "subjective" characteristics do not actually depend on a human observer; e.g. a flower will be blue even if you're not looking at it, and gunpowder is flammable even when not exploding. Sensory qualities are mentioned in answer chocies B and D, but only D gets at the idea of a human observer determining how sensory characteristics—in this case, a parallel example of sound made—exist when no observer is present. D is correct.

13 The teacher's lectures tended to _____ instead of delving into the grand ideas of history.

- ■ minutiae
- ☐ ontologies
- ■ esoterica
- ☐ hermeneutics
- ☐ hypotheses
- ☐ abstractions

We need to pick out a noun for the blank that contrasts well against "the grand ideas of history." The correct answers, "minutiae" and "esoterica," both describe relatively unimportant small-scale details, the scale of which contrasts against the "grand" scale of the other ideas.

---

14 The report put the _____ of the problem on the city's police and minimized the fault of other groups.

- ☐ solving
- ■ onus
- ☐ focus
- ■ burden
- ☐ resolution
- ☐ exegesis

The word that goes in the blank has to mean something along the lines of "brunt" or "majority," as the police were seen as being at fault more than the other groups. "Focus" might seem like a decent answer, but "onus" and "burden" are the best pair. Both refer to *weighty responsibility*.

---

15 Caught completely unaware by his professor's question, Alfred timidly ventured a guess, which much to his relief, his professor _____.

- ☐ queried
- ☐ derided
- ■ corroborated
- ☐ ignored
- ☐ dismissed
- ■ validated

"Queried" means *questioned*; if the professor asked follow-up questions about Alfred's question, this would not necessarily reassure Alfred. If we were to choose "derided," the professor would have *made fun of* Alfred's answer, and Alfred would once more probably not be relieved. We might be tempted to choose "dismissed" and "ignored" since they are close in meaning, but neither of these fit the context of the sentence and would reasonably cause Alfred to feel relieved. "Corroborated" and "validated," though, are much better options. Both imply that the professor supported Alfred's answer, which would reasonably make Alfred feel relieved.

16 The talk show host drew _____ from critics for his harshly confrontational interviews that were often painful to watch.

- ☐ suspicion
- ☐ acclimation
- ■ censure
- ☐ ardor
- ■ flak
- ☐ praise

"Harshly confrontational interviews" don't exactly sound pleasant, so chances are, we'll need to pick out a pair of negative words. "Suspicion" is negative, but it doesn't make much sense in the sentence's context and none of the other answer options are similar in meaning to it. "Acclimation" and "praise" are similar in meaning, but have positive connotations that don't mesh well with the rest of the sentence. The best options are "censure" and "flak," both of which mean *strong criticism*.

17 Her answers rarely _____ the facts of the case, increasing the police's suspicion.

- ■ jibed with
- ☐ proved
- ☐ led to
- ■ dovetailed with
- ☐ opposed
- ☐ differed from

What relationship between a person's story and "the facts of the case" might result in the police growing more suspicious? If a story rarely matched the known facts, that would seem pretty suspicious. So, we need to pick out a word or phrase that means something like "accorded with" or "matched." It's easy to make a logical misstep and miss the negative reversal conveyed by "rarely"; if you did this, you might have picked "opposed" and "differed from." "Proved" and "led to" each convey too strong of a causal relationship between the person's story and the facts. The best answer is "jibed with" and "dovetailed with," both of which mean *matched* in this context.

---

18 What evidence does the author offer to justify his choice to omit dreams described in scientific publications?

A. He doesn't give direct evidence, but he suggests that his text will explain this decision.

B. Those dreams came from patients diagnosed with various psychological conditions, so they introduced additional uncontrolled variables.

C. Scientific publications edit the dreams they publish and may omit parts that would greatly change the author's interpretation of said dreams.

D. The author had to interview people about their dreams to make sure that the questions people were prompted with were unbiased.

(B) conflates the author's discussion of why he didn't use dreams described by his own patients with why he didn't use dreams from scientific literature. (C) conflates his discussion of editing the dreams he did use (his own) with the avoidance of using those published in scientific literature. (D) is not supported by the passage.

The best answer is (A). In Sentence 2, the author states, "The work itself will demonstrate why all dreams related in scientific literature or collected by others had to remain useless for my purpose." He doesn't give evidence here, but tells the reader that the evidence exists and that he presents it in "the work itself"—his text.

---

19 Select the sentence in which the author most directly elicits the reader's sympathy.

> Sentence 8: "I can only express the hope that the reader of this work, putting himself in my difficult position, will show patience, and also that anyone inclined to take offense at any of the reported dreams will concede freedom of thought at least to the dream life."

The author most directly elicits the reader's sympathy when he addresses the reader in Sentence 8. Here, Freud asks the reader to put him- or herself in his shoes and, understanding all of the difficulties he had to deal with and that he has discussed in the passage up until this point, give him a bit of leeway. If you weren't sure what was meant by "eliciting sympathy," this sentence would still be a reasonable selection because you know it has to do with the "most [direct]" version of a specific author-reader interaction.

---

20 **Select all answers that apply.**

The passage supports which of these inferences?

A. In the work that follows, the author discusses embarassing things that occurred in his dreams and leaves nothing out.

B. The author believes himself to have no "intermixture of neurotic characters."

C. Some readers might be offended by the content of some of the dreams the author discusses.

(A) is not correct because the writer contradicts this statement in Sentence 7. After discussing how he had to use only his own dreams as evidence, he adds, "I disguised some of my indiscretions through omissions and substitutions." Sentences 3–5 support (B):

> [3] In choosing my examples, I had to limit myself to considering my own dreams and those of my patients who were under psychoanalytic treatment. [4] I was restrained from utilizing material derived from my patients' dreams by the fact that during their treatment, the dream processes were subjected to an undesirable complication—the intermixture of neurotic characters. [5] On the other hand, in discussing my own dreams . . .

The author here states that he had to use either his patients' or his own dreams, and that he couldn't use his patients due to "the intermixture of neurotic characters." Since the author did not reject his own dreams for the same reason, this allows us to infer that the author doesn't think he has any "intermixture of neurotic characters."

Sentence 8, the concluding sentence of the passage, supports (C) when the author says, "I can only express the hope that . . . anyone inclined to take offense at any of the reported dreams will concede freedom of thought at least to the dream life. The author is taking the time to address readers that potentially might be offended by the dreams his work relates, so it's reasonable to make this inference, especially because the author himself makes it!

For each question, indicate the best answer using the directions given. If a question has answer choices with ovals, then the correct answer consists of a single choice. If a question has answer choices with square boxes, then the correct answer consists of one or more answer choices. Read the directions for each question carefully.

1

Quantity A	Quantity B
The y-intercept of the line $y = 3x - 4$	The x-intercept of the line $y - 3.5 = 0.5(x - 3)$

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

---

2 A car dealer sold two trucks for $40,000 each, resulting in a 25% profit on one car and a 20% loss on the other car.

Quantity A	Quantity B
The dealer's net gain	The dealer's net loss

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

---

3 In a particular seven-sided polygon, the sum of four equal interior angles, each equal to a degrees, is equivalent to the sum of the remaining three interior angles.

Quantity A	Quantity B
$a°$	$110°$

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

4

The arithmetic mean of a, b, c, and d is 14.

Quantity A	Quantity B
32	The arithmetic mean of $a + 3b + 2d$ and $a - b + 2c - 48$

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

5

Quantity A	Quantity B
0	$(-1)^{137}$

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

6

You have a 7.5-inch tall right cylindrical glass with a three-inch radius. You also have an ice cube tray that makes perfectly cubic ice cubes with half-inch sides. You put three ice cubes in your glass.

Quantity A	Quantity B
The volume of the ice cubes	The volume of soda that you can add to the glass without spilling

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

7

Quantity A	Quantity B
The minimum number of handshakes that can occur among a dozen people if each person only shakes each other person's hand once	The number of ways that three people out of five can be seated at a table

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

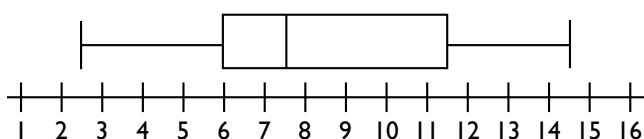○ The relationship cannot be determined from the given data.

---

8 If *v* is divisible by 2, 3, and 15, which of the following is also divisible by these numbers?

Multiple answers may be correct. Select all that apply.

A. $v + 5$
B. $v + 15$
C. $v + 20$
D. $v + 30$
E. $v + 90$

---

9 What is the interquartile range of the data depicted in the following box-and-whisker plot?

A. 5.0
B. 5.5
C. 6.5
D. 7.5

---

10 A small circle with radius 5 lies inside a larger circle with radius *x*. What is the area of the region inside the larger circle but outside of the smaller circle in terms of *x*?

A. $2\pi x - 5\pi$
B. $\pi x^2 - 5\pi$
C. $2\pi x - 25\pi$
D. $\pi x^2 - 25\pi$

---

11 If *m* and *n* are both odd integers, which of the following is or are not necessarily odd?

Multiple answers may be correct. Select all that apply.

A. $2m - n$
B. mn
C. $\dfrac{m + n}{2}$
D. $m^2 n$
E. $\dfrac{m^2 + n^2}{2}$

---

12 Satoshi and Reginald together have 46 bottlecaps. If they were to receive 6 bottle caps each, Satoshi would have three times as many bottle caps as Reginald. How many bottlecaps does Satoshi have?

On test day, you will be asked to type your answer into a blank text box. Note your response to check later.

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The following information applies to questions 13–15.

A study was conducted to determine the effectiveness of a getting a flu shot at preventing the flu. 1000 patients were studied: 500 who got a flu shot at least three months ago, and 500 who had not received a flu shot. The patients were then asked if they had caught the flu in the past two months.

Table 1: Number of Patients Who Caught The Flu

Patient Age Group	Vaccinated	Unvaccinated
Under 18	18	63
18–30	4	32
31–50	5	29
51–70	4	51
Over 70	19	75

13 In the study, a patient who received a flu shot was how much less likely to catch the flu than an unvaccinated patient?

A. 60% less likely

B. 65% less likely

C. 75% less likely

D. 80% less likely

14 Suppose the scientists who performed the study create a pie chart that includes all 1000 patients and that reflects a patient's odds of catching the flu depending on vaccination status and age group.

What would be the measure of the central angle of the portion of the chart representing vaccinated patients of all age groups who caught the virus?

A. 15°

B. 18°

C. 24°

D. 36°

15 The health department wants a public service announcement to focus on the age group with the greatest difference in percentage of people who got flu shots and caught the flu and people who did not get flu shots and caught the flu. On which of the following demographics should their public service announcement focus?

A. Under 18

B. 31–50

C. 51–70

D. Over 70

16   A traffic light hangs t feet from the ground, over a street. A man standing on the exterior edge of the shadow of the traffic light is h feet tall, and his shadow is s feet long. How far is the man standing from the spot on the street directly under the traffic light?

A. $\dfrac{Th}{s}$

B. $\dfrac{sT}{h}$

C. $\dfrac{hs}{T}$

D. hsT

---

17   150 seniors attend a high school. The school offers a philosophy class on ethics and one on metaphysics. There are 27 seniors in the ethics class and 32 in the metaphysics class. 95 seniors have elected to take neither class. How many seniors are enrolled in both philosophy courses this term?

On test day, you will be asked to type your answer into a blank text box. Note your response to check later.

---

18   If $a(x) = 2x^3 + x$, and $b(x) = -2x$, what is $a(b(2))$?

A. -132

B. 128

C. -503

D. 503

---

19   Find the length of a line segment whose endpoints are located at the coordinates $(7, -4)$ and $(-5, -1)$.

A. 12

B. 13

C. 14

D. 15

---

20   A tank containing 5,000 gallons of water springs a leak. It leaks one hundred gallons of water every fifteen minutes, but it is simultaneously filled at a rate of fifty gallons per hour. After how much time will the tank be empty?

A. 13.56 hours

B. 14.04 hours

C. 14.18 hours

D. 14.29 hours

End of Section   **STOP**   **End of Section**

Answer Key: Quantitative Reasoning Section #1

	Answer	Section / Concept Tested
1	C	*Algebra — Properties of Linear Graphs*
2	B	*Arithmetic — Percentages*
3	A	*Geometry — Angles, Parallel Lines, and Perpendicular Lines*
4	C	*Data Analysis — Central Tendency: Mean, Median, Mode*
5	A	*Algebra — Rules of Exponents and Variables*
6	B	*Geometry — Geometric Solids*
7	A	*Data Analysis — Factorials, Combinations, and Permutations*
8	D and E	*Arithmetic — Number Theory: LCM and GCF*
9	B	*Data Analysis — Range, Quartiles, Standard Deviation* *Data Analysis — Box-and-Whisker Plots*
10	D	*Geometry — Inscription and Circumscription*
11	C and E	*Arithmetic — Integers, Factors, and Basic Operations*
12	63	*Algebra — Systems of Linear Equations*
13	D	*Arithmetic — Percentages*
14	B	*Data Analysis — Circle Graphs and Pie Charts* *Geometry — Circles: Arcs, Chords, and Sectors*
15	C	*Arithmetic — Percentages*
16	B	*Geometry — Congruent Figures and Similar Figures*
17	4	*Data Analysis — Sets, Set Notation, and Venn Diagrams*
18	A	*Algebra — Function Notation*
19	B	*Geometry — Length and Midpoint*
20	D	*Algebra — Word Problems: Working with Rates*

1

Quantity A	Quantity B
The y-intercept of the line $y = 3x - 4$	The x-intercept of the line $y - 3.5 = 0.5(x - 3)$

○ Quantity A is greater.

○ Quantity B is greater.

● The two quantities are equal.

○ The relationship cannot be determined from the given data.

The key to these quantitative comparison problems is to figure out the worth of both quantities, or figure out whether evaluating the quantities is even possible. In this case, evaluating the quantities is a fairly straightforward case of figuring out the intercepts of two different lines, which is possible; therefore, you can already discount "the relationship cannot be determined from the given data."

Quantity A is in $y = mx + b$ form, where b is the y-intercept. Therefore, the y-intercept is 4.

To solve for the x-intercept of Quantity B, consider that the x-axis exists at the line $y = 0$, and therefore to find out where the line crosses the x-axis, you can set $y = 0$ and solve for x.

$$-3.5 = 0.5(x - 3)$$
$$-3.5 = 0.5x - 1.5$$
$$-2 = 0.5x$$
$$-4 = x$$

Both Quantity A and Quantity B equal –4; therefore the two quantities are equal.

---

2 A car dealer sold two trucks for $40,000 each, resulting in a 25% profit on one car and a 20% loss on the other car.

Quantity A	Quantity B
The dealer's net gain	The dealer's net loss

○ Quantity A is greater.

● Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

For the profitable car, the dealer bought it for x, where $1.25x = 40,000$. Therefore, $x = \$32,000$.

For the unprofitable car, the dealer bought it for y, where $0.80y = 40,000$. Therefore, $\$y = 50,000$.

Thus, the dealer's net profit is $8,000, and the dealer's net loss is $10,000. The car dealer's loss is greater than his profit, so Quantity B is larger.

3

In a particular seven-sided polygon, the sum of four equal interior angles, each equal to *a* degrees, is equivalent to the sum of the remaining three interior angles.

Quantity A	Quantity B
a°	110°

● Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

First, we need to figure out the sum of interior angles in a seven-sided polygon. We can use the formula $180°(n - 2)$, where *n* is the number of sides of the polygon.

$$180°(n - 2) = 180°(7 - 2) = 180°(5) = 900°$$

Three interior angles (call them *b*, *c*, *d*) are unknown, but we are told that the sum of them is equal to the sum of four other equivalent angles (which we'll designate *a*):

$$4a = b + c + d$$

Furthermore, all of these angles must sum up to 900°, so we can write out the following:

$$4a = b + c + d = 900°$$

We may not be able to find *b*, *c*, or *d*, indvidually, but the problem does not call for that, and we need only use their relation to *a*, as stated in the first equation with them. Utilizing this in the second equation, we find that *a* equals 112.5°, making Quantity A larger than Quantity B.

$$4a + 4a = 900°$$

$$a = \frac{900}{8} = 112.5°$$

4

The arithmetic mean of *a*, *b*, *c*, and *d* is 14.

Quantity A	Quantity B
32	The arithmetic mean of $a + 3b + 2d$ and $a - b + 2c - 48$

○ Quantity A is greater.

○ Quantity B is greater.

● The two quantities are equal.

○ The relationship cannot be determined from the given data.

$$\sum_{i=1}^{n} \frac{x_i}{n}$$

The definition of an arithmetic mean of a set of values is given as the sum of all the values divided by the total count of values, shown at left. Here, x_i represents the ith value in a set, and *n* is the number of values in the set.

Quantity B can thus be defined and simplified as follows:

$$\frac{(a + 3b + 2d) + (a - b + 2c - 48)}{2}$$

$$\frac{2a + 2b + 2c + 2d - 48}{2}$$

$$a + b + c + d - 24$$

We are told that the mean of a, b, c, and d is 14, which can be written as:

$$\frac{a + b + c + d}{4} = 14$$

and then as

$$a + b + c + d = 56$$

Plugging this value into our definition of Quantity B, we can find its numerical value: $56 - 24 = 32$, so Quantity A = 32 = Quantity B.

5	Quantity A	Quantity B	● Quantity A is greater.
	0	$(-1)^{137}$	○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

This question comes down to figuring out whether Quantity A is positive or negative—that is, whether our expression "lands" on a negative or a positive. Raising a negative number to an even power gets you a positive number, since a negative times a negative is a positive. Raising a negative number to an odd power gets you an odd number, since it'll be times one more multiple number than a positive even power. You can see this pattern emerge in counting up from a power of 2:

$(-1)^2 = -1 \times -1 = 1$

$(-1)^3 = (-1)^2 \times -1 = 1 \times -1 = -1$

$(-1)^4 = (-1)^2 \times (-1)^2 = 1$

$(-1)^5 = (-1)^2 \times (-1)^2 \times -1 = 1 \times 1 \times -1 = -1$

So, We don't actually need to mutliply –1 by itself 136 times to figure out the answer. Since 137 is an odd number, we know our answer will be negative: –1. –1 < 0, so Quantity A > Quantity B.

6 You have a 7.5-inch right cylindrical tall glass with a three-inch radius. You also have an ice cube tray that makes perfectly cubic ice cubes with 2.5-inch sides. You put seven ice cubes in your glass.

○ Quantity A is greater.

● Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

Quantity A	Quantity B
The volume of the ice in your glass	The volume of soda that you can add to the glass without spilling

Each ice cube's volume is the length of one of its sides cubed, so Quantity A equals the following:

$$V_{ice\ cube} = s^3 = (2.5\ \text{in})^3 = 15.625\ \text{in}^3$$
$$V_{ice} = 15.625 \times 7 = 109.375\ \text{in}^3$$

To find Quantity B, we need to subtract the volume of ice from the total volume of the glass.

$$V_{glass} = \pi r^2 h = \pi(3)^2(7) = 9\pi(7.5) = 67.5\pi = 67.5 \times 3.14 = 211.95\ \text{in}^3$$
$$V_{soda} = V_{glass} - V_{ice} = 211.95 - 109.375 = 102.575\ \text{in}^3$$

Quantity A = $V_{total\ ice}$ = 109.375 in³ and Quantity B = V_{soda} = 102.575 in³. Quantity A > Quantity B.

7

Quantity A	Quantity B
The minimum number of handshakes that can occur among a dozen people if each person only shakes each other person's hand once	The number of ways that three people out of five can be seated at a table

● Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

Quantity A is a combination problem of the form "choose 2 from 15" because the sets of handshakes do not matter in order: "A shakes B's hand" is the same as "B shakes A's hand."

$$C_r = \frac{n!}{r!(n-r)!}\ \frac{12!}{2!(12-2)!} = \frac{12!}{2!(10!)} = \frac{12 \times 11 \times \cancel{10!}}{2 \times 1 \cdot (\cancel{10!})} = \frac{12 \times 11}{2} = \frac{12}{2} \times 11 = 6 \times 11 = 66$$

Quantity B is a permutation problem, since order matters: the order in which people are selected from the group changes the makeup of the remaining people in the group.

$$P_{5,3} = \frac{n!}{(n-r)!} = \frac{5!}{(5-3)!} = \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} = 60$$

Quantity A is 66 and Quantity B is 60, so Quantity A > Quantity B and A is correct.

8 If v is divisible by 2, 3 and 15, which of the following is also divisible by these numbers?

> Multiple answers may be correct. Select all that apply.

A. $v + 5$

B. $v + 15$

C. $v + 20$

D. $v + 30$

E. $v + 90$

Since v is divisible by 2, 3 and 15, v must be a multiple of 30. Any number that is divisible by both 2 and 15 must be divisible by their product, 30, since this is the least common multiple.

Out of all the answer choices, $v + 30$ and $v + 90$ are the only ones that equal multiples of 30.

----

9 What is the interquartile range of the data depicted in the following box-and-whisker plot?
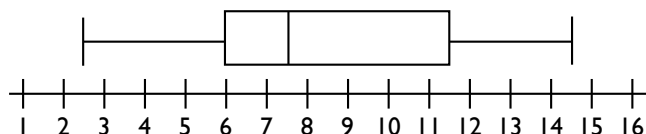
A. 5.0

B. 5.5

C. 6.5

D. 7.5

To find the interquartile range of a set of data, you need to subtract the first quartile value (Q1) value from the third quartile value. (Q3). When data is displayed as a box-and-whisker plot, the left edge of the box is Q1, and the right edge of the box is Q3. For our diagram, Q1 is 6 and Q3 is 11.5. $11.5 - 6 = 5.5$, so B is the correct answer.

----

10 A small circle with radius 5 lies inside a larger circle with radius x. What is the area of the region inside the larger circle but outside of the smaller circle in terms of x?

A. $2\pi x - 5\pi$

B. $\pi x^2 - 5\pi$

C. $2\pi x - 25\pi$

D. $\pi x^2 - 25\pi$

The area of a circle equation, $A = \pi r^2$, is going to be relevant in this problem. The area of the larger circle is going to simply be this equation with x as the variable for radius instead of r: πx^2. We're told that the smaller circle's radius is 5, so let's go ahead and calculate its area, since we have all the variables we need to do so. The area outside the smaller circle but inside the bigger circle is the area leftover if we subtract the smaller circle's area from the bigger circle's area, so the correct answer is D.

$$A = \pi r^2 = \pi(5)^2 = 25\pi$$

$$A_{between\ circles} = A_{bigger\ circle} - A_{smaller\ circle} = \pi x^2 - 25\pi$$

11 If m and n are both odd integers, which of the following is or are not necessarily odd?

A. $2m - n$

B. mn

C. $\dfrac{m + n}{2}$

D. $m^2 n$

E. $\dfrac{m^2 + n^2}{2}$

Multiple answers may be correct. Select all that apply.

With many questions like this, it might be easier to plug in numbers rather than dealing with theoretical variables; however, given that this question asks for the expression that is not always even or odd but only not *necessarily* odd, the theoretical route might be our only choice; therefore, our best approach is to simply analyze each answer choice.

$2m - n$: Since is odd, multiplying it by 2 will give an even number. Since n is odd, subtracting it from our even number will give an odd number, since an even number minus and odd number is always odd; therefore, this answer will always be odd.

mn : Since both numbers are odd, their product will also always be odd.

$\dfrac{m + n}{2}$: Since both numbers are odd, their sum will be even; however, dividing an even number by another even number (2 in our case) does not always produce an even or an odd number. For example, 5 and 7 are both odd. Their sum, 12, is even. Dividing by 2 gives 6, an even number; however, 5 and 9 are also both odd. Their sum, 14, is even, but dividing by 2 gives 7, an odd number; therefore, this expression isn't necessarily always odd or always even, and is therefore our answer.

$m^2 n$: Since m is odd, m^2 is also odd, since an odd number multiplied by an odd number yields an odd product. Since n is also odd, multiplying it by m^2 will again yield an odd product, so this expression is always odd.

$\dfrac{m^2 + n^2}{2}$: Squaring either odd variable yields an odd result, and adding them together results in an even result. Dividing an even number can yield an odd or even result. For example, consider if $m = 3$ and $n = 5$. We'd get $9 + 25 = 36$, and 36 divided by 2 is 18, an even number. But if $m = 5$ and $n = 7$, we get $25 + 49 = 74$, and 72 divided by 2 is 37, an odd number.

The correct answers are C and E.

12   Satoshi and Reginald together have 46 bottlecaps. If they were to receive 6 bottle caps each, Satoshi would have three times as many bottle caps as Reginald. How many bottlecaps does Satoshi have?

Correct Answer: 63

The mathematical approach to this problem would be to set up a system of equations; one which represents the current relationship of Satoshi's and Reginald's number of bottle caps:

$S + R = 46$

And one which represents the relationship 6 bottle caps later, wherein Satoshi would have three times as many bottlecaps as Reginald:

$(S + 6) = 3(R + 6)$

Subtracting the first equation from the second equation provides:

$(S + 6) - S = 3(R + 6) - (R + 46)$

which simplifies to:

$6 = 2R - 28$

Solving this yields the value for Reginald's number of bottle caps, which is 17, which can be plugged back into the very first equation to find Satoshi's number of bottle caps, which is 63.

There is, however, another way to find this solution, which is by first looking at the answer choices. Since we know that Satoshi's number of bottlecaps plus six must be divisible by 3 (since it is three times Reginald's new number of caps), we can quickly eliminate the choices 86, 143, and 52. Furthermore, since Satoshi has 46 more bottlecaps than Reginald, it'd be impossible for him to have 21 bottlecaps. This leaves only the 63 option.

- -

13   In the study, a vaccinated patient was how much less likely to catch the flu than an unvaccinated patient?

A. 60% less likely   **C.** 75% less likely

B. 65% less likely   **D.** 80% less likely

According to the table, 50 unvaccinated patients and 250 unvaccinated patients caught the flu. Next, we need to determine the percentage relationship between the number of unvaccinated versus vaccinated patients that caught the virus.

$18 + 4 + 5 + 4 + 19 = 50$

$63 + 32 + 29 + 51 + 75 = 250$

$$\frac{\text{Vaccinated}}{\text{Unvaccinated}} = \frac{50}{250} = 0.20 \times 100 = 20\%$$

$100\% - 20\% = 80\%$

Because only 20% as many vaccinated patients exposed to the cold virus caught the cold, this reflects an 80% reduction in the likelihood of a patient to catch the cold after receiving the vaccine.

14

Suppose the scientists who performed the study create a pie chart that includes all 1000 patients and that reflects a patient's odds of catching the flu depending on vaccination status and age group.

What would be the measure of the central angle of the portion of the chart representing vaccinated patients of all age groups who caught the virus?

A. 15°
B. 18°
C. 24°
D. 36°

First, we must determine what proportion of the 1000 patients were vaccinated and caught the virus. The total number of patients who were vaccinated and caught the virus is 50.

$18 + 4 + 5 + 4 + 19 = 50$

The proportion of the patients is represented by dividing this group by the total number of participants in the study.

$$\frac{50}{1000} = 0.05$$

Next, we need to figure out how that proportion translates into a proportion of a pie chart. There are 360° in a pie chart. Multiply 360° by our proportion to reach the solution.

$360° \times 0.05 = 18°$

The measure of the central angle of the sector for vaccinated patients who caught the virus is 18°.

15

The health department wants a public service announcement to focus on the age group with the greatest difference in percentage of people who got flu shots and caught the flu and people who did not get flu shots and caught the flu. On which of the following demographics should their public service announcement focus?

A. Under 18
B. 31–50
C. 51–70
D. Over 70

For each of the listed age ranges, we need to figure out the proportion of people who did and did not get the flu, and then compare these quantities to one another to find the category with the largest difference between them.

Under 18	31–50	51–70	Over 70
$\frac{V}{U} = \frac{18}{63} \approx 0.286$	$\frac{V}{U} = \frac{5}{29} \approx 0.172$	$\frac{V}{U} = \frac{4}{51} \approx 0.078$	$\frac{V}{U} = \frac{19}{75} \approx 0.253$
$1 - 0.286 = 0.714$	$1 - 0.172 = 0.828$	$1 - 0.078 = 0.922$	$1 - 0.253 = 0.747$
$0.714 - 0.286 = 0.428$	$0.828 - 0.172 = 0.656$	$0.922 - 0.078 = 0.844$	$0.747 - 0.253 = 0.494$

The category 51–70 has the greatest difference between the proportion of vaccinated and unvaccinated people who got the flu, so C is the correct answer.

16 A traffic light is suspended t feet above a street. A man standing on the exterior edge of the shadow of the traffic light is h feet tall; his shadow is s feet long. How far is the man standing from the spot on the street directly under the traffic light?

**A.** $\dfrac{Th}{s}$

B. $\dfrac{sT}{h}$

C. $\dfrac{hs}{T}$

D. hsT

We can set this problem up like a set of similar triangles. The first triangle is created by the traffic light, the spot beneath the traffic light, and the spot where the traffic light's shadow is (which is also where the man is standing). The height of this triangle is T and its base is the unknown, X. The second triangle is created by the top of the man's head, his feet, and the end of his shadow. The height of this triangle is h and the base is s. We can set up a proportion and rearrange it for X:

$$\frac{height}{base} = \frac{T}{X} = \frac{h}{s}$$

Simply cross-multiply to solve: $X = \dfrac{sT}{h}$
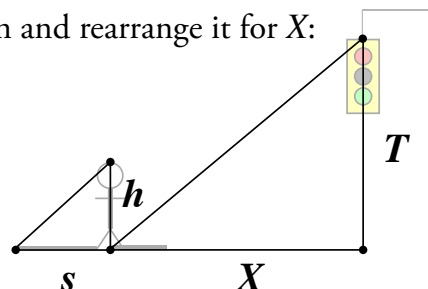
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17 150 seniors attend a high school. The school offers a philosophy class on ethics and one on metaphysics. There are 27 seniors in the ethics class and 32 in the metaphysics class. 95 seniors have elected to take neither class. How many seniors are enrolled in both philosophy courses this term?

Correct Answer: 4

We're told that there are 150 seniors at this particular high school. Each of those 150 have to fall into one of four categories: taking ethics only, taking metaphysics only, taking neither ethics nor metaphysics, or taking both ethics and metaphysics. We're told how many seniors are in each category apart for one: how many are enrolled in both classes.

150 = Ethics Only + Metaphysics Only + Neither Class + Both Classes

$150 = 27 + 32 + 95 + x$

$150 = 154 + x$

$-4 = x$

That's a perplexing result until we interpret our equation. If there are only 150 seniors, but the other side of the equation sums to 154, there have to be 4 students taking both classes. They've been "double counted" as part of both the ethics-only category and as part of the metaphysics-only categories, so a negative number is needed to account to omit the duplicate counts and arrive at a correct total.

----

18 If $a(x) = 2x^3 + x$, and $b(x) = -2x$, what is $a(b(2))$?

A. −132

B. 128

C. −503

D. 503

When functions are set up within other functions like in this problem, the function closest to the given variable is performed first. The value obtained from this function is then plugged in as the variable in the outside function. Since $b(x) = -2x$ and $x = 2$, the value we obtain from $b(x)$ is -4. We then plug this value in for x in the $a(x)$ function. So $a(x)$ then becomes $2(-4)^3 + (-4)$, which equals -132. A is the correct answer.

---

19 Find the length of a line segment whose endpoints are located at the coordinates $(7, -4)$ and $(-5, -1)$.

A. 12

B. 13

C. 14

D. 15

According to the distance formula, the linear distance d between two points (x_a, y_a) and (x_b, y_b) is given by the following formula, which is a generalization of the Pythagorean theorem $a^2 + b^2 = c^2$ for any two points on the coordinate plane.

$$d = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

The two points given in this problem are $(7, -4)$ and $(5, -1)$. Although the question asks for the length of a line segment given its two endpoints, the distance formula is applicable here because the geometric meaning of "distance between two points" is the length of a line segment. Hence, we can use the distance formula to determine the distance between these two endpoints as shown:

$$d = \sqrt{((7) - (-5))^2 + ((-4) - (1))^2}$$
$$d = \sqrt{(12)^2 + (-5)^2}$$
$$d = \sqrt{144 + 25}$$
$$d = \sqrt{169}$$
$$d = 13$$

Hence, the line segment bounded by the points $(7, -4)$ and $(5, -1)$ is 13 units long. B is correct.

---

20 A tank containing 5,000 gallons of water springs a leak. It leaks one hundred gallons of water every fifteen minutes, but it is simultaneously filled at a rate of fifty gallons per hour. After how much time will the tank be empty?

A. 13.56 hours

B. 14.04 hours

C. 14.18 hours

D. 14.29 hours

Solving this problem is going to involve two general steps: calculating a net rate of water loss for the tank from the rates of water being added and water being lost, and then using that single net rate of water loss and the information about how much water the tank begins with to calculate the time it will take to completely drain.

To find a net rate of water loss, we just need to find the sum of the leakage rate and the fill rate. This is somewhat complicated by the fact that we're given the leakage rate in gallons per fifteen minutes, and not hours, but we can easily account for that.

$$-\frac{100 \text{ gal}}{15 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = \frac{400 \text{ gal}}{1 \text{ hr}}$$

If you simply realized that 100 gallons ever 15 minutes means 400 gallons per hour (multiplied by 4, since there are 4 blocks of 15 minutes in an hour), you may have skipped writing out this math. That's ok!

Now we need to factor in the water being added to the tank at a rate of 50 gallons per hour:

$$-\frac{400 \text{ gal}}{1 \text{ hr}} + \frac{50 \text{ gal}}{1 \text{ hr}} = -\frac{350 \text{ gal}}{1 \text{ hr}}$$

Now we have one compound rate to work with, and can bring in the information about the number of gallons of water initially in the tank. We'll need to flip our rate fraction upside down so the units cancel correctly and we're left with a final answer of hours. D is the correct answer!

$$5000 \text{ gal} \times \frac{1 \text{ hr}}{350 \text{ gal}} = 14.285714286 \approx 14.29 \text{ hours}$$

For each question, indicate the best answer using the directions given. If a question has answer choices with ovals, then the correct answer consists of a single choice. If a question has answer choices with square boxes, then the correct answer consists of one or more answer choices. Read the directions for each question carefully.

1

r is s% of 80

s is r% of 125

Quantity A	Quantity B
s	r

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

2

$x > 0$

$y > 0$

Quantity A	Quantity B
$(x + y)^2$	$x^2 + 4xy + y^2$

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

3

A square is inscribed within a circle with a radius of $3\sqrt{2}$ cm. Use 3.14 for the value of π.

Quantity A	Quantity B
The area of the circle that is not covered by the square	20 cm²

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

4

Quantity A	Quantity B
$\sqrt{320} - \sqrt{45}$	$\sqrt{243} - \sqrt{48}$

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

5

Quincy has $20,000 to invest in one of two bank accounts and wants to earn as much money as possible in interest.
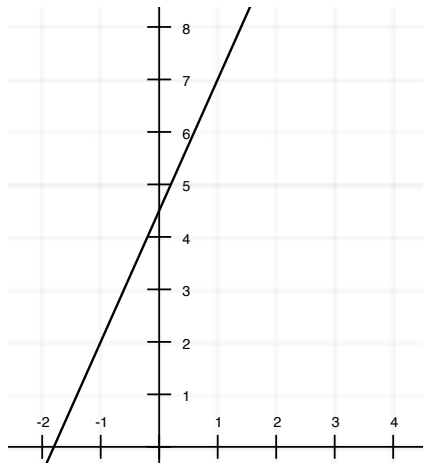
Quantity A	Quantity B
The amount of interest earned on Quincy's money in a savings account that earns 7.5% interest.	The amount of interest earned on Quincy's money in CD that earns 5.25% compounded monthly.

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

6

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

Quantity A	Quantity B
The x-intercept of the line perpendicular to the depicted line on which (3,4) is a point	10

7

Sheryl and Bonnie are competing in an archery tournament. Each person gets to shoot four arrows at a target, and the best shot counts. Sheryl hits the bullseye 42% of the time, and Bonnie hits it 35% of the time. Round to two decimal places.

Quantity A	Quantity B
The probability that Sheryl will hit the bullseye at least once in her first three tries	The probability that Bonnie will hit the bullseye at least once in her four tries

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

8

$$\frac{x}{y} = \frac{3}{7}$$

Quantity A	Quantity B
x	y

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

---

9 At a certain company, one quarter of the employees take the bus to work and one third drive. Of the remaining employees, half walk, one third ride a bike, and the rest take the subway. Out of the total number of employees, what fraction ride a bike to work?

A. $\dfrac{1}{12}$

B. $\dfrac{8}{15}$

C. $\dfrac{2}{21}$

D. $\dfrac{5}{36}$

---

10 Which of the following is or are (a) possible value(s) for x in the inequality $|2x - 2| > 20$?

> Multiple answers may be correct.
> Select all that apply.

A. −12.5

B. −9

C. 4

**D.** 12

E. 20

---

11 An ant begins at the center of a pie with a 12" radius. Walking out to the edge of pie, it then proceeds along the outer edge for a certain distance. At a certain point, it turns back toward the center of the pie and returns to the center point. Its whole trek was 55.3 inches. What is the approximate size of the angle through which it traveled?

A. 128.21°

B. 91.44°

C. 81.53°

D. 149.52°

The following information applies to questions 13–15.

A successful business decided to expand and open a new branch in a neighboring country fifteen months ago. Data about the profit per month of the new branch of the business is shown below.
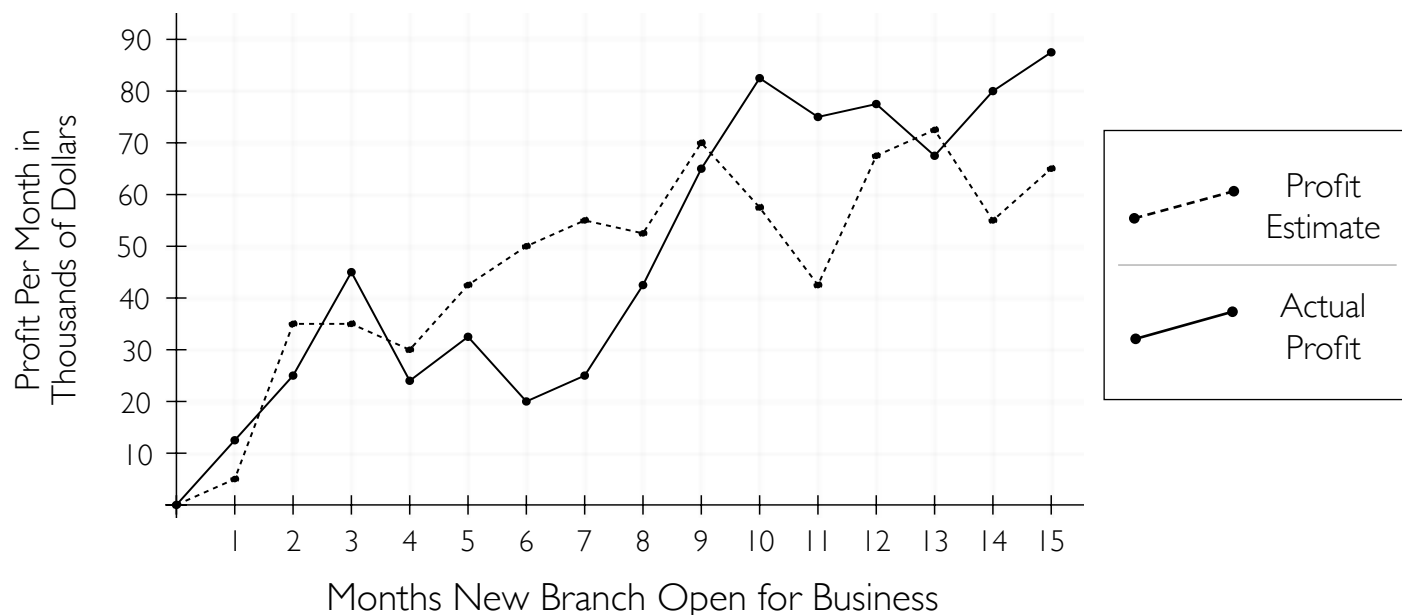
12 What was the percent increase in profit of the new branch of the business between the sixth month after it opened and the fourteenth month after it opened?

On test day, you will be asked to type your answer into a blank text box. Note your response to check later.

13 If the business's profits had continued to decrease steadily after month 4 at the same rate as they were predicted to fall between months 2 and 4 after the new branch opened, what would be the new branch's profit after 6 months?

A. 10.2

B. 10.8

C. 11.0

D. 11.4

14 What was the maximum amount of money by which the predicted profits and the actual profits differed over the course of the shown 15 months?

A. $25,500

B. $30,000

C. $30,500

D. $42,500

15 Which of the following equations represents the equation $y = 4x^2 - 2$ shifted down 2 units and right 3 units?

A. $y = 4(x + 2)^2 - 5$

B. $y = 4(x - 2)^2 - 5$

C. $y = 4(x - 3)^2 - 4$

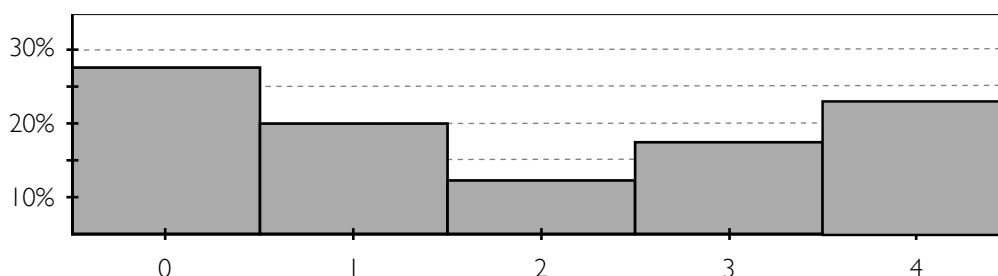D. $y = 4(x + 3)^2 - 4$

---

16 Solve for x:

$$\frac{x}{\sqrt{0.04}} = \sqrt{0.16}$$

A. 0.08

B. 0.2

C. 0.64

D. 0.4

---

17 Consider the probability distribution shown above. What is the mean of the random variable being measured?
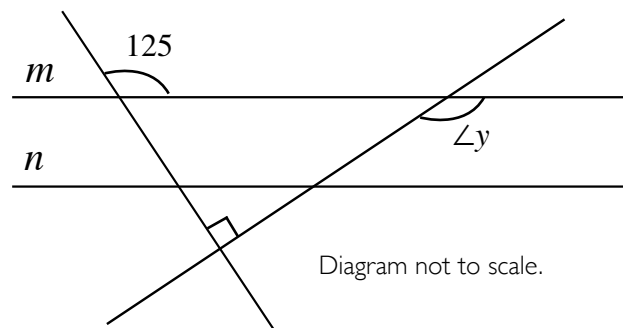
A. 1.725

B. 1.875

C. 1.950

D. 1.975

---

18 What is the value of angle y?

Diagram not to scale.

On test day, you will be asked to type your answer into a blank text box. Note your response to check later.

19   A farmer has 34 ft of fence and wants to fence in his sheep. He wants to build a rectangular pen with one side formed by the side of his barn. He wants the area of the pen to be 120 ft². Which of the following could be the length of the side of the pen opposite the barn?

> Multiple answers may be correct. Select all that apply.

A.   8 ft

B.   10 ft

C.   12 ft

D.   18 ft

E.   24 ft

---

20   Simplify the following expression.

$$\frac{(x^4)^{-7}}{x^2 x^4}$$

A. $\dfrac{1}{x^{34}}$

B. x^{34}

C. $\dfrac{1}{x^{22}}$

D. x^{22}

End of Section   **STOP**   **End of Section**

	Answer	Section / Concept Tested
1	B	*Arithmetic — Percentages*
2	B	*Algebra — Quadratic Equations and the Quadratic Formula*
3	A	*Geometry — Inscription and Circumscription*
4	A	*Arithmetic — Solving Exponents and Roots*
5	A	*Algebra — Word Problems: Simple Interest and Compound Interest*
6	A	*Algebra — Properties of Linear Graphs* *Geometry — Angles, Parallel Lines, and Perpendicular Lines*
7	B	*Data Analysis — Calculating Probability*
8	D	*Arithmetic — Decimals, Ratios, and Proportions* *Arithmetic — Fractions*
9	D	*Arithmetic — Fractions*
10	A, D, and E	*Algebra — Linear Equations and Inequalities*
11	D	*Geometry — Area and Perimeter: Triangles, Quadrilaterals, and Circles*
12	300%	*Arithmetic — Percentages* *Data Analysis — Scatterplots, Trends, and Line Graphs*
13	B	*Arithmetic — Percentages* *Algebra — Word Problems: Working with Rates* *Data Analysis — Scatterplots, Trends, and Line Graphs*
14	D	*Data Analysis — Scatterplots, Trends, and Line Graphs*
15	C	*Algebra — Properties of Quadratic Graphs*
16	A	*Arithmetic — Decimals, Ratios, and Proportions* *Arithmetic — Solving Exponents and Roots*
17	B	*Data Analysis — Random Variables, Sampling, and Expected Values*
18	145°	*Geometry — Angles, Parallel Lines, and Perpendicular Lines*
19	B and E	*Algebra — Quadratic Equations and the Quadratic Formula* *Algebra — Systems of Linear Equations*
20	A	*Algebra — Rules of Exponents and Variables*

1

r is s% of 80
s is r% of 125

Quantity A
s

Quantity B
r

○ Quantity A is greater.

● Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

To attempt this problem, translate the statements into mathematical equations. When you see the word 'is', it very often means 'equals:

s is r% of 80:

$$s = \frac{r}{100} 80 = \frac{4}{5}r$$

Now before we continue, look at this statement. We see that s is fourth-fifths of r. That alone should tell us that r is greater! On the GRE, once you find the answer like this, fill it in and move on. In the interest of a full explanation, we can continue with the calculations to see what these values are, if only for the sake of robustness.

r is s% of 125

$$r = \frac{s}{100} 125 = \frac{5}{4}s$$

Again, this validates our result. We may not know what r and s are, but we know how they relate to each other, and that is enough. Quantity B is greater.

2

$x > 0$
$y > 0$

Quantity A
$(x + y)^2$

Quantity B
$x^2 + 4xy + y^2$

○ Quantity A is greater.

● Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

To approach this problem, consider the two quantitites. They are in different forms, so expand Quantity A:

$(x + y)^2$
$x^2 + 2xy + y^2$

Quantity A: $x^2 + 2xy + y^2$
Quantity B: $x^2 + 4xy + y^2$

Now, for the purpose of comparison, subtract shared terms from each quantity:

Quantity A': 0
Quantity B': $2xy$

Both x and y are negative, non-zero values. Since $2xy$ is a product of two negative values, it must be positive. Quantity B must be greater than Quantity A.

----

3 A square is inscribed within a circle with a radius of $3\sqrt{2}$ cm. Use 3.14 for the value of π.

● Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

Quantity A	Quantity B
The area of the circle that is not covered by the square	20 cm²

First, find the area of the circle.

$A = \pi(3\sqrt{2})^2$
$A = \pi \times 9 \times 2$
$A = 18\pi$

Next, find the length of one side of the square using the Pythagorean theorem. Two radii from the center of the circle to adjacent corners of the square will create a right angle at the center of the circle. The radii will be the legs of the triangle, and the side of the square will be the hypotenuse.

$(3\sqrt{2})^2 + (3\sqrt{2})^2 = c^2$
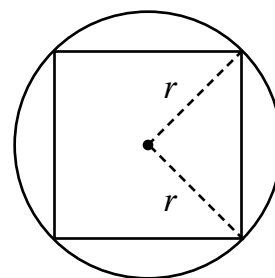$(9 \times 2) + (9 \times 2) = c^2$
$18 + 18 = c^2$
$36 = c^2$
$c = 6$

Find the area of the square.

$A = c^2 = 6^2 = 36$

Subtracting the area of the square from the area of the circle, we find that the area of the circle not covered by the square is greater than 20 cm², so Quantity A > Quantity B and A is the correct answer.

$18\pi - 36 = 18(3.14) - 36 = 56.52 - 36 = 20.52$

4

Quantity A	Quantity B
$\sqrt{320} - \sqrt{45}$	$\sqrt{243} - \sqrt{48}$

● Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

Begin by breaking down each of the roots in question. Often, for the GRE, your answer arises out of doing such basic "simplification moves".

Quantity A

$\sqrt{320} - \sqrt{45}$

This is the same as

$\sqrt{64 \times 5} - \sqrt{9 \times 5}$, which can be reduced to:

$8\sqrt{5} - 3\sqrt{5} = 5\sqrt{5}$

Quantity B

$\sqrt{243} - \sqrt{48}$

This is the same as

$\sqrt{81 \cdot 3} - \sqrt{16 \cdot 3}$, which can be reduced to:

$9\sqrt{3} - 4\sqrt{3} = 5\sqrt{3}$

Now, since we know that $\sqrt{5}$ must be greater than $\sqrt{3}$,, we know that $5\sqrt{5} > 5\sqrt{3}$; therefore, Quantity A > Quantity B and A is the correct answer.

---

5 Quincy has $20,000 to invest in one of two bank accounts and wants to earn as much money as possible in interest in the first year he invests.

Quantity A	Quantity B
The amount of interest earned on Quincy's money in a savings account that earns 7.5% interest.	The amount of interest earned on Quincy's money in CD that earns 5.25% compounded monthly.

● Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

To generate numerical values for Quantities A and B, you'll need to employ the interest formulas. Quantity A doesn't mention a compouding schedule, but Compound B's account is "compounded monthly." Thus, you'll need the formula for simple interest in solving for Quantity A and the formula for compound interest in solving for Quantity B.

For Quantity A:

$$I_A = P_A \times r_A \times t_A$$
$$I_A = 20{,}000 \times 0.075 \times 1$$
$$I_A = 1500$$

For Quantity B:

$$A_B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A_B = 20{,}000\left(1+\frac{0.0525}{12}\right)^{12\times1}$$

$$A_B = 21075.64$$

For Quantity B, the compound interest formula yields a value that includes the original principle, so we need to subtract $20,000 to find the interest earned.

$$I_B = A_B - P = 21075.64 - 20000.00 = 1075.64$$

Quantity A's account would earn $1500.00 in one year, whereas Quantity B's account, while compounded more often, would only earn $1075.64 due to a lower interest rate. Quantity A > Quantity B, so A is correct.

6
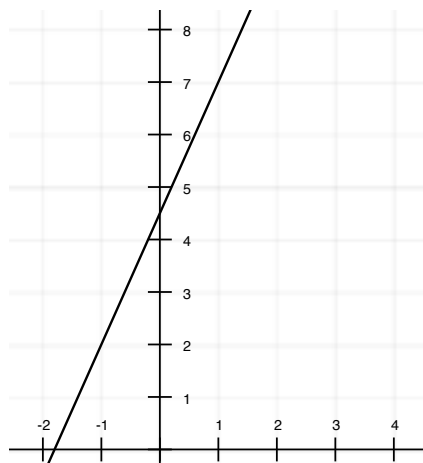
● Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

Quantity A	Quantity B
The x-intercept of the line perpendicular to the depicted line on which (3,4) is a point	10

This question requires a few steps. We have to somehow write the equation of the line shown in the graph, then determine the equation of a perpendicular line that goes through a specific point, and finally, we have to find the x-intercept of that second line and compare it to ten. We can find the equation of the graphed line using two points that it definitively passes through. The graph shows that the line passes through (−1, 2) and (1,7). Let's find the slope of the line through these points:

$$\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 2}{1 - (-1)} = \frac{5}{2}$$

We can see that the graphed line crosses the y-axis at 4.5, but we can also solve for this value by substituting in one of our points and solving for *b*:

$$y = \frac{5}{2}x + b$$

$$2 = \frac{5}{2}(-1) + b$$

$$2 = -\frac{5}{2} + b$$

$$2 = -2.5 + b$$

$$b = 4.5$$

Now that we know our line's slope and y-intercept, we can write its equation: $y = 2.5x + 4.5$

Next, we need to calculate the slope of a line perpendicular to this one going through a specific point. The perpendicular part is easy enough—we just find the reciprocal of the slope and change its sign. Since our slope was $\frac{5}{2}$, the perpendicular slope will be $-\frac{2}{5}$, or −0.4. To find the equation of the perpendicular line that goes through the point (3,4), we just need to substitute in that point to an equation with our new slope and solve for the new y-intercept.

$$y = -0.4x + b$$

$$4 = -0.4(3) + b$$

$$4 = -1.2 + b$$

$$b = 5.2$$

Now we can write the equation of the perpendicular line: $y = -0.4x + 5.2$

Solving for the x-intercept just means plugging in a value of 0 for *y* and seeing what x-value we get:

$$0 = -0.4x + 5.2$$

$$-5.2 = -0.4x$$

$$x = \frac{-5.2}{-0.4} = 13$$

13 > 10, so Quantity A > Quantity B, and A is correct.

7

Sheryl and Bonnie are competing in an archery tournament. Each person gets to shoot four arrows at a target, and the best shot counts. Sheryl hits the bullseye 42% of the time, and Bonnie hits it 35% of the time. Round to two decimal places.

○ Quantity A is greater.

● Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the given data.

Quantity A	Quantity B
The probability that Sheryl will hit the bullseye at least once in her first three tries	The probability that Bonnie will hit the bull-seye at least once in her four tries

This is most easily solved by looking at it from an inverse perspective, and instead of calculating the odds of hitting the bullseye in a given number of tries for each person, finding the probability that each person will not hit the bullseye at all in the given number of tries.

Quantity A: There are two options in this scenario: hitting the bullseye or missing it. If Sheryl hits the bullseye 42% of the time, she misses 58% of the time.

$1 - 0.42 = 0.58$

and the probability she misses three times will be

$(0.58)^3 = 0.195112 \approx 0.20$

The probability Sheryl hitting the bullseye at least once in three tries is the complement of this, or:

$1 - 0.20 = 0.80$

$0.80 \times 100 = 80\%$

Quantity B: The same applies here, though we're calculating Bonnie's odds out of four tries, and she hits the bullseye only 35% of the time, so the probability she'll miss is

$1 - 0.35 = 0.65$

and the probability of her missing all four times will be

$(0.65)^4 = 0.17850625 \approx 0.18$

So, the probability of Bonnie hitting the target once out of four tries is

$1 - 0.18 = 0.82$

$0.82 \times 100 = 82\%$

80% (Quantity A) < 82% (Quantity B), so B is the correct answer.

8

$$\frac{x}{y} = \frac{3}{7}$$

Quantity A	Quantity B
x	y

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

● The relationship cannot be determined from the given data.

Although it seems as though "Quantity B is greater" is the correct answer at first glance, a further analysis indicates that this answer is a trap. If x and y are negative numbers, such as –3 and –7, then x would be the larger number, since it would be closer to zero. Similarly, y is larger if both x and y are positive numbers. Thus, it cannot be determined which variable is larger simply based on the information given.

---

9   At a certain company, one quarter of the employees take the bus to work and one third drive. Of the remaining employees, half walk, one third ride a bike, and the rest take the subway. Out of the total number of employees, what fraction ride a bike to work?

A. $\dfrac{1}{12}$

B. $\dfrac{8}{15}$

C. $\dfrac{2}{21}$

D. $\dfrac{5}{36}$

First we want to find the fraction of employees that neither take the bus nor drive, so we'll add the fractions that do take the bus or drive and subtract that result from the total.

Bus: $\dfrac{1}{4}$

Drive: $\dfrac{1}{3}$

Remaining: $1 - (\dfrac{1}{4} + \dfrac{1}{3}) = 1 - (\dfrac{7}{12}) = \dfrac{5}{12}$

Now we need the fraction representing one third of these remaining employees (the fraction that ride a bike). Since "of " means multiply, we'll multiply. The correct answer is D.

$$\frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$$

10 Which of the following is or are (a) possible value(s) for x in the inequality $|2x - 2| > 20$?

Multiple answers may be correct. Select all that apply.

A. –12.5

B. –9

C. 4

D. 12

E. 20

For this problem, we must take into account the absolute value. First, we solve for $2x - 2 > 20$. But we must also solve for $2x - 2 < 20$.

$2x - 2 > 20 \quad 2x - 2 < -20$

$2x > 22 \quad\quad 2x < -18$

$x > 11 \quad\quad\quad x < -9$

Therefore, $x > 11$ and $x < -9$.

Possible values for x listed amongst the answer choices would be –12.5 (A) since that is less than –9, and 12 (D) and 20 (E), since those are each greater than 11. Note that the value –9 (B) is not a possible value for x because the inequality sign given does not include an equal sign.

----

11 An ant begins at the center of a pie with a 12" radius. Walking out to the edge of pie, it then proceeds along the outer edge for a certain distance. At a certain point, it turns back toward the center of the pie and returns to the center point. Its whole trek was 55.3 inches. What is the approximate size of the angle through which it traveled?

A. 128.21°

B. 91.44°

C. 81.53°

D. 149.52°

To solve this, we must ascertain the following:

    1) The arc length through which the ant traveled.
    2) The percentage of the total circumference in light of that arc length.
    3) The percentage of 360° proportionate to that arc percentage.

To begin, let's note that the ant travelled $12 + 12 + x$ inches, where x is the outer arc distance. (It traveled the radius twice, remember); therefore, we know that $24 + x = 55.3$, or $x = 31.3$. Now, the total circumference of the circle is $2\pi r$ or 24π. The arc is $\dfrac{31.3}{24\pi}$ percent of the total circumference; therefore, the percentage of the angle is $\dfrac{360 \times 31.3}{24\pi}$. Since the answers are approximations, use 3.14 for π. This would be 149.52°, so D is correct.

----

12   What was the percent increase in profit of the new branch of the business between the sixth month after it opened and the fourteenth month after it opened?

Correct Answer: 300%

To solve this problem, we need to identify two points on the line graph: the profits in month 6 and the profits in month 14. Make sure you're reading the correct line—the solid line, which is the actual profits, not the dashed line, which is the estimated profits. The graph shows that the company's new branch had a profit of 20,000 dollars after six months and a profit of 80,000 dollars after 14 months. As long as you're consistent, you can just work with the numbers 20 and 80 instead of interpreting them as twenty thousand and eighty thousand.

To calculate the percent increase, we just find the difference between the profit in month 14 and in month 6 and make that the numerator of a fraction with the profit in month 6 as the denominator. Then, we multiply our decimal value by 100 to convert it to a percent. The correct answer is that the branch's profits increased by 300%.

$$\left(\frac{\text{Profit}_{New} - \text{Profit}_{Old}}{\text{Profit}_{Old}} \times 100 \right)\% = \left(\frac{80 - 20}{20} \times 100 \right)\% = \left(\frac{60}{20} \times 100 \right)\% = (3 \times 100)\% = 300\% \text{ increase}$$

---

13   If the business's profits had continued to decrease steadily after month 4 at the same rate as they were predicted to fall between months 2 and 4 after the new branch opened, what would be the new branch's profit after 6 months?

A. 10.2
B. 10.8
C. 11.0
D. 11.4

We first have to figure out the rate of decrease between the second and fourth month. Then, we can extrapolate using the fourth month's value to what the profit would be in the sixth month. First, we need to find the percent decrease, but instead of converting a decimal to a percent, we can leave it as a decimal, since we're going to use it as a multiplier. The estimate of the branch's profits as 35 in the second month and 30 in the fourth month. Make sure you're reading the dashed line, not the solid one! Now we can calculate the rate of decrease:

$$\frac{\text{Profit}_{New} - \text{Profit}_{Old}}{\text{Profit}_{Old}} = \frac{35 - 30}{30} = \frac{5}{30} = 0.6$$

At this point, we have to use this rate to extrapolate. The branch's profits were estimated to be 30 in the fourth month, so we just need to multiply this value by 0.6 twice:

$$30 \times 0.6 \times 0.6 = 10.8$$

If the profits kept decreasing as described, we would estimate them to be 10.8 in the sixth month, making B the correct answer.

14   What was the maximum amount of money by which the predicted profits and the actual profits differed over the course of the shown 15 months?

A. $25,500

B. $30,000

C. $30,500

D. $42,500

This problem is an exercise in comparing values on the graph. Three months on the x-axis stick out as potentially involving the greatest difference between the lines: 6, 7, and 11. In month 6, the actual profits are 20, and the estimate is 50. That's a difference of 30. For month 7, the profits are 25 and the estimate is 55. That's another difference of 30. Month 11's estimate is about 42.5, and its profits are 75. That's the point at which the lines are the farthest apart, so D is the correct answer.

---

15   Which of the following equations represents the equation $y = 4x^2 - 2$ shifted down 2 units and right 3 units?

A. $y = 4(x + 2)^2 - 5$

B. $y = 4(x - 2)^2 - 5$

C. $y = 4(x - 3)^2 - 4$

D. $y = 4(x + 3)^2 - 4$

To shift a linear equation down two units, you need to subtract two units outside of parentheses containing the x variable. The equation we're given is $y = 4x^2 - 2$, an equation that doesn't include any parentheses. Thus, subtracting two from this equation outside of the parentheses would subtract two from –2, yielding –4. Focusing only on those answer choices that contain –4, we can eliminate A and B. To shift a linear equation right three units, we need to subtract three from the x-variable within parentheses, as shifts left correlate with variables added to x within parentheses and shifts right correlate with variables subtracted from x within parentheses. Thus, the shifted equation is $y = 4(x - 3)^2 - 4$, and C is the correct answer.

16

Solve for x:

$$\frac{x}{\sqrt{0.04}} = \sqrt{0.16}$$

A. 0.08

B. 0.2

C. 0.64

D. 0.4

Just like any other equation, isolate your variable. Start by multiplying both sides by $\sqrt{0.04}$:
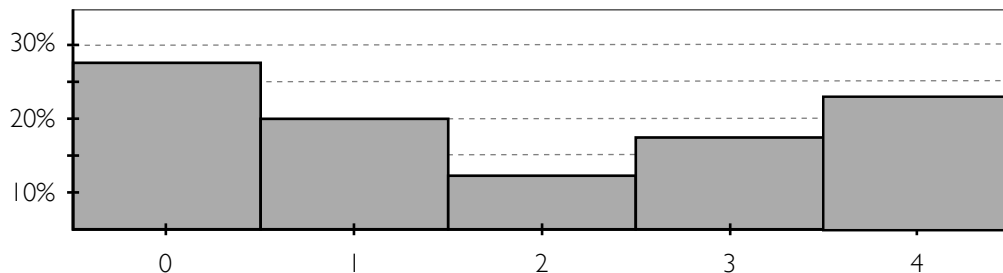
$x = \sqrt{0.16} \times \sqrt{0.04}$

Now, this is the same as

$x = \sqrt{0.0064}$

You know that $\sqrt{64}$ is 8. You can intelligently rewrite this problem as

$x = \sqrt{0.0064} = \sqrt{\dfrac{1}{10,000} \times 64}$, which is the same as

$\sqrt{\dfrac{1}{10,000} \times 64} = \sqrt{\dfrac{1}{10,000}} \times \sqrt{64} = \dfrac{1}{100} \times 8 = 0.08$

---

17   Consider the relative frequency distribution shown above. What is the mean of the random variable being measured?

A. 1.725
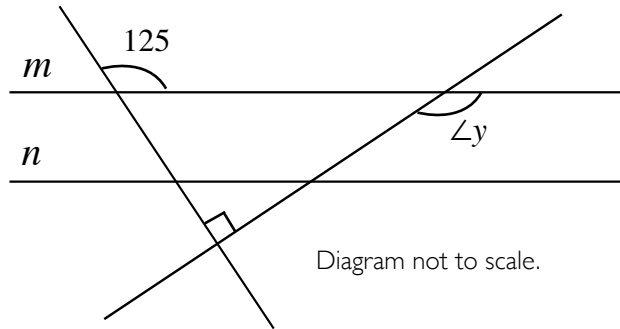
B. 1.875

C. 1.950

D. 1.975

To calculate the mean of the variable measured in the histogram, we need to multiply each value on the x-axis by its relative probability–that is, the chances it has of showing up if you pick a variable at random from those shown. This is represented by each bar's value as a decimal. The correct answer is B.

$0(P(0)) + (1(P(1)) + 2(P(2)) + 3(P(3)) + 4(P(4))$

$0(0.275) + 1(0.2) + 2(0.125) + 3(0.175) + 4(0.225)$

$0 + 0.2 + 0.25 + 0.525 + 0.9 = 1.875$

# Explanations: Quantitative Reasoning #2

18 What is the value of angle *y*?

125

m

∠*y*

n

Diagram not to scale.

Correct Answer: 145°

We can figure out the measure of angle *y* using the complementary and supplementary rules of geometry, which are applicable because lines *m* and *n* are parallel. In addition, we'll need to employ the fact that the sum of all angles within a triangle sum is 180˚.

The angle directly across from the given 125˚ angle is also 125˚. Then, the 125˚ angle and the angle on the same line crossing both lines *m* and *n* have to sum to 180˚, since they form a linear angle (a straight line). That means that the angle in the upper-left corner of line *n*'s first intersection is 55˚. The angle opposite that one has to be 55˚ also, and this is part of a right triangle. All triangles angles' sum to 180˚, and we know that this triangle contains a 55˚ angle, a 90˚ angle, and one more angle. $55˚+90˚+x=180˚$, so that unknown angle has to be 45˚. The angle opposite that one has to be 45˚ as well, and angle y forms a linear angle (straight line) with that one, so they have to sum to 180˚. $y + 45˚=180˚$, so angle $y = 135˚$ and the correct answer is C.
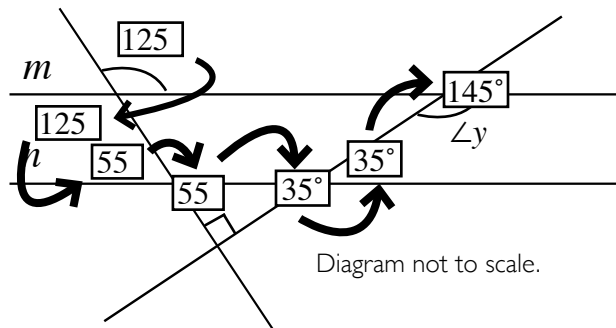
125

m

125

n   55

55   35°

35°

145°

∠*y*

Diagram not to scale.

19 A farmer has 34 ft of fence and wants to fence in his sheep. He wants to build a rectangular pen with one side formed by the side of his barn. He wants the area of the pen to be 120 ft². Which of the following could be the length of the side of the pen opposite the barn?

A. 8 ft

B. 10 ft

C. 12 ft

D. 18 ft

E. 24 ft

Multiple answers may be correct.
Select all that apply.

Set up two equations from the given information:

$120 = xy$

$34 = 2x + y$

Rearrange the first equation into $y = \dfrac{120}{x}$, and then substitute this into the second equation to create one equation with one variables. At this point, we can solve for x.

$34 = 2x + \dfrac{120}{x}$

$34x = 2x^2 + 120$

$0 = 2x^2 - 34x + 120$

$x^2 - 17x + 60$

We can factor this:

$(x - 5)(x - 12) = 0$

Thus $x = 5$ or $x = 12$. Note that this is not the side opposite the barn, though: we need to solve for y in both cases.

For $x = 5$: $120 = (5)y$

$y = 24$

For $x = 12$: $120 = (12)y$

$y = 10$

24 and 10 both appear in the answer choices, so B and E are the correct answers.

---

20 Simplify the following expression.

$$\frac{(x^4)^{-7}}{x^2 x^4}$$

A. $\dfrac{1}{x^{34}}$

B. x^{34}

C. $\dfrac{1}{x^{22}}$

D. x^{22}

To solve this problem, we must first understand some of the basic concepts of exponents. When multiplying exponents with the same base, one would simply add the powers together with the same base to obtain the result. For example, $x^a \times x^b = x^{a+b}$.

When raising exponents to a certain power, one would simply multiply the power the exponent is being raised to with the exponent itself to obtain the new exponent. The base also gets raised to the same power as it normally would and the new exponent gets put on afterward. For example, $(x^a)^b = x^{a\times b}$.

Now that we have covered the basic concepts of exponent manipulation, we can now solve the problem. The top part of the expression give to us is $(x^4)^{-7}$. As we have stated before, when an exponent is raised to a certain power, you simply multiply the power and the exponent together. This results in $x^{4\times-7} = x^{-28}$. The new expression becomes

$$\frac{x^{-28}}{x^2 x^4}$$

Solving for the denominator of the equation, we previously stated that when multiplying exponents together with the same base, we simply add the exponents together and keep the same base. Therefore $x^2 x^4 = x^{2+4} = x^6$.

The new expression becomes $\dfrac{x^{-28}}{x^6}$; however $x^{-28} = \dfrac{1}{x^{28}}$, therefore the expression becomes $\dfrac{1}{x^{28} x^6} = \dfrac{1}{x^{34}}$. The correct answer is A.

Score Conversion Tables for Full-Length Test

Raw Verbal Score	Approximate Verbal Reasoning Score	Approximate Percentile Score
40	170	99
39	169	99
38	168	98
37	167	97
36	166	96
35	165	95
34	164	93
33	163	91
32	162	89
31	161	86
30	160	83
29	159	80
28	158	77
27	157	73
26	156	69
25	155	65
24	154	61
23	153	57
22	152	53
21	151	49

Raw Verbal Score	Approximate Verbal Reasoning Score	Approximate Percentile Score
20	150	44
19	149	40
18	148	36
17	147	32
16	146	28
15	145	24
14	144	21
13	143	18
12	142	15
11	141	12
10	140	10
9	139	8
8	138	6
7	137	5
6	136	4
5	135	3
4	134	2
3	133	1
2	132	1
1	131	1
0	130	1

Raw Verbal Score	Approximate Quantitative Reasoning Score	Approximate Percentile Score
40	170	99
39	169	98
38	168	97
37	167	96
36	166	94
35	165	92
34	164	90
33	163	88
32	162	85
31	161	82
30	160	79
29	159	77
28	158	74
27	157	71
26	156	68
25	155	64
24	154	60
23	153	56
22	152	52
21	151	48

Raw Verbal Score	Approximate Quantitative Reasoning Score	Approximate Percentile Score
20	150	43
19	149	39
18	148	35
17	147	31
16	146	27
15	145	23
14	144	20
13	143	17
12	142	14
11	141	11
10	140	8
9	139	6
8	138	4
7	137	4
6	136	3
5	135	3
4	134	2
3	133	2
2	132	1
1	131	1
0	130	1

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